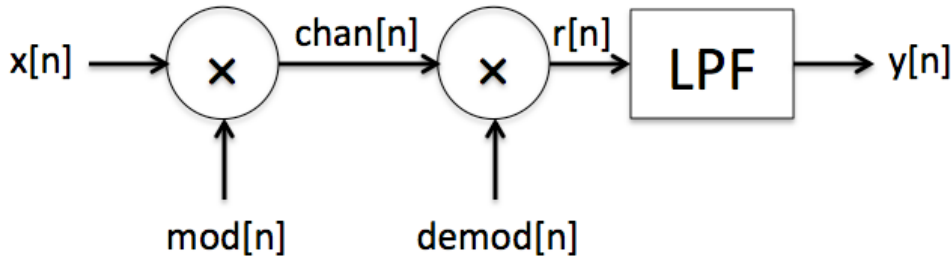


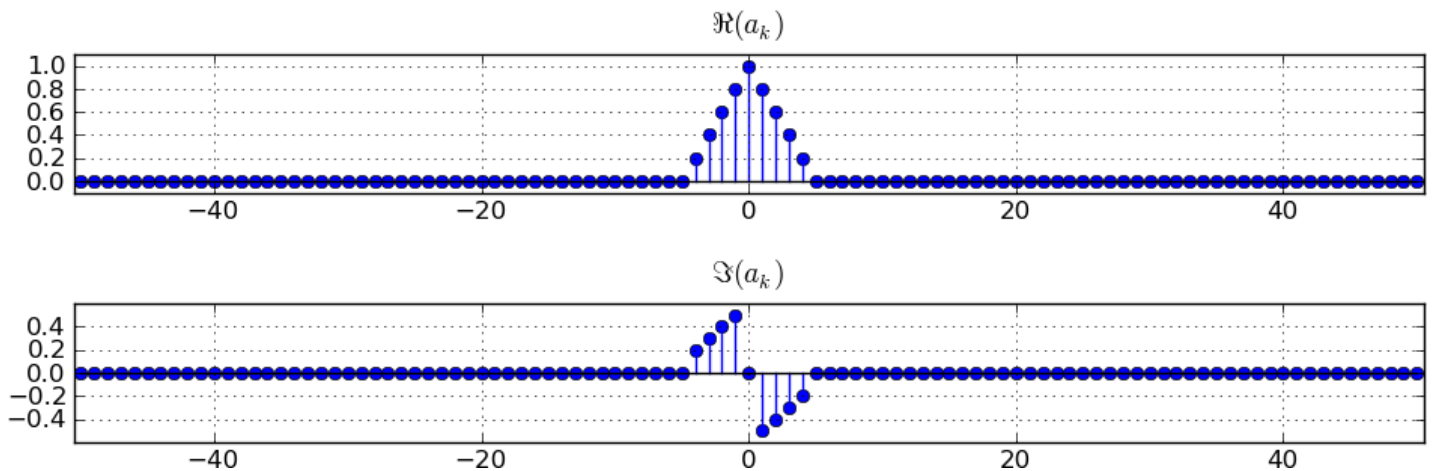
6.02 Practice Problems: Modulation & Demodulation

Problem 1.

Here's our "standard" modulation-demodulation system diagram: at the transmitter, signal $x[n]$ is modulated by signal $\text{mod}[n]$ and the result ($\text{chan}[n]$) is sent to the receiver where the incoming signal is demodulated by $\text{demod}[n]$ and sent through a low-pass filter to produce $y[n]$.



Suppose the band-limited signal $x[n]$ has the spectral coefficients shown below, where $N=101$:



A. Let $\text{mod}[n] = \cos(15(2\pi/101)n)$. Sketch a plot of the spectral coefficients for $\text{chan}[n]$, showing both the real and imaginary components. Please label center frequencies and peak amplitudes.

Show Answer

B. Let $\text{demod}[n] = \cos(15(2\pi/101)n)$, i.e., a sinusoid of the same frequency and phase as $\text{mod}[n]$. Sketch a plot of the spectral coefficients for $r[n]$, showing both the real and imaginary components. Please label center frequencies and peak amplitudes.

Show Answer

C. Give the appropriate cutoff frequency and gain for the low-pass filter LPF so that $y[n]$ is identical to $x[n]$.

Show Answer

D. Let $\text{mod}[n] = \sin(15(2\pi/101)n)$, i.e., like part (A) except the modulating signal is a sine instead of a cosine. Sketch a plot of the spectral coefficients for $\text{chan}[n]$, showing both the real and imaginary components. Please label center frequencies and peak amplitudes.

