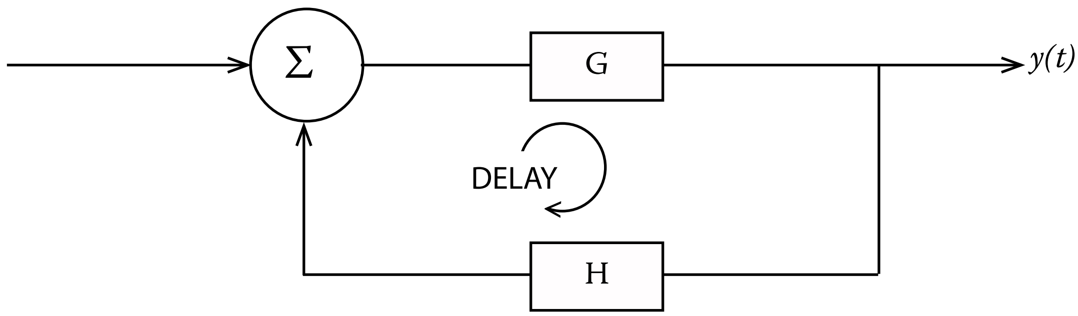
6.302 Feedback Systems

Recitation 2: Block Diagrams and Physical Modeling

Prof. Joel L. Dawson

First, a word about loop transmission, and why it is important.



Remember from recitation 1: delay complicates the design of feedback systems. We call *G(s)H(s)* the loop transmission *L(s)*. Recalling what we know from 6.003, what happens when we pass a sinusoid $e^{j\omega t}$ through *L(s)*? We get:

(Assume negative phase shift)

$$\begin{aligned} L(j\omega)e^{j\omega} &= |L(j\omega)|e^{-j\phi(\omega)}e^{j\omega t} \\ &= |L(j\omega)|e^{j\omega t - j\phi(\omega)} \\ &= |L(j\omega)|\, e^{j\omega\underbrace{\left(t-\frac{\phi(\omega)}{\omega}\right)}_{\text{frequency-dependent delay!!}}} \end{aligned}$$

This is out first clue that the details of the loop transmission will be very important to us in our study of feedback systems.

Now on to the main topic, which is the modeling of physical systems using block diagrams.

In this class we study linear dynamical systems. A classical "state space" description of an unforced dynamical system looks something like this:

$$\frac{d}{dt}\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \underset{\substack{n\times n \\ matrix}}{\begin{bmatrix} \quad A \quad \\ \\ \\ \end{bmatrix}}\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

The v_i are the state variables, and the matrix A is constant.

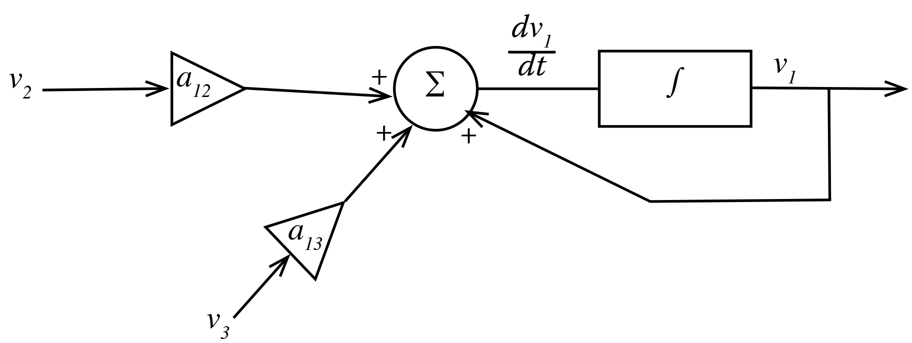
We can represent this expression as either a system of equations

$$\frac{dv_1}{dt} = a_{11}v_1 + a_{12}v_2 + \ldots. + a_{1v}v_v$$

$$\frac{dv_2}{dt} = a_{21}v_1 + a_{22}v_2 + \ldots. + a_{2v}v_v$$

$$\vdots$$

or in the form of a block diagram.



...and so on.

Block diagrams are a powerful tool for understanding physical systems of all kinds. Notice that in this particular example, the block diagram makes it immediately clear that feedback is involved.

