6.301 Solid-State Circuits

Recitation 24: Charge Control w/Demo
Prof. Joel L. Dawson

We're almost at the finish line! Charge control is the last major topic that we will cover this term.
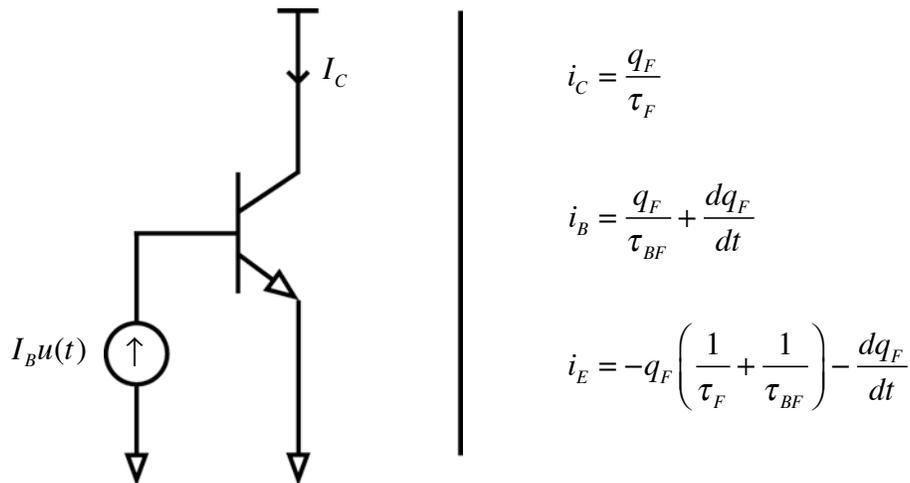
Why charge control? Hasn't the hybrid-π model served us well this whole semester? It certainly has. But to see some of the holes that it has left, ask yourself the following questions:

- How does one analytically treat a device that has entered saturation?

- Are there any dynamics associated with suddenly changing regions of operation?

- What really happens if we suddenly cut-off the base current of a BJT?

The list goes on, and the point is that the answers to these questions can be useful. Transistors, after all, aren't used only for small signal amplification. Sometimes they are useful as switches, either for realizing digital logic or in digital-to-analog converters.

We should emphasize that charge control, like open circuit time constants, is not good for giving us precise numerical answers. It <u>does</u> give us design insight, telling us, for instance, what topologies should yield high switching speeds.

To review, recall that in lecture we looked at finding $I_c(t)$ in the following situation:

$$i_C = \frac{q_F}{\tau_F}$$

$$i_B = \frac{q_F}{\tau_{BF}} + \frac{dq_F}{dt}$$

$$i_E = -q_F\left(\frac{1}{\tau_F} + \frac{1}{\tau_{BF}}\right) - \frac{dq_F}{dt}$$

Since everything depends on q_F, we first figure out $q_F(t)$:

$$i_B = \frac{q_F}{\tau_{BF}} + \frac{dq_F}{dt} = I_B u(t)$$

Using basic differential equations, we solved this and found:
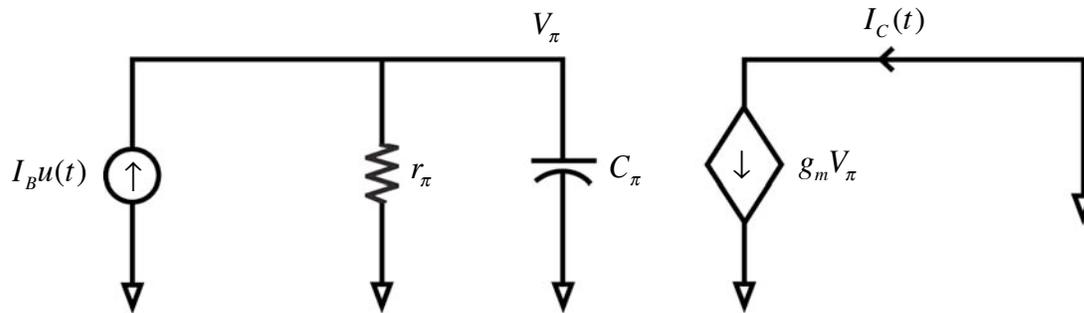
$$q_F = \tau_{BF} I_B \left(1 - e^{-t/\tau_{BF}}\right)$$

$$I_C = \frac{q_F}{\tau_F} = \frac{\tau_{BF}}{\tau_F} I_B \left(1 - e^{-t/\tau_{BF}}\right) = \beta I_B \left(1 - e^{-t/\tau_{BF}}\right)$$

Now, for the class exercise we're going to solve this a different way.