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E. Let $\text{demod}[n] = \cos(15(2\pi/101)n)$, i.e., a sinusoid of the same frequency and but $\pi/2$ out of phase with $\text{mod}[n]$. Sketch a plot of the spectral coefficients for $r[n]$, showing both the real and imaginary components. Please label center frequencies and peak amplitudes.

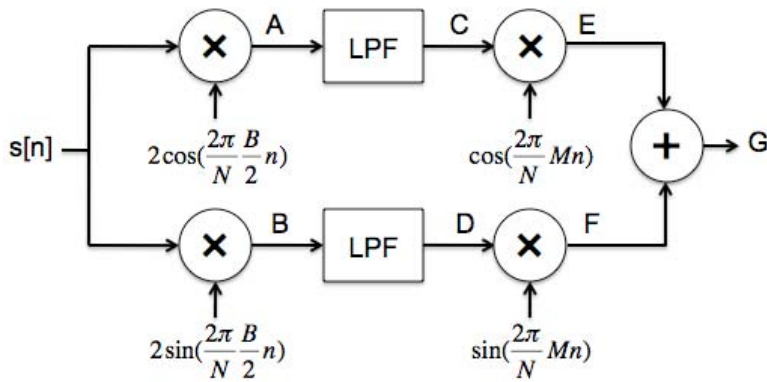
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F. Assuming the parameters of the LPF are set as determined in part (C), describe $y[n]$.

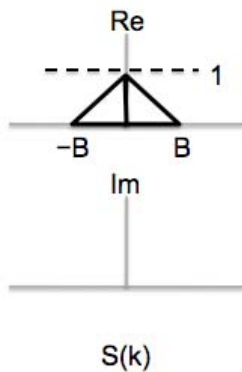
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Problem 2. Single-sideband (SSB) modulation is a modulation technique designed to minimize the amount of footprint used to transmit an

amplitude modulated signal. Here's one way to implement an SSB transmitter.



- A. Starting with a band-limited signal $s[n]$, modulate it with two carriers, one phase shifted by $\pi/2$ from the other. The modulation frequency is chosen to be $B/2$, i.e., in the middle of the frequency range of the signal to be transmitted. Sketch the real and imaginary parts of the Fourier coefficients for the signals at points A and B. The figure below shows the Fourier coefficients for the signal $s[n]$.



Show Answer

- B. The modulated signal is now passed through a low-pass filter with a cutoff frequency of $B/2$. Sketch the real and imaginary parts of the Fourier coefficients for the signals at points C and D.

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- C. The signal is modulated once again to shift it up to the desired transmission frequency. Sketch the real and imaginary parts of the Fourier coefficients for the signals at points E and F.

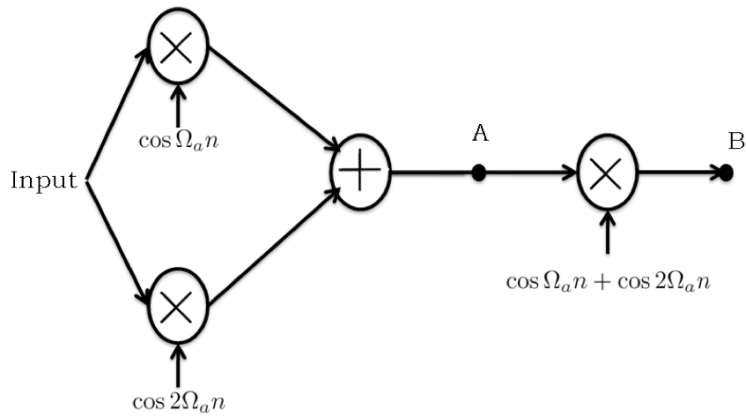
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- D. Finally the two signals are summed to produce the signal to be sent over the air. Sketch the real and imaginary parts of the Fourier coefficients for the signal at point G.

