Chapter 3
Programs and Data

Object-oriented programming is a popular way of organizing programs, which groups together data with the procedures that operate on them, thus facilitating some kinds of modularity and abstraction. In the context of our PCAP framework, object-oriented programming will give us methods for capturing common patterns in data and the procedures that operate on that data, via classes, generic functions, and inheritance.

In this chapter, we will try to develop a deep understanding of object-oriented programming by working through the mechanism by which an interpreter evaluates a computer program. The first part of the chapter will focus on interpretation of typical expxressions, starting from the simplest single-statement programs and working up through list structures and procedures. Many of the observations made through this process apply to styles of program organization as well as object-oriented programming. Once we understand how an interpreter evaluates standard expressions, we will move to objects and classes. Although we use Python as an example, the discussion in this chapter is intended to be illustrative of principles of computer languages, more generally.

In many computer languages, including Python, programs are understood and executed by a computer program called an *interpreter*. Interpreters are surprisingly simple: the rules defining the meaning or *semantics* of a programming language are typically short and compact; and the interpreter basically encodes these rules and applies them to any legal expressionn in the language. The enormous richness and complexity of computer programs comes from the composition of primitive elements with simple rules. The interpreter, in essence, defines the semantics of the language by capturing the rules governing the value or behavior of program primitives, and of what it means to combine the primitives in various ways. We will study the meaning of computer programs by understanding how the interpreter operates on them.

An interpreter is made up of four pieces:

- The *reader* or *tokenizer* takes as input a string of characters and divides them into *tokens*, which are numbers (like `-3.42`), words (like `while` or `a`), and special characters (like `:`).
- The *parser* takes as input the string of tokens and understands them as constructs in the programming language, such as while loops, procedure definitions, or return statements.
- The *evaluator* (which is also sometimes called the *interpreter*, as well) has the really interesting job of determining the value and effects of the program that you ask it to interpret.
- The *printer* takes the value returned by the evaluator and prints it out for the user to see.

Programs should never be a mystery to you: you can learn the simple semantic rules of the language and, if necessary, simulate what the interpreter would do, in order to understand *any* computer program which you are facing. Of course, in general, one does not want to work through

the tedious process of simulating the interpreter, but this foundation of understanding the interpreter's process enables you to reason about the evaluation of any program.

3.1 Primitives, Composition, Abstraction, and Patterns

We will start by thinking about how the PCAP framework applies to computer programs, in general. We can do this by filling in **table 3.1**, exploring the PCAP ideas in data, procedures, and objects.

Data

The primitive data items in most programming languages are things like integers, floating point numbers, and strings. We can combine these into data structures (we discuss some basic Python data structures in **section 3.3**) such as lists, arrays, dictionaries and records. Making a data structure allows us, at the most basic level, to think of a collection of primitive data elements as if it were one thing, freeing us from details. Sometimes, we just want to think of a collection of data, not in terms of its underlying representation, but in terms of what it represents. So, we might want to think of a set of objects, or a family tree, without worrying whether it is an array or a list in its basic representation. *Abstract data types* provide a way of abstracting away from representational details and allowing us to focus on what the data really means.

Procedures

The primitive procedures of a language are things like built-in numeric operations and basic list operations. We can combine these using the facilities of the language, such as `if` and `while`, or by using function composition (`f(g(x))`). If we want to abstract away from the details of how a particular computation is done, we can define a new function; defining a function allows us to use it for computational jobs without thinking about the details of how those computational jobs get done. You can think of this process as essentially creating a new primitive, which we can then use while ignoring the details of how it is constructed. One way to capture common patterns of abstraction in procedures is to abstract over procedures themselves, with higher-order procedures, which we discuss in detail in **section 3.4.6**.

Objects

Object-oriented programming provides a number of methods of abstraction and pattern capture in both data and procedures. At the most basic level, objects can be used as records, combining together primitive data elements. More generally, they provide strategies for jointly abstracting a data representation and the procedures that work on it. The features of *inheritance* and *polymorphism* are particularly important, and we will discuss them in detail later in this chapter.