Let's do some modelling. →

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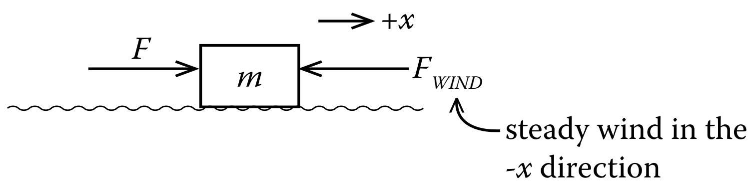
EXAMPLE 1:



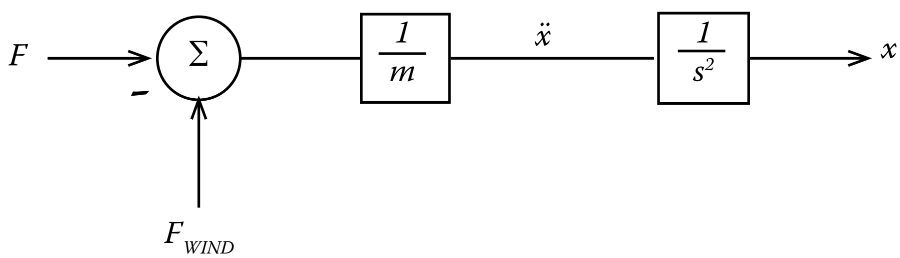
Output variable of interest: x (position)

Equations approach:

$$m\ddot{x} = -F - F_{WIND}$$

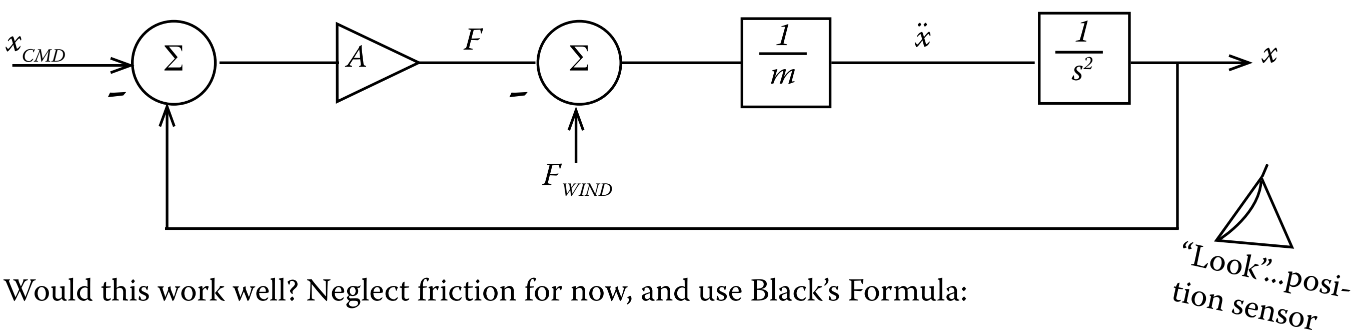
$$x = \int dt \int dt \left(\frac{F}{m} - \frac{F_{WIND}}{m} \right)$$

Block diagram (which we can write down without doing equations first!) :



In our minds, if we wish, we can think of this as an analog circuit. In this case, F and μmg could be voltages, and the $\frac{1}{ms^2}$ could be realized as a pair of integrations. If we built a circuit like this as a way of modelling a physical system, we would have a rudimentary example of an analog computer.

Now we've got our block diagram. Suppose that we wanted to control the position of this block using force as an input. One could imagine a <u>feedback</u> contoller:



Would this work well? Neglect friction for now, and use Black's Formula:

$$\frac{X}{X_{CMD}} = \frac{A/ms^2}{1+A/ms^2} = \frac{A}{ms^2+A} \Rightarrow \text{POLES at } \pm j\sqrt{\tfrac{A}{m}}$$

OSCILLATES!