CLASS EXERCISE: Consider the hybrid-π model

Derive the collector current as a function of time. You may find the following reminders helpful:

(1) $g_m r_\pi = \beta_F$

(2) $r_\pi C_\pi = \left(\frac{\beta_F}{g_m}\right)(g_m \tau_F) = \beta_F \tau_F = \tau_{BF}$

(Workspace)
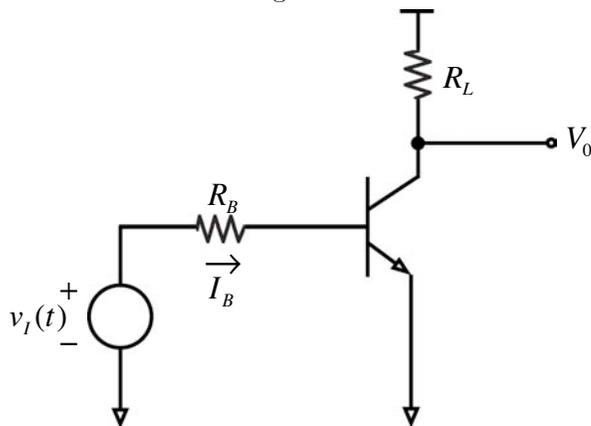
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The equivalence is a little surprising, isn't it? Think about why this worked out…noticing that $g_m r_\pi$ and $r_\pi C_\pi$ are both independent of collector current.

Very well. We promised design insight as a result of our derivations, and we're coming to a first example. According to our charge control model, turning on a transistor is simply a matter of placing the right amount of charge in the base.
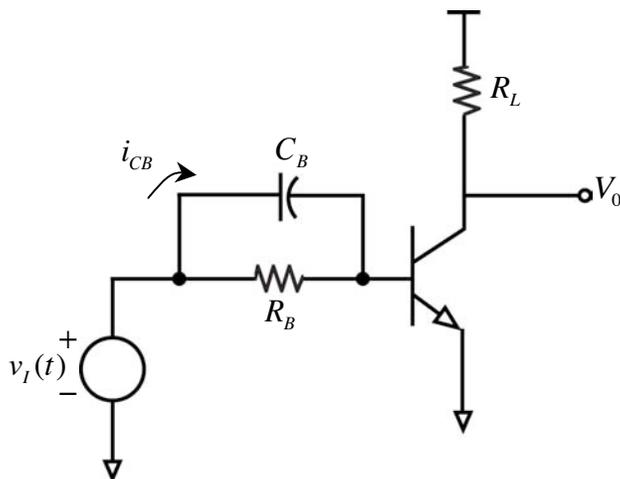
When we tried the following circuit in lecture:

$$v_I(t) = \left(V_I + V_{BE}\right)u(t)$$

There was a limit to how rapidly we could change the amount of charge in the case specifically:

$$\left|\frac{dq_F}{dt}\right| \le \frac{V_I}{R_B}$$

But what if we could arrange to put charge into the base very quickly? Consider:

$$v_I(t) = \left(V_I + V_{BE}\right)u(t)$$

Hmm. Conceptually, the voltage across the capacitor changes instantaneously from zero to V_I:

$$i_{CB} = C_B \frac{dV_{CB}}{dt} = C_b V_I \delta(t)$$

Therefore, the base current can be written:

$$i_B(t) = \frac{V_I}{R_B} u(t) + C_B V_I \delta(t)$$

Using charge control, we have

$$i_B = \frac{q_F}{\tau_{BF}} + \frac{dq_F}{dt} = \frac{V_I}{R_B} u(t) + C_B V_I \delta(t)$$