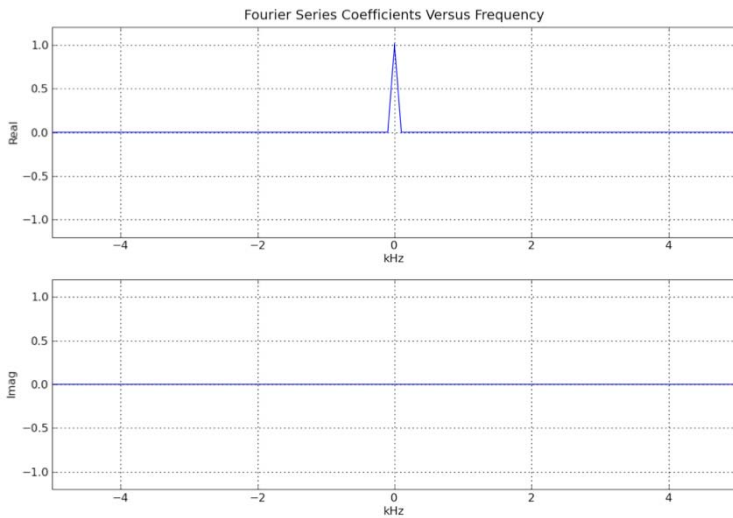
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Problem 3.

Consider the simple modulation-demodulation system below, where all signals are assumed periodic with period $N = 10000$ and the sampling frequency, f_s , is 10000 samples per second. In addition, $\Omega_a = 2\pi(f_a/f_s) = (1000 \cdot 2\pi)/10000$.



The Fourier Series coefficients versus frequency for the input to the modulation-demodulation system are plotted below for the case $N=10000$ and $f_s=10000$. Note that the Fourier coefficients are nonzero only for $-100 \leq k \leq 100$.



Please plot the Fourier series coefficients versus frequency for the signals at location A and B in the above diagram. Be sure to label key features such as values and coefficient indices for peaks. Also indicate the width of the peaks.

[Show Answer](#)

Problem 4.

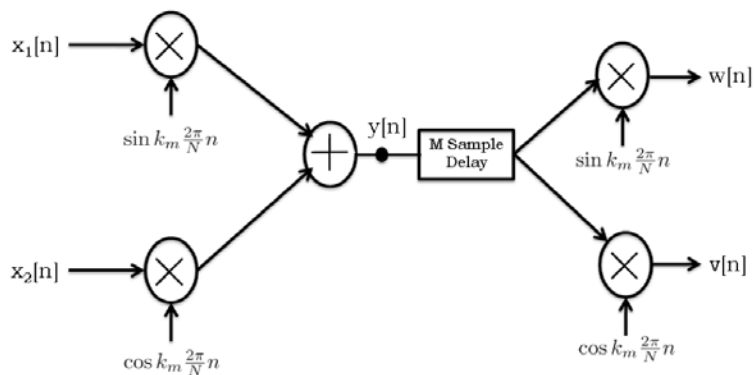
In this modulation problem you will be examining periodic signals and their associated discrete-time Fourier series (DTFS) coefficients. Recall that a periodic signal $x[n]$ with period N has DTFS coefficients given by

$$a_k = (1/N) \sum_{\tilde{n}=\langle N \rangle} x[n] e^{-j k (2\pi/N) n}$$

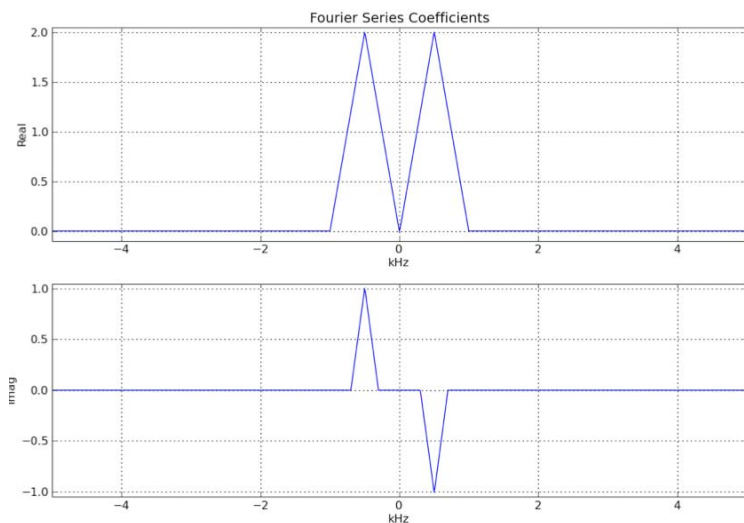
and that the signal $x[n]$ can be reconstructed from the DTFS coefficients using

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{j k (2\pi/N) n}$$

All parts of this question pertain to the following modulation-demodulation system, where all signals are periodic with period $N = 10000$. Please also assume that the sample rate associated with this system is 10000 samples per second, so that k is both a coefficient index and a frequency. In the diagram, the modulation frequency, k_m , is 500.