	Procedures	Data
Primitives	`+`, `*`, `==`	numbers, strings
Means of **combination**	`if`, `while`, `f(g(x))`	lists, dictionaries, objects
Means of **abstraction**	`def`	ADTS, classes
Means of capturing **patterns**	higher-order procedures	generic functions, inheritance

Table 3.1 Primitives, combination, abstraction, patterns framework for computer programs

3.2 Expressions and assignment

We can think of most computer programs as performing some sort of transformation on data. Our program might take as input the exam scores of everyone in the class and generate the average score as output. Or, in a transducer model, we can think about writing the program that takes the current memory state of the transducer and an input, and computes a new memory state and output.

To represent data in a computer, we have to encode it, ultimately as sequences of binary digits (0s and 1s). The memory of a computer is divided into 'words', which typically hold 32 or 64 bits; a word can be used to store a number, one or several characters, or a pointer to (the address of) another memory location.

A computer program, at the lowest level, is a set of primitive instructions, also encoded into bits and stored in the words of the computer's memory. These instructions specify operations to be performed on the data (and sometimes the program itself) that are stored in the computer's memory. In this class, we will not work at the level of these low-level instructions: a high-level programming language such as Python lets us abstract away from these details. But it is important to have an abstract mental model of what is going on within the computer.

3.2.1 Simple expressions

A cornerstone of a programming language is the ability to evaluate expressions. We will start here with arithmetic expressions, just to get the idea. An expression consists of a sequence of 'tokens' (words, special symbols, or numerals) that represent the application of operators to data elements. Each expression has a value, which can be computed recursively by evaluating primitive expressions, and then using standard rules to combine their values to get new values.

Numerals, such as `6` or `-3.7` are primitive expressions, whose values are numeric constants. Their values can be *integers*, within some fixed range dictated by the programming language, or *floating point numbers*. Floating point numbers are used to represent non-integer values, but they are different, in many important ways, from the real numbers. There are infinitely many real numbers within a finite interval, but only finitely many floating-point numbers exist at all (because they all must be representable in a fixed number of bits). In fact, the usual laws of real arithmetic (transitivity, associativity, etc.) are violated in floating-point arithmetic, because the results of any given sub-computation may not be representable in the given number of bits.

We will illustrate the evaluation of expressions in Python by showing short transcripts of interactive sessions with the *Python shell*: the shell is a computer program that

- Prompts the user for an expression, by typing »>,
- Reads what the user types in, and converts it into a set of tokens,
- Parses the tokens into a data structure representing the syntax of the expression,
- Evaluates the parsed expression using an interpreter, and
- Prints out the resulting value

So, for example, we might have this interaction with Python:

```
>>> 2 + 3
5
>>> (3 * 8) - 2
22
>>> ((3 * 8) - 2) / 11
2
>>> 2.0
2.0
>>> 0.1
0.10000000000000001
>>> 1.0 / 3.0
0.33333333333333331
>>> 1 / 3
0
```

There are a couple of things to observe here. First, we can see how floating point numbers only approximately represent real numbers: when we type in `0.1`, the closest Python can come to it in floating point is `0.10000000000000001`. The last interaction is particularly troubling: it seems like the value of the expression `1 / 3` should be something like `0.33333`. However, in Python, if both operands to the `/` operator are integers, then it will perform an integer division, truncating any remainder.[12]

These expressions can be arbitrarily deeply nested combinations of primitives. The rules used for evaluation are essentially the same as the ones you learned in school; the interpreter proceeds by applying the operations in precedence order[13], evaluating sub-expressions to get new values, and then evaluating the expressions those values participate in, until a single value results.

3.2.2 Variables

We cannot go very far without *variables*. A variable is a name that we can *bind* to have a particular value and then later use in an expression. When a variable is encountered in an expression, it is evaluated by looking to see to what value it is bound.

An interpreter keeps track of which variables are bound to what values in *binding environments*. An environment specifies a mapping between variable names and values. The values can be integers, floating-point numbers, characters, or pointers to more complex entities such as procedures or larger collections of data.

Here is an example binding environment:

[12] This behavior will no longer be the default in Python 3.0.

[13] Please Excuse My Dear Aunt Sally (Parentheses, Exponentiation, Multiplication, Division, Addition, Subtraction)

b	3
x	2.2
foo	-1012

Each row represents a binding: the entry in the first column is the variable name and the entry in the second column is the value it to which it is bound.

