Quiz 1

Your name:____

Circle the name of your Tutorial Instructor:

Ashish	Carole	Christos	Eric
George	Jack	Nick	Tina

- This quiz is **closed book**. There is an Appendix giving the definitions of standard properties of relations.
- There are four (4) problems totaling 100 points. Problems are labeled with their point values.
- Put your name on the top of **every** page – *these pages may be separated for grading*.
- Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem. Please keep your entire answer to a problem on that problem's page.
- You may assume any of the results presented in class or in the lecture notes.
- **Be neat and write legibly**. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it.
- GOOD LUCK!

DO NOT WRITE BELOW THIS LINE

Problem	Points	Grade	Grader
1	25		
2	20		
3	20		
4	35		
Total	100		

Problem 1 (25 points). Consider the following system specifications[1].

1. The system is in multiuser state iff it is operating normally.
2. If the system is operating normally, then the kernel is functioning.
3. The kernel is not functioning or the system is in interrupt mode.
4. If the system is not in multiuser state, then it is in interrupt mode.
5. The system is not in interrupt mode.

(a) (0 points) To make sense of these confusing conditions, let's introduce four Boolean variables.

$$M ::= \text{in Multiuser state} \tag{1}$$
$$N ::= \text{operating Normally} \tag{2}$$
$$K ::= \text{Kernel is functioning} \tag{3}$$
$$I ::= \text{in Interrupt mode} \tag{4}$$

Translate the five statements in the specification into propositional logic notation: $\wedge, \vee, \neg, \longrightarrow, \longleftrightarrow$

1. ____

2. ____

3. ____

4. ____

5. ____

[1] Rosen, Exercise 1.1.35

(b) (0 points) Are these system specifications consistent? ________. Prove it!

Problem 2 (20 points). For each of the following logical formulas with domain of discourse the natural numbers, $\mathbb{N}$, indicate whether it is a possible formulation of

- I: the **I**nduction Axiom,
- S: the **S**trong Induction Axiom,
- L: the **L**east Number Principle (also known as Well-ordering), or
- N: **N**one of these.

For example, the ordinary Induction Axiom could be expressed by the following formula, so it gets labelled "I".

$$(P(0) \land [\forall k\ P(k) \longrightarrow P(k+1)]) \longrightarrow \forall k\ P(k) \qquad \underline{\ \ I\ \ }$$

This is a multiple choice problem: do not explain your answer.

(a) (0 points) $(P(b) \land [\forall k \geq b\ P(k) \longrightarrow P(k+1)]) \longrightarrow \forall k \geq b\ P(k)$ ____

(b) (0 points) $(P(b) \land [\forall k \leq b\ P(k) \longrightarrow P(k+1)]) \longrightarrow \forall k \leq b\ P(k)$ ____

(c) (0 points) $[\forall b\ (\forall k < b\ P(k)) \longrightarrow P(b)] \longrightarrow \forall k\ P(k)$ ____

(d) (0 points) $(\exists n\ P(n)) \longrightarrow \exists n\ \forall k < n\ \overline{P(k)}$ ____

(e) (0 points) $\forall n\ [P(n) \longrightarrow (\exists n\ P(n) \land \forall k < n\ \overline{P(k)})]$ ____

Problem 3 (20 points). Classify each of the following binary relations as

- E: An equivalence relation.
- T: A Total order,
- P: A Partial order that is not total.
- S: A Symmetric relation that is not transitive.
- N: None of the above.

This is a multiple choice problem: do not explain your answer.

(a) (0 points) The relation xRy between times of day such that x and y are at most twenty minutes apart. ____

(b) (0 points) The relation xRy between times of day such that x is more than twenty minutes later than y. ____

(c) (0 points) The relation xRy over all words in this sentence such that x does not appear after y. (Consider "x", "y", and "xRy" to be words.) ____

(d) (0 points) The relation xRy over all words in this sentence such that word x does not appear after word y. ____

(e) (0 points) The relation xRy over all words in this sentence such that the final appearance of y occurs after x. ____

Problem 4 (35 points). To encourage collaborative study, the 6.042 staff is considering assigning each student to a study group with two or three other students. Prove that as long as the enrollment is large enough, the class can always be divided into such study groups.

Appendix

A binary relation, R, on a set A is

- *reflexive* iff xRx,
- *symmetric* iff $xRy \longrightarrow yRx$,
- *anti-symmetric* iff $xRy \wedge yRx \longrightarrow x = y$,
- *transitive* iff $xRy \wedge yRz \longrightarrow xRz$,

for all $x, y, z \in A$.

- R is an *equivalence relation* iff it is reflexive, symmetric and transitive.
- R is a *partial order* iff it is transitive and anti-symmetric.
- R is a *total order* iff it is a partial order and for all $x \neq y \in A$ either xRy or yRx.