Slide 1

Mathematics for Computer Science
MIT 6.042J/18.062J

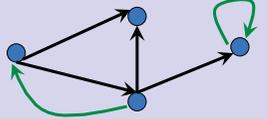# Simple Graphs: Degrees

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Slide 2

Types of Graphs
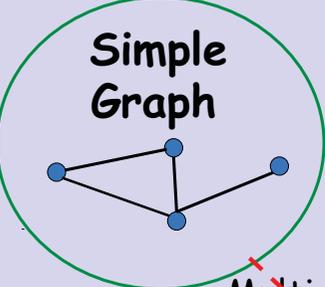
Simple Graph

Directed Graph

last week

this week

Multi-Graph

Slide 3
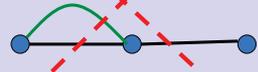
A simple graph:

Definition:

A simple graph G consists of
- a nonempty set, V, of vertices, and
- a set, E, of edges such that
  each edge has two endpoints in V

Slide 4

A Simple Graph

edge

vertices, V

undirected edges, E

$::= \{ \bullet, \circ \}$

"adjacent"

1

Vertex degree

degree of a vertex is
# of incident edges

$\deg(\bullet) = 2$

Vertex degree

degree of a vertex is
# of incident edges
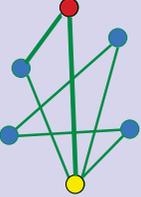
$\deg(\circ) = 4$
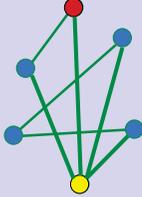
Impossible Graph

Is there a graph with
vertex degrees 2,2,1?

NO!
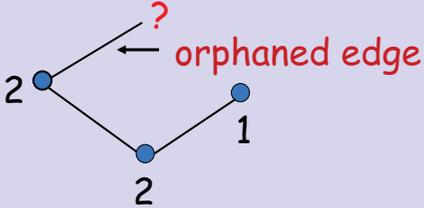
?
← orphaned edge

2

1

2

Handshaking Lemma

sum of degrees is
twice # edges

$$2|E| = \sum_{v \in V} \deg(v)$$

Proof: Each edge contributes
2 to the sum on the right

Handshaking Lemma

sum of degrees is
twice # edges

$$2|E| = \sum_{v \in V} \deg(v)$$

2+2+1 = odd,
so impossible

Sex in America: Men more Promiscuous?

- Univ Chicago, *The Social Organization of Sexuality*, 1994: men have 74% more women partners than women have men
- NBC News, *American Sex Survey*, 2004: men have 233% more women partners
- U.S. National Center for Health Statistics study, 2007: men have 175% more women partners
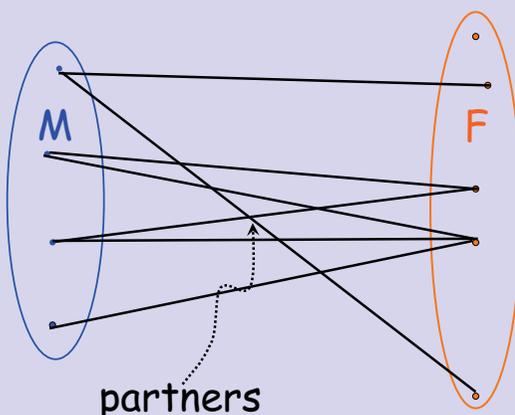
Sex in America: Men more Promiscuous?

Studies claim different %'s but agree that
men average many more partners than women.

Graph theory shows
this is nonsense

Sex Partner Graph

M

F

partners

3

Counting pairs of partners

$$\text{avg degree}(M) ::= \frac{\sum\limits_{m \in M} \deg(m)}{|M|}$$

$$\text{avg degree}(F) ::= \frac{\sum\limits_{f \in F} \deg(f)}{|F|}$$

Counting pairs of partners

$$\frac{\text{avg degree}(M)}{\text{avg degree}(F)} = \frac{\dfrac{\cancel{\sum\limits_{m \in M} \deg(m)}}{|M|}}{\dfrac{\cancel{\sum\limits_{f \in F} \deg(f)}}{|F|}}$$

$$\sum_{m \in M} \deg(m) = |E| = \sum_{f \in F} \deg(f)$$

Counting pairs of partners

$$\frac{\text{avg degree}(M)}{\text{avg degree}(F)} = \frac{|F|}{|M|}$$

Average number of partners

$$\text{avg-deg}(M) = 1.035 \cdot \text{avg-deg}(F)$$

Averages differ solely by
ratio of females to males.
No big difference
Nothing to do with promiscuity

Why are surveys wrong?

Maybe people are lying:

- Males exaggerate?

- Females deny?

Maybe Males have partners outside the study

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Spring 2015