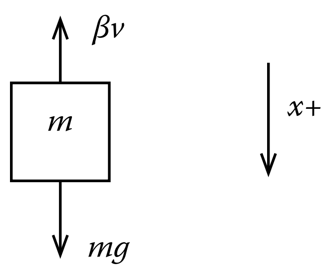
As we go along in 6.302, we'll study ways to deal with situations like this, i.e. have a control strategy that does not lead to oscillation. For now, remember the steps that we took:

1. Looked at the PHYSICS of the situation.
2. Wrote out a block diagram.
3. Began formulating our control solution.

IN-CLASS EXAMPLE:

Suppose we have a free body falling through a viscous medium, and we are concerned with its velocity over time.

Free body diagram:



Draw a block diagram expressing the velocity in terms of all known information.

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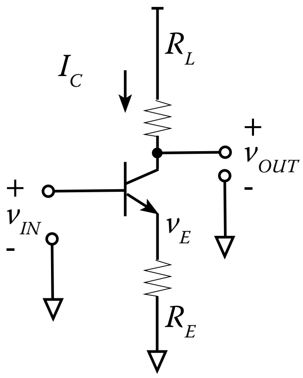
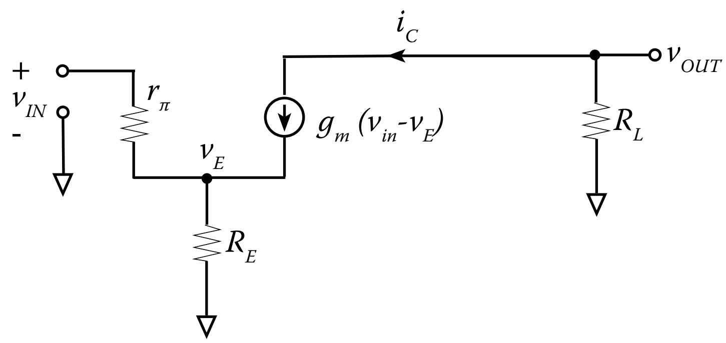
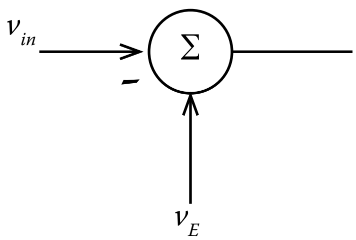
EXAMPLE 2: Emitter degeneration

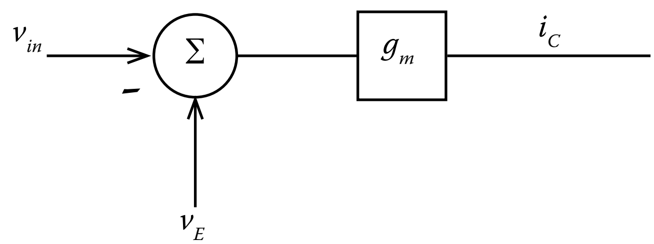




From 6.002 we know how to solve this using KCL and KVL. Can we generate a block diagram directly? (for simplicity, ignore base current)



1



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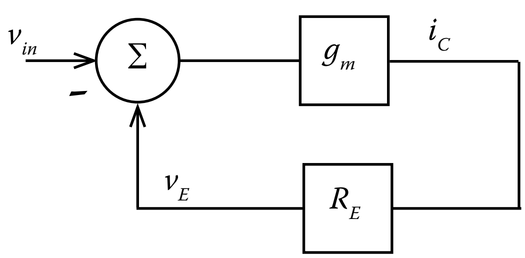

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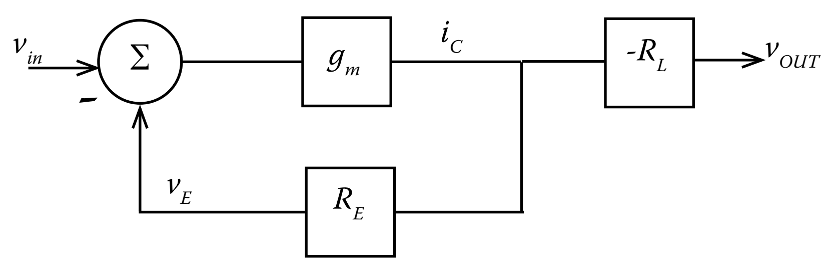


