


Mathematics for Computer Science
MIT 6.042J/18.062J

Random Variables Uniform, Binomial

Albert R Meyer May 6, 2013 binom-uniform.1



Uniform Random Variables


...all values equally likely

"threshold" variable was uniform:

$$\Pr[Z = 0] = \dots = \Pr[Z = 6]$$

$$= \frac{1}{7}$$

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Uniform Distribution

D ::= outcome of fair die roll
 $\Pr[D=1] = \Pr[D=2] = \dots = \Pr[D=6] = 1/6$

S ::= 4-digit lottery number
 $\Pr[S = 0000] = \Pr[S = 0001] = \dots$
 $= \Pr[S = 9999] = 1/10000$

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Equal Pairs of Uniform Variables


Lemma. If R_1, R_2, R_3 have the same range, are mutually independent, and R_1 is uniform, then

$$[R_1=R_2], [R_2=R_3], [R_1=R_3]$$

are pairwise independent.


Obviously NOT 3-way indep.

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Equal Pairs of Uniform Variables

R_1 is independent of $[R_2 = R_3]$ & has probability p of equaling each value
 So it equals a common value of R_2 & R_3 with probability p
 That is,


$$\Pr[R_1=R_2 \mid R_2=R_3] = \Pr[R_1=R_2] = p$$


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Binomial Random Variable

$B_{n,p} ::= \#$ heads in n mutually indep flips.
 Coin may be biased. So 2 parameters
 $n ::= \#$ flips, $p ::= \Pr\{\text{head}\}$
 for $n=5, p=2/3$
 $\Pr[\text{HHTTH}] =$
 $\Pr[\text{H}] \cdot \Pr[\text{H}] \cdot \Pr[\text{T}] \cdot \Pr[\text{T}] \cdot \Pr[\text{H}]$
 (by independence)




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Binomial Random Variable

$B_{n,p} ::= \#$ heads in n mutually indep flips.
 Coin may be biased. So 2 parameters
 $n ::= \#$ flips, $p ::= \Pr\{\text{head}\}$
 for $n=5, p=2/3$
 $\Pr[\text{HHTTH}] =$


$$\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{3}$$


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Binomial Random Variable

$B_{n,p} ::= \#$ heads in n mutually indep flips.
 Coin may be biased. So 2 parameters
 $n ::= \#$ flips, $p ::= \Pr\{\text{head}\}$
 for $n=5, p=2/3$
 $\Pr[\text{HHTTH}] = \left(\frac{2}{3}\right)^3 \cdot \left(\frac{1}{3}\right)^2$




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Binomial Random Variable

$B_{n,p}$::= # heads in n mutually indep flips.
 Coin may be biased. So 2 parameters
 n ::= # flips, p ::= $\Pr\{\text{head}\}$
 $\Pr[\text{each sequence w/i H's, n-i T's}] =$


$$p^i (1-p)^{n-i}$$


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Binomial Random Variable

$B_{n,p}$::= # heads in n mutually indep flips.
 Coin may be biased. So 2 parameters
 n ::= # flips, p ::= $\Pr\{\text{head}\}$
 $\Pr[\text{get } i \text{ H's, n-i T's}] = \# \text{seq's} \cdot \text{pr}[\text{seq}]$


$$\binom{n}{i} p^i (1-p)^{n-i}$$


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Binomial Random Variable

$B_{n,p}$::= # heads in n mutually indep flips.
 Coin may be biased. So 2 parameters
 n ::= # flips, p ::= $\Pr\{\text{head}\}$
 $\Pr[B_{n,p} = i] = \# \text{seq's} \cdot \text{pr}\{\text{seq}\}$

$$\binom{n}{i} p^i (1-p)^{n-i}$$


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Density & Distribution

Probability Density Function
 of random variable R ,

$$\text{PDF}_R(a) ::= \Pr[R = a]$$

so

$$\text{PDF}_{B_{n,p}}(i) = \binom{n}{i} p^i (1-p)^{n-i}$$


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| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Density & Distribution
 Probability Density Function
 of random variable R ,

$$\text{PDF}_R(a) ::= \Pr[R = a]$$

so

$$\text{PDF}_U(v) = \text{constant}$$

for v in range of uniform U



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| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Density & Distribution
 Probability Density Function
 of random variable R ,

$$\text{PDF}_R(a) ::= \Pr[R = a]$$

Cumulative Distribution

$$\text{CDF}_R(a) ::= \Pr[R \leq a]$$



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| | | | |
|----|---|----|----|
| 6 | 9 | 13 | 7 |
| 12 | | 10 | 5 |
| 3 | 1 | 4 | 14 |
| 15 | 8 | 11 | 2 |

Density & Distribution

Key observation:

The Probability Density &
 Cumulative Distribution
 Functions of R , **do not**
depend on the **sample space**



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