

6.441 Transmission of Information

Problem Set 3

Spring 2010

Due date: February 25

Problem 1 Consider a D -ary alphabet $0, 1, \dots, D - 1$. Let the weight of a codeword be the sum of the values of all the letters in the codeword. Give an algorithm for finding an algorithm that establishes a Huffman code that minimizes the mean weight of the codewords.

Problem 2 We call a run of length n a set of n consecutive independent tosses of a fair coin. We perform m independent runs of length n . We index the 2^n possible runs. We create a PMF $p_i = \frac{N_i}{m}$ where N_i is the number of times the run i was observed.

1) What is the probability that a run appears in on set of m independent runs but not in another, mutually independent, set of m runs? Suppose that we create a binary Shannon-Fano-Elias coding using the PMF derived from a set of m runs to code another set of m runs. Assume that if a run in the second set of runs does not occur in the PMF derived from the first of runs, then the length of that codeword is m .

2) What is the mean of the codeword length? Compare this mean to that in the case where the Shannon-Fano-Elias code is built using the true PMF.

3) What is its variance? Calculate the difference between this mean and that in the case where the Shannon-Fano-Elias code is built using the true PMF.

4) Repeat 2) and 3) when the two sets of m runs are chosen to be the same. Let us now consider that the coin is no longer fair (but distinct trials remain independent). The probability of the coin coming up heads is now $1 > p > \frac{1}{2}$

5) For fixed n , determine m large enough so that the difference in 3) with the new p is less than half a standard deviation for the case where the Shannon-Fano-Elias code is built using the true PMF.

Problem 3 Problem 5.25 in Cover and Thomas (first or second edition).

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