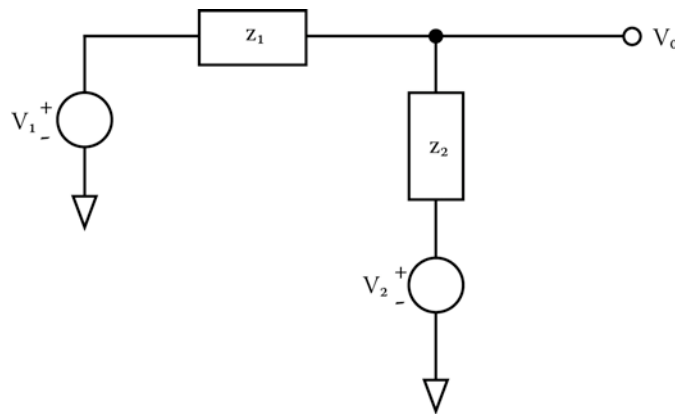# 6.301 Solid State Circuits

Recitation 3: AC Coupling, and Single-Transistor Amplifiers

Prof. Joel L. Dawson

When first presented, AC coupling is kind of a mysterious thing. After seeing it enough times, most people come to terms with it in an intuitive way, which is certainly good enough. But it's useful as an exercise to come to understand it in a way that is more mathematically explicit.
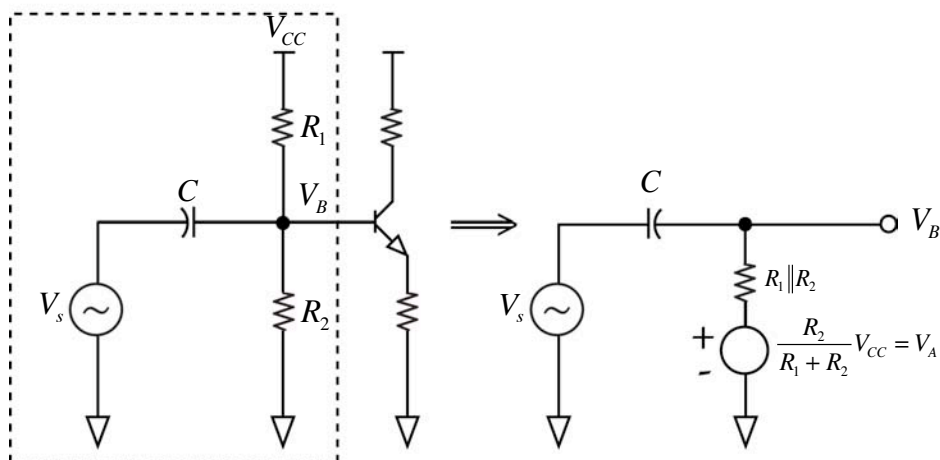
<u>CLASS EXERCISE</u>: Consider the following two-source network:

(1) Write a general expression for V_0 using superposition.

(2) Approximate V_0 in the cases $|z_1| << |z_2|$ and $|z_2| << |z_1|$.

(Workspace)

Now let's take a look at AC coupling.

6.301 Solid State Circuits

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Using superposition, we can write V_B fairly rapidly:

$$V_B = \frac{1}{(R_1 \| R_2)Cs + 1}V_A + \frac{R_1 \| R_2}{\frac{1}{Cs} + R_1 \| R_2}V_S$$