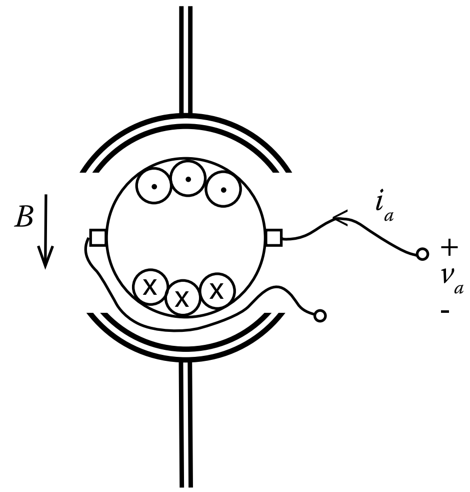
4

Black's Formula: $\frac{v_{out}}{v_{in}} = -\left(\frac{g_m}{1+g_m R_E}\right) R_L$

for $g_m R_E >> 1.$ $\quad \frac{v_{out}}{v_{in}} \approx -\frac{R_L}{R_E}$

Familiar result from circuit theory.

DC MOTORS AND GEAR TRAINS



B field could be constant, or controlled by the user. i_a is the current in the armature windings.

Force on a current-carrying wire in a magnetic field is

$$F = \int I(\vec{dl} \times \vec{B}) = i_a L B$$

wire length

Torque T on the rotor is therefore

$$T = \vec{r} \times \vec{F}$$
$$T = N \cdot RLBi_a$$

Let's lump all of the constants we can't contol, *NRLB,* into one constant K_t:

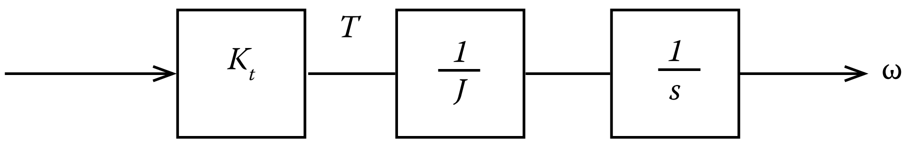
$$T = K_t i_a$$

Perfect. This paves the way for us to write down the mechanical dynamics of a shaft attached to this motor. Recall:

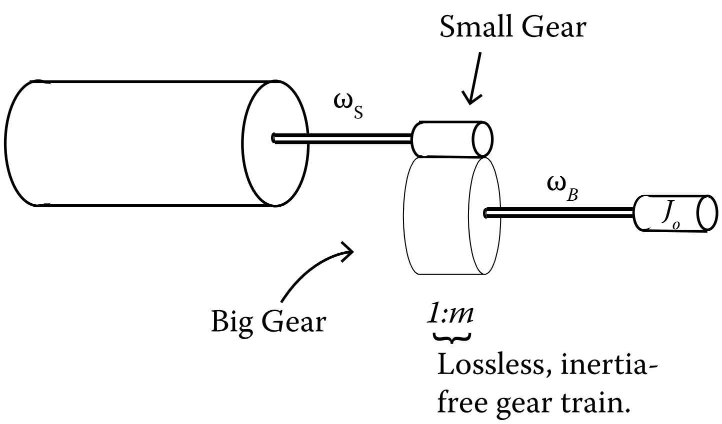
$$T = \alpha J = J\omega = J\Theta$$

Where α = angular acceleration, ω = angular velocity, Θ is rotational position, and J is the moment of inertia of the shaft.

Doing a current source drive, then, we could write the shaft speed ω as



For your labs, we'll ask you to consider the effect of a gear train:



For every turn of the BIG GEAR, the SMALL GEAR turns *m* times. This implies $\omega_S = m\omega_B$.

Now we ask, what moment of inertia J_I does the motor "see"? We can figure this out using a very powerful method of physical reasoning: utilize conservation of energy.

Let's say the output shaft was spun up from rest. Its net change in kinetic energy is therefore:

$$\Delta KE = \tfrac{1}{2} J_0 \omega_B^2$$

To the motor, then, it must appear that it did $\frac{1}{2} J_0 \omega_B^2$ worth of work, even though its shaft speed is $\omega_S = m\omega_B$. We require:

$$\tfrac{1}{2} J_I (m\omega_B)^2 = \tfrac{1}{2} J_0 \omega_B^2$$

$$J_I = \frac{J_0}{m^2}$$

See text for another way to arrive at the same result.