When you start up the Python shell, you immediately start interacting with a local binding environment. You can add a binding or change an existing binding by evaluating an assignment statement of the form:

```
<var> = <expr>
```

where `<var>` is a variable name (a string of letters or digits or the character `_`, not starting with a digit) and `<expr>` is a Python expression.[14]

Expressions are always evaluated in some environment.

We might have the following interaction in a fresh Python shell:

```
>>> a = 3
>>> a
3
>>> b
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
NameError: name 'b' is not defined
>>>
```

We started by assigning the variable `a` to have the value `3`. That added a binding for `a` to the local environment.

Next, we evaluated the expression `a`. The value of an expression with one or more variable names in it cannot be determined unless we know with respect to what environment it is being evaluated. Thus, we will always speak of evaluating expressions *in* an environment. During the process of evaluating an expression in some environment `E`, if the interpreter comes to a variable, it looks up that variable in `E`: if `E` contains a binding for the variable, then the associated value is returned; if it does not, then an error is generated. In the Python shell interaction above, we can see that the interpreter was able to find a binding for `a` and return a value, but it was not able to find a binding for `b`.

Why do we bother defining values for variables? They allow us to re-use an intermediate value in a computation. We might want to compute a formula in two steps, as in:

```
>>> c = 952**4
>>> c**2 + c / 2.0
6.7467650588636822e+23
```

[14] When we want to talk about the abstract form or `syntax` of a programming language construct, we will often use *meta-variables*, written with angle brackets, like `<var>`. This is meant to signify that `<var>` could be any Python variable name, for example.

They will also play a crucial role in abstraction and the definition of procedures. By giving a name to a value, we can isolate the use of that value in other computations, so that if we decide to change the value, we only have to change the definition (and not change a value several places in the code).

It is fine to reassign the value of a variable; although we use the equality symbol = to stand for assignment, we are not making a mathematical statement of equality. So, for example, we can write:

```
>>> a = 3
>>> a = a + 1
>>> a
4
```

Exercise 3.1. What is the result of evaluating this sequence of assignment statements and the last expression? Determine this by hand-simulating the Python interpreter. Draw an environment and update the stored values as you work through this example.

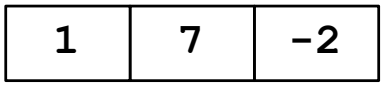
```
>>> a = 3
>>> b = a
>>> a = 4
>>> b
```

3.3 Structured data

We will often want to work with large collections of data. Rather than giving each number its own name, we want to organize the data into natural structures: grocery lists, matrices, sets of employee medical records. In this section, we will explore a simple but enormously useful and flexible data structure, which is conveniently built into Python: the list. The precise details of how lists are represented inside a computer vary from language to language. We will adopt an abstract model in which we think of a list as an ordered sequence of memory locations that contain values. So, for example, in Python, we can express a list of three integers as:

```
>>> [1, 7, -2]
[1, 7, -2]
```

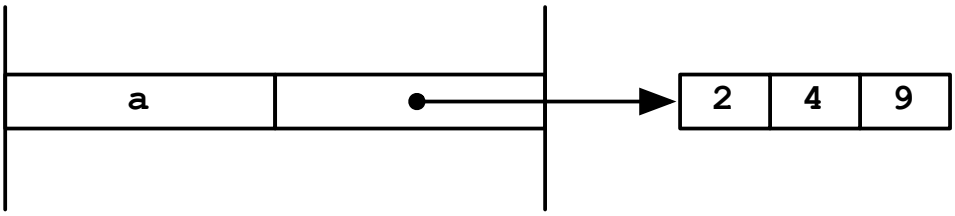
which we will draw in an abstract memory diagram as:



We can assign a list to a variable:

```
>>> a = [2, 4, 9]
```

A binding environment associates a name with a single fixed-size data item. So, if we want to associate a name with a complex structure, we associate the name directly with a 'pointer' to (actually, the memory address of) the structure. So we can think of `a` as being bound to a 'pointer' to the list:



Now that we have lists, we have some new kinds of expressions, which let us extract components of a list by specifying their indices. An index of 0 corresponds to the first element of a list. An index of -1 corresponds to the last element (no matter how many elements there are).[15] So, if `a` is bound as above, then we would have:

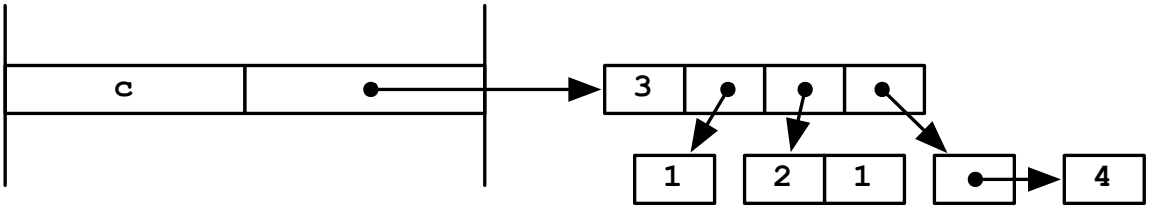
```
>>> a[0]
2
>>> a[2]
9
>>> a[-1]
9
>>> a[3]
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
IndexError: list index out of range
```

Note that if we attempt to access an element of the list that is not present (in this case, the fourth element of a three-element list), then an error is generated.

Lists can be nested inside one another. The Python expression:

```
>>> c = [3, [1], [2, 1], [[4]]]
```

creates a list that looks, in memory, like this:

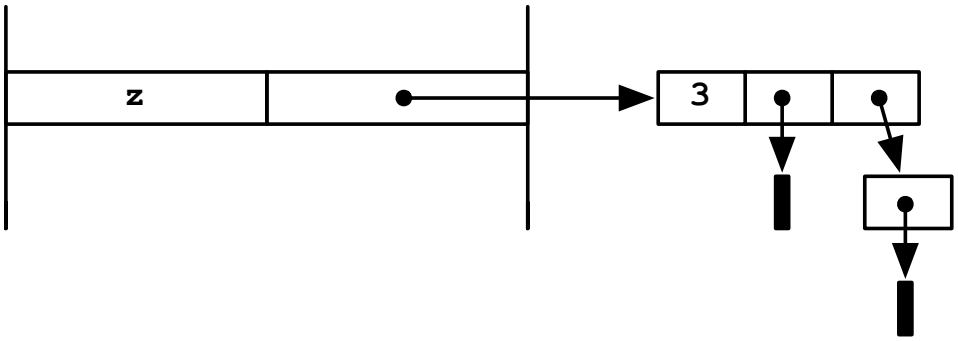


It is also possible to have an empty list, which is written in Python as `[]`. We will draw it in our memory diagrams as a small black box. So, for example, this list

```
>>> z = [3, [], [[]]]
```

looks like this in memory:

[15] See the Python tutorial for much more on list indexing.



Python has a useful function, `len`, which takes a list as an argument and returns its length. It does not look inside the elements of the list—it just returns the number of elements at the top level of structure. So, we have

```
>>> len([1, 2, 3])
3
>>> len([[1, 2, 3]])
1
```

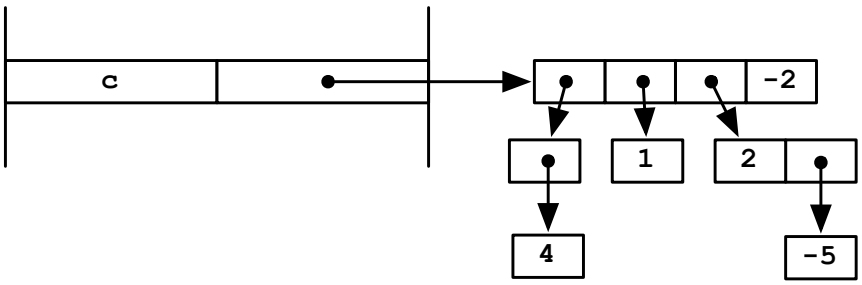
Exercise 3.2. Draw a diagram of the binding environment and memory structure after the following statement has been evaluated:

```
a = [[1], 2, [3, 4]]
```