$$= \frac{1}{(R_1 \| R_2)Cs + 1}V_A + \frac{(R_1 \| R_2)Cs}{(R_1 \| R_2)Cs + 1}V_S$$
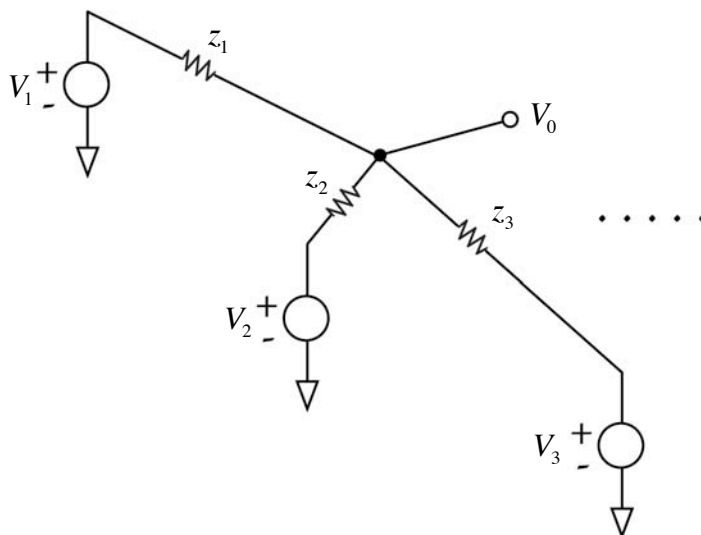
So the AC coupling setup low-pass filters the supply and high-pass filters the signal source!

For frequencies $\omega \ll \dfrac{1}{(R_1 \| R_2)C}$, $V_B \approx V_A$

For frequencies $\omega \gg \dfrac{1}{(R_1 \| R_2)C}$, $V_B \approx V_S$

This is exactly what we wanted!

For you network theory buffs, we can generalize this development to an arbitrary number of sources:

$$V_0 = \sum_{i=1}^{n} \frac{\left(\sum_{j \neq i} \dfrac{1}{z_j}\right)^{-1}}{z_i + \left(\sum_{j \neq i} \dfrac{1}{z_j}\right)^{-1}}V_i = \sum_{i=1}^{n} \frac{V_i}{z_i \left(\sum_{j \neq i} \dfrac{1}{z_j}\right) + 1}$$

Notice that if there is one source impedance z_i that is small compared to all of the other source impedances, its corresponding source dominates and $V_0 \approx V_i$. This is why one tends to think of low-impedance voltage sources as "strong."
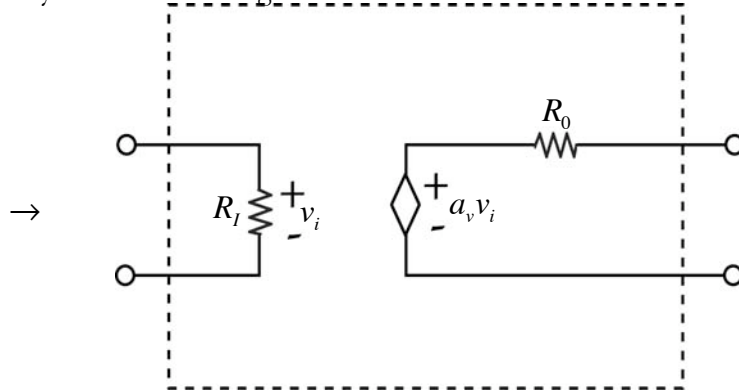
6.301 Solid State Circuits
Recitation 3: AC Coupling, and Single-Transistor Amplifiers
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Now let's embark on our study of single-transistor stages. Our overall goal will be to collapse the details of the stage into three easy-to-remember figures of merit:

- Input impedance
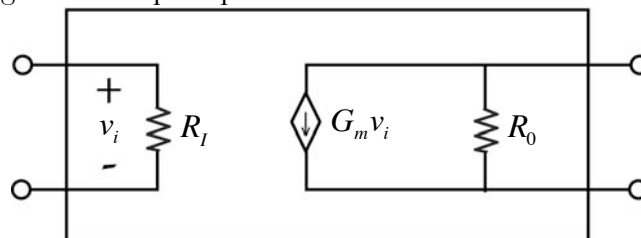- Gain   →
- Output impedance

