Mathematics for Computer Science

MIT 6.042J/18.062J

Random Variables Uniform, Binomial

Uniform Random Variables

...all values equally likely

"threshold" variable was uniform:

$$\Pr[Z = 0] = \cdots = \Pr[Z = 6]$$

$$= \frac{1}{7}$$

Uniform Distribution

D ::= outcome of fair die roll

Pr[D=1] = Pr[D=2] =···= Pr[D=6] = 1/6

S ::= 4-digit lottery number

Pr[S = 0000] = Pr[S = 0001] = ···

= Pr[S = 9999] = 1/10000

Equal Pairs of Uniform Variables

Lemma. If R_1,R_2,R_3 have the same range, are mutually independent, and R_1 is uniform, then

$[R_1=R_2]$, $[R_2=R_3]$, $[R_1=R_3]$

are pairwise independent.

Obviously NOT 3-way indep.

Equal Pairs of Uniform Variables

R_1 is independent of $[R_2 = R_3]$ & has probability p of equaling each value

So it equals a common value of R_2 & R_3 with probability p

That is,

$$\Pr[R_1=R_2 \mid R_2=R_3] = \Pr[R_1=R_2] = p$$

Binomial Random Variable

$B_{n,p}$ ::= # heads in n mutually indep flips.

Coin may be biased. So 2 parameters

n ::= # flips, p ::= Pr{head}

for $n=5$, $p=2/3$

$$\Pr[HHTTH] = \Pr[H]\cdot\Pr[H]\cdot\Pr[T]\cdot\Pr[T]\cdot\Pr[H]$$

(by independence)

Binomial Random Variable

$B_{n,p}$ ::= # heads in n mutually indep flips.

Coin may be biased. So 2 parameters

n ::= # flips, p ::= Pr{head}

for $n=5$, $p=2/3$

$$\Pr[HHTTH] = \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{2}{3}$$

Binomial Random Variable

$B_{n,p}$ ::= # heads in n mutually indep flips.

Coin may be biased. So 2 parameters

n ::= # flips, p ::= Pr{head}

for $n=5$, $p=2/3$

$$\Pr[HHTTH] = \left(\frac{2}{3}\right)^3\cdot\left(\frac{1}{3}\right)^2$$

Binomial Random Variable

$B_{n,p}$ ::= # heads in n mutually indep flips.

Coin may be biased. So 2 parameters

n ::= # flips, p ::= Pr{head}

Pr[each sequence w/i H's, n-i T's] =

$$p^i(1-p)^{n-i}$$

Binomial Random Variable

$B_{n,p}$ ::= # heads in n mutually indep flips.

Coin may be biased. So 2 parameters

n ::= # flips, p ::= Pr{head}

Pr[get i H's, n-i T's] = #seq's · pr[seq]

$$\binom{n}{i}p^i(1-p)^{n-i}$$

Binomial Random Variable

$B_{n,p}$ ::= # heads in n mutually indep flips.

Coin may be biased. So 2 parameters

n ::= # flips, p ::= Pr{head}

Pr[$B_{n,p} = i$] = #seq's · pr{seq}

$$\binom{n}{i}p^i(1-p)^{n-i}$$

Density & Distribution

Probability Density Function

of random variable R,

$$\mathrm{PDF}_R(a) ::= \Pr[R = a]$$

so $\mathrm{PDF}_{B_{n,p}}(i) = \binom{n}{i}p^i(1-p)^{n-i}$

Density & Distribution

Probability Density Function of random variable R,

$$\mathrm{PDF}_R(a) ::= \Pr[R = a]$$

so

$$\mathrm{PDF}_U(v) = \text{constant}$$

for v in range of uniform U

Density & Distribution

Probability Density Function of random variable R,

$$\mathrm{PDF}_R(a) ::= \Pr[R = a]$$

Cumulative Distribution

$$\mathrm{CDF}_R(a) ::= \Pr[R \leq a]$$

Density & Distribution

Key observation:

The Probability Density & Cumulative Distribution Functions of R, do not depend on the sample space