6.055J / 2.038J The Art of Approximation in Science and Engineering
Spring 2008

6.055J/2.038J (Spring 2008)

Homework 3

*Do the following warmups and problems. Due in class on **Friday, 14 Mar 2008**.*

***Open universe:** Collaboration, notes, and other sources of information are **encouraged**. However, avoid looking up answers until you solve the problem (or have tried hard). That policy helps you learn the most from the problems.*

***Bring a photocopy to class on the due date**, trade it for a solution set, and figure out or ask me about any confusing points. Your work will be graded lightly: P (made a reasonable effort), D (did not make a reasonable effort), or F (did not turn in).*

Warmups

1. **Explain a Unix pipeline**
   What does this pipeline do?

   ```
   ls -t | head | tac
   ```

   [Hint: If you are not familiar with Unix commands, use the `man` command on Athena or on any nearby Unix or GNU/Linux system.]

2. **Symmetry for algebra**
   Use symmetry to find $(a - b)^3$.

Problems

3. **Highway vs city driving**
   In lecture we derived a measure of how important drag is for a car moving at speed v for a distance d:

   $$\frac{E_{\text{drag}}}{E_{\text{kinetic}}} \sim \frac{\rho v^2 A d}{m_{\text{car}} v^2}.$$

   a. Show that the ratio is equivalent to the ratio

   $$\frac{\text{mass of the air displaced}}{\text{mass of the car}}$$

   and to the ratio

   $$\frac{\rho_{\text{air}}}{\rho_{\text{car}}} \times \frac{d}{l_{\text{car}}},$$

   where ρ_{car} is the density of the car (i.e. its mass divided by its volume) and l_{car} is the length of the car.

b. Make estimates for a typical car and find the distance d at which the ratio becomes significant (say, roughly 1). How does the distance compare with the distance between exits on the highway and between stop signs or stoplights on city streets?

4. **Symmetry for second-order systems**

   This problem analyzes the frequency of maximum gain for an *LRC* circuit or, equivalently, for a damped spring–mass system. The gain of such a system is the ratio of the input amplitude to the output amplitude as a function of frequency.

   If the output voltage is measured across the resistor, and you drive the circuit with a voltage oscillating at frequency ω, the gain is (in a suitable system of units):

   $$G(\omega) = \frac{j\omega}{1 + j\omega/Q - \omega^2},$$

   where $j = \sqrt{-1}$ and Q is quality factor, a dimensionless measure of the damping.

   Do not worry if you do not know where that gain formula comes from. The purpose of this problem is not its origin, but rather using symmetry to maximize its magnitude.

   a. Show that the magnitude of the gain is

   $$|G(\omega)| = \frac{\omega}{\sqrt{\left(1 - \omega^2\right)^2 + \omega^2/Q^2}}.$$

   b. Find a variable substitution (a symmetry operation) $\omega_{\text{new}} = f(\omega)$ that turns $|G(\omega)|$ into $|H(\omega_{\text{new}})|$ such that G and H are the same function (i.e. they have the same structure but with ω in G replaced by ω_{new} in H).

   c. Use the form of that symmetry operation to maximize $|G(\omega)|$ without using calculus.

   d. *[Optional, for masochists!]* Maximize $|(G\omega)|$ using calculus.

5. **Gravity on the moon**

   In this problem you use a scaling argument to estimate the strength of gravity on the surface of the moon.

   a. Assume that a planet is a uniform sphere. What is the proportionality between the gravitational acceleration g at the surface of a planet and the planet's radius R and density ρ?

   b. Write the ratio $g_{\text{moon}}/g_{\text{earth}}$ as a product of dimensionless factors as in the analysis of the fuel efficiency of planes.

   c. Estimate those factors and estimate the ratio $g_{\text{moon}}/g_{\text{earth}}$, then estimate g_{moon}. [Hint: To estimate the radius of the moon, whose angular size you can estimate by looking at it, you might find it useful to know that the moon is $4 \cdot 10^8$ m distant from the earth.]

    d. Look up g_{moon} and compare the value to your estimate, venturing an explanation for any discrepancy.

6. **Checking plane fuel-efficiency calculation**
   This problem offers two more methods to estimate the fuel efficiency of a plane.

   a. Use the cost of a plane ticket to estimate the fuel efficiency of a 747, in passenger–miles per gallon.

   b. According to Wikipedia, a 747-400 can hold up to $2 \cdot 10^5 \ell$ of fuel for a maximum range of $1.3 \cdot 10^4$ km. Use that information to estimate the fuel efficiency of the 747, in passenger–miles per gallon.

   How do these values compare with the rough result from lecture, that the fuel efficiency is comparable to the fuel efficiency of a car?

7. **Invent your own problem**
   Invent your own problem whose solution might use symmetry, proportional reasoning, or a Unix pipeline.

Optional

8. **Design a Unix pipeline**
   Make a pipeline that prints the ten most common words in the input stream, along with how many times each word occurs. They should be printed in order from the the most frequent to the less frequent words. [Hint: First **tr**anslate any non-alphabetic character into a newline. Useful utilities include `tr` and `uniq`.]