We write things this way to simplify our lives when we go to a cascade of multiple stages:
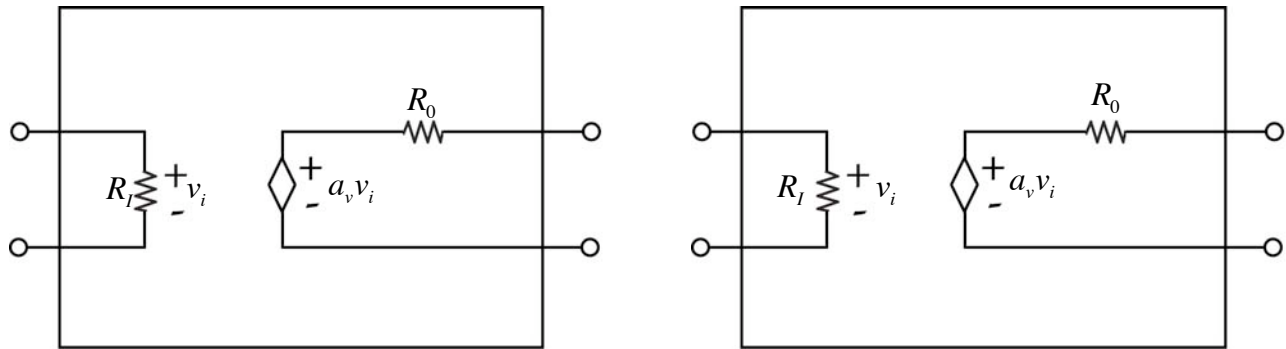
Overall gain: $a_v \left(\dfrac{R_I}{R_0 + R_I} \right) a_v = a_v^{\ 2} \left(\dfrac{R_I}{R_0 + R_I} \right)$

"Interstage Loading"

Note that sometimes it will be convenient to use the Norton equivalents model. We'll probably do this when modeling stages within op-amps:
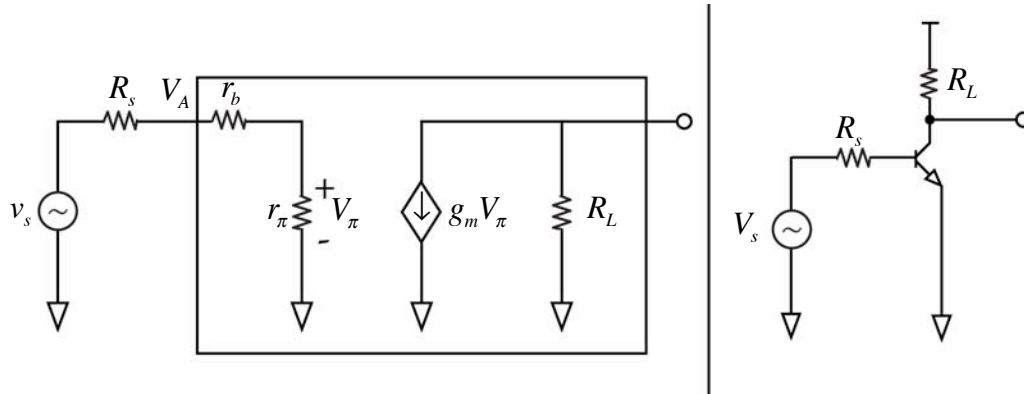
$$a_v = G_m R_0$$

As a warm-up, the familiar Common Emitter Stage:

