Massachusetts Institute of Technology

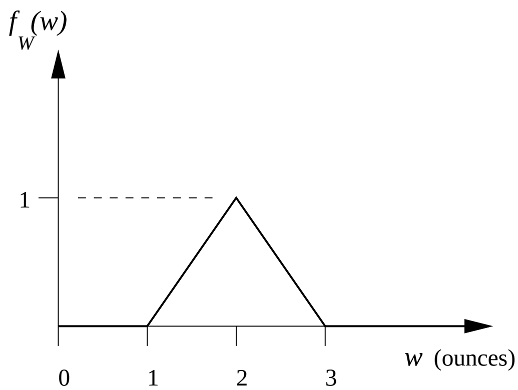
Department of Electrical Engineering & Computer Science

6.041/6.431: Probabilistic Systems Analysis

(Spring 2006)

Problem Set 12:
Topic: Central Limit Theorem
Due: No due date

1. The weight of a Pernotti Parabolic Pretzel, W, is a continuous random variable described by the probability density function



$$f_W(w) = \begin{cases} 0 & w \leq 1 \\ w-1 & 1 \leq w \leq 2 \\ 3-w & 2 \leq w \leq 3 \\ 0 & 3 \leq w \end{cases}$$

What is the probability that 102 pretzels (with independent weights) will have a total weight of more than 200 ounces? Find an approximate answer.

2. Your friend challenges you to a coin flipping contest. However you know this friend to be of questionable moral character. In fact you know that he usually caries a weighted coin that comes up heads with probability 0.55, along with a fair coin. You demand that he flip one coin 1000 times, and if it comes up heads more than 525 times, then you will play using the other coin. Assuming he uses the fair coin, find an approximation for the probability that you will think it is the biased coin.

3. On any given flight, an airline's goal is to have the plane be as full as possible, without overbooking. If, on average, 10cancel their tickets, all independently of each other, what is the probability that a particular flight with maximum capacity 300 people will be overbooked if the airline sells 320 tickets? Find an approximate answer.

4. The length in meters, X, of each section of pipe we obtain is an independent discrete random variable with PMF:

$$p_X(k) = (\frac{1}{2})^k \qquad k = 1, 2, \ldots$$

$$E[X] = 2 \qquad \text{var}(X) = 2$$

   (a) Suppose we obtain 400 sections of pipe. Determine the value of a bound, w, such that the total length of the sections we obtain will be <u>greater</u> than w with a probability of approximately 0.841.

   (b) Determine n, the number of sections of pipe needed such that the probability we obtain at least 200 meters of pipe is approximately 0.841.