A. Suppose the DTFS coefficients for the signal $y[n]$ in the modulation/demodulation diagram are as plotted below.



Assuming that $M = 0$ for the M -sample delay (no delay), please plot the DTFS coefficients for the signals w and v in the modulation/demodulation diagram. Be sure to label key features such as values and coefficient indices for peaks.

Show Answer

B. Assuming the DTFS coefficients for the signal $y[n]$ are the same as in part A, please plot the DTFS coefficients for the signal x_1 in the modulation/demodulation diagram. Be sure to label key features such as values and coefficient indices for peaks.

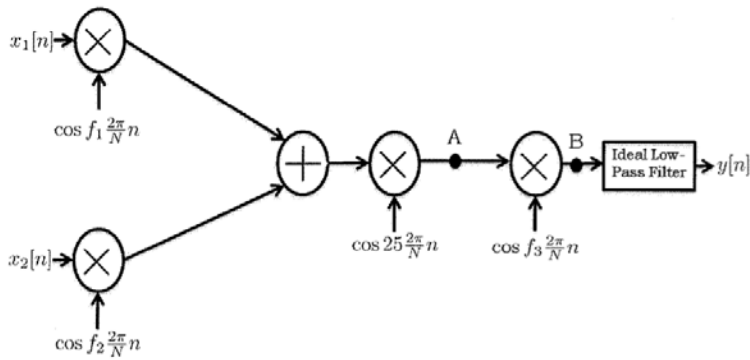
Show Answer

C. If the M -sample delay in the modulation/demodulation diagram has the right number of samples of delay, then it will be possible to nearly perfectly recover $x_1[n]$ by low-pass filtering $v[n]$. Please determine the smallest positive number of samples of delay that are needed and the cut-off frequency for the low-pass filter. Please be sure to justify your answer.

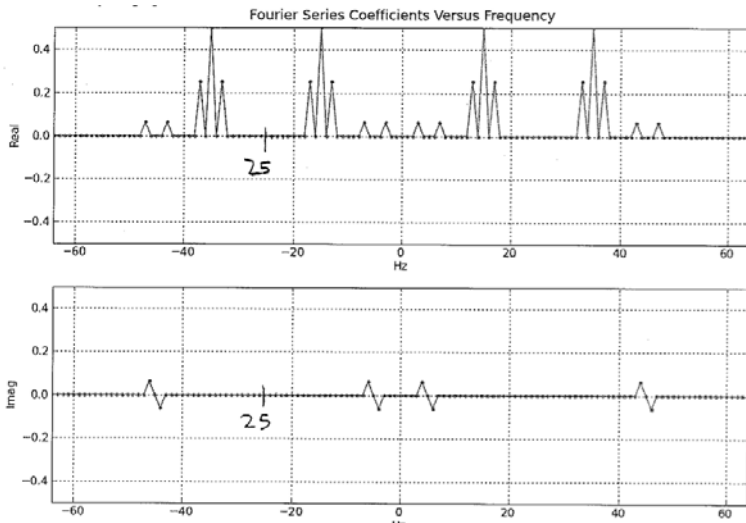
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Problem 5.

In answering this question, please refer to the modulation-demodulation system diagrammed below. Assume that $N=128$ and that the sampling frequency, f_s , is 128 samples per second.



A. Below are the plots of the real and imaginary parts of the Fourier coefficients vs. frequency for point A of the modulation-demodulation diagram. Please plot the real and imaginary parts of the Fourier coefficients vs. frequency for the signal at point B, assuming $f_3 = 25$. You only need to plot the Fourier coefficients in the range -25 to 25 . Please be sure to label critical frequencies and values in your graph.



Show Answer

B. Referring to plot given in part (A), if

$$x_1[n] = \alpha + \beta \cos(2(2\pi/N)n) \text{ and}$$

$$x_2[n] = (1/2) \cos(2(2\pi/N)n) + (1/2) \sin((2\pi/N)n)$$

please determine the two modulation frequencies, f_1 and f_2 , and the two amplitudes, α and β .

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C. Now suppose

$$x_1[n] = (1/2) \cos(2(2\pi/N)n) + (1/2) \sin((2\pi/N)n)$$

$$x_2[n] = 0$$

and $f_1 = 15$ (not one of the answers to part B!). For what values of $f_3 < 64$ (there is more than one) will $y[n] = x_1[n]$, assuming that the low-pass filter has been designed correctly? In addition, what should the magnitude be for the low frequency response of the low-pass filter?

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6.02 Introduction to EECS II: Digital Communication Systems
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