The solution is

$$q_F(t) = \tau_{BF} \frac{V_I}{R_B}\left(1 - e^{-t/\tau_{BF}}\right) u(t) + C_B V_I e^{-t/\tau_{BF}}$$

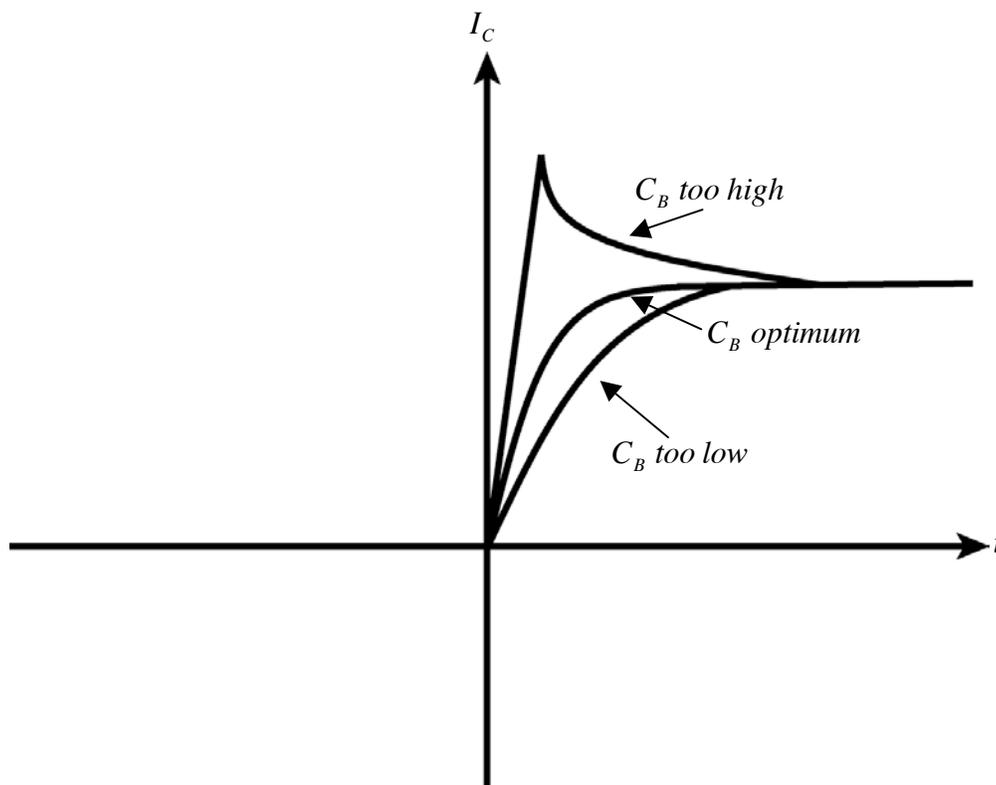Notice that if we choose $C_B = \dfrac{\tau_{BF}}{R_B}$,

$$q_F(t) = \tau_{BF} \frac{V_I}{R_B} u(t)$$

i_C reaches its final value instantaneously! For this reason, C_B is sometimes called a speed-up capacitor. Step responses for different values of C_B would look like the following:
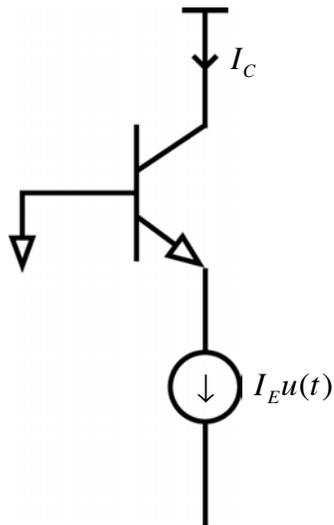

Real circuits do exactly this! C_B may require some trimming, but this is a trick that works in the real world.

Charge control also gives us insight into "emitter switching." That is,

Challenge: Find I_C and I_B. We'll talk more about this case later.

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