6.301 Solid State Circuits
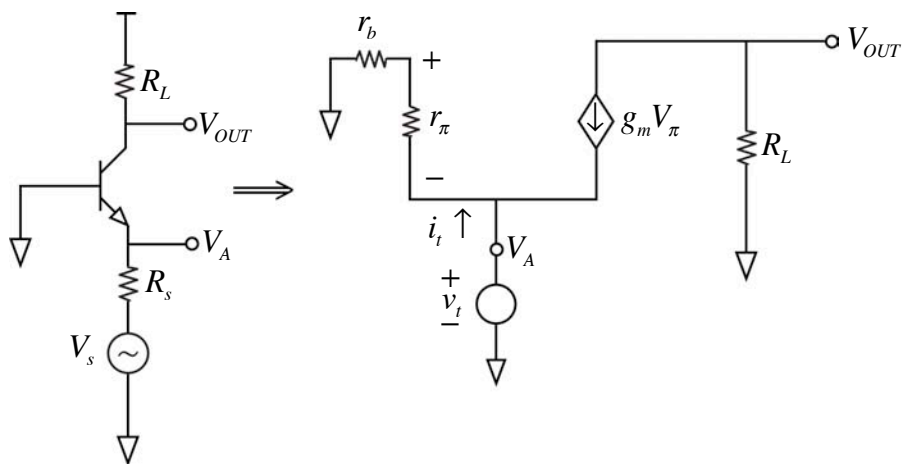Recitation 3: AC Coupling, and Single-Transistor Amplifiers
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Input Impedance: $\quad R_I = r_b + r_\pi$

Output Impedance: $\quad R_0 = R_L$

Voltage Gain: $\quad V_\pi = \dfrac{r_\pi}{r_b + r_\pi} V_A \rightarrow V_0 = -g_m R_L \left(\dfrac{r_\pi}{r_b + r_\pi} \right) V_A$

$$\Rightarrow \quad a_v = -\left(\dfrac{r_\pi}{r_b + r_\pi} \right) g_m R_L$$

Common Base Stage

6.301 Solid State Circuits

Recitation 3: AC Coupling, and Single-Transistor Amplifiers

Prof. Joel L. Dawson

First, we can calculate the input impedance using a test voltage source:

$$i_t = \frac{v_t}{r_b + r_\pi} + g_m\left(\frac{r_\pi}{r_\pi + r_b}\right)v_t$$

$$= \frac{1}{r_\pi + r_b}(1 + g_m r_\pi)v_t$$

But $g_m r_\pi = \beta$. So

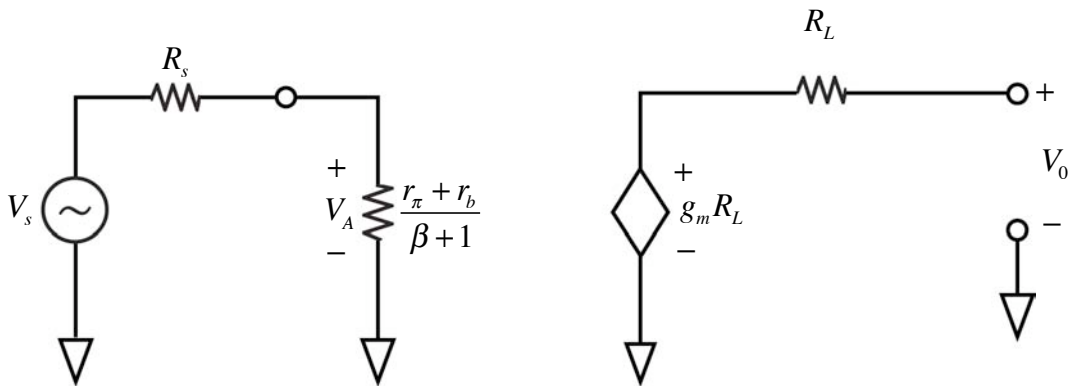$$R_{IN} = \frac{v_t}{i_t} = \frac{r_\pi + r_b}{\beta + 1} \approx \frac{r_\pi}{\beta} \approx \frac{1}{g_m}$$

Output impedance is easy: $R_0 = R_L$

And for the gain:

$$V_\pi = \frac{r_\pi}{r_b + r_\pi}V_A \rightarrow V_{OUT} = g_m\left(\frac{r_\pi}{r_b + r_\pi}V_A\right)R_L$$

$$\frac{V_{OUT}}{V_A} = g_m R_L\left(\frac{r_\pi}{r_b + r_\pi}\right) \approx g_m R_L\left(\frac{r_\pi}{r_\pi}\right) = g_m R_L$$

So our model for the common base stage becomes

Now notice two things. First, the input impedance to the common base stage is <u>low</u>.

$$R_{IN} = \frac{r_\pi + r_b}{\beta + 1} \approx \frac{1}{g_m}$$

Notice that the impact this has on the gain from V_S to V_0

$$V_0 \approx \frac{\frac{1}{g_m}}{\frac{1}{g_m} + R_S} V_S \cdot g_m R_L$$

$$\frac{V_0}{V_S} = \frac{1}{1 + g_m R_S} g_m R_L \approx \frac{g_m R_L}{g_m R_S} = \frac{R_L}{R_S}$$

Second, notice how the dependant current source has the effect of "transforming" resistances in the base so that they appear smaller when seen from the emitter:
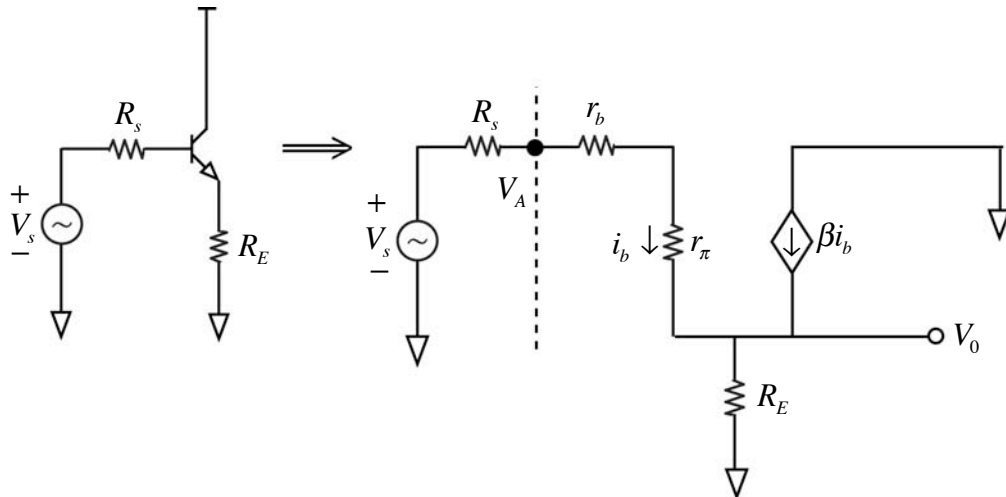
$$R_{IN} = \frac{r_\pi + r_b}{\beta + 1}$$

Summary of common base stage: High gain, low input impedance, high output impedance.

6.301 Solid State Circuits

Recitation 3: AC Coupling, and Single-Transistor Amplifiers
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Emitter Follower Stage

Input Impedance: $\quad R_I = r_b + r_\pi + (\beta + 1) R_E$

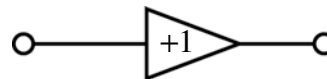Resistances in the emitter seem larger when viewed from the base.

Output Impedance: $\quad R_0 = R_E \parallel \left(\dfrac{R_S + r_b + r_\pi}{\beta + 1} \right)$

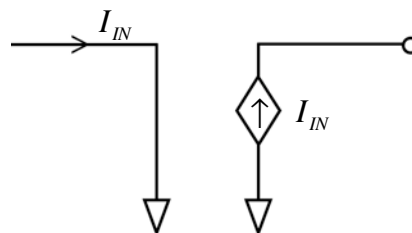Notice how the source resistance effects the R_0 if this stage.

Voltage Gain: $\quad a_v = \dfrac{(\beta + 1) R_E}{r_b + r_\pi + (\beta + 1) R_E} \approx 1$

Summary of Emitter Follower Stage: Unity gain, high input impedance, low output impedance

Emitter followers make good voltage buffers:

Common base stages make good current buffers:

6.301 Solid-State Circuits
Fall 2010