Exercise 3.3. Draw a diagram of the binding environment and memory structure after the following statement has been evaluated:

```
a = [[[]]]
```

Exercise 3.4. Give a Python statement which, when evaluated, would give rise to this memory structure:



What is the value, in this environment, of the following expressions:

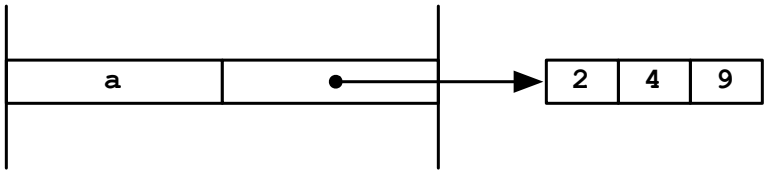
- `c[1]`
- `c[-1]`
- `c[2][1]`

3.3.1 List mutation and shared structure

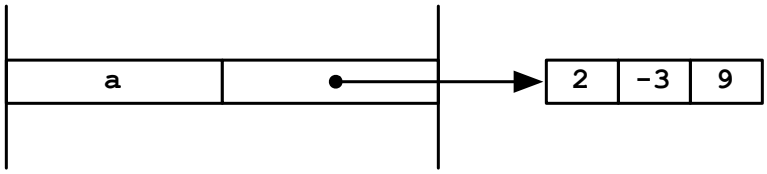
Lists are *mutable* data structures, which means that we can actually change the values stored in their elements. We do this by using element-selection expressions, like `a[1]` on the left-hand side of an assignment statement. So, the assignment

```
a[1] = -3
```

assigns the second element of `a` to be `-3`. In more detail, the left-hand side of this expression evaluates to a pointer to a specific location in memory (just as `a`'s value is a pointer to a location in memory); then the assignment statement changes the value stored there by inserting the value of the right-hand side of the expression. If that statement were evaluated in this environment,



then the resulting environment would be:



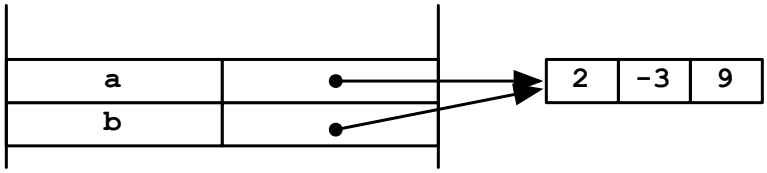
We have permanently changed the list named `a`.

In this section, we will explore the consequences of the mutability of lists; programs that change list structure can become *very* confusing, but you can always work your way through what is happening by drawing out the memory diagrams.

Continuing the previous example, let us remember that `a` is bound directly to a pointer to a list (or a sequence of memory cells), and think about what happens if we do:

```
>>> b = a
```

Now, `a` and `b` are both names for *the same list structure*, resulting in a memory diagram like this:



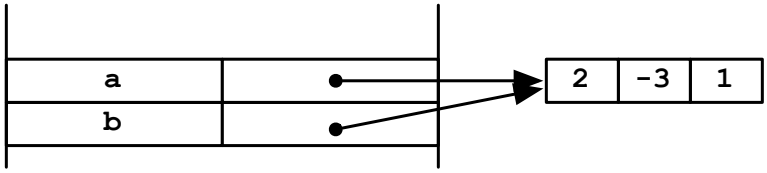
Now, we can reference parts of the list through `b`, and even change the list structure that way:

```
>>> b[0]
2
>>> b[2] = 1
```

Notice that, because `a` and `b` point to the same list, changing `b` changes `a`!

```
>>> a
[2, -3, 1]
```

Here is the memory picture now:

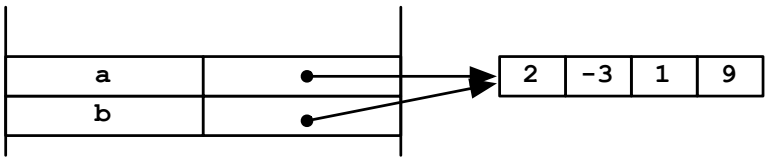


This situation is called *aliasing*: the name `b` has become an alias for `a`. Aliasing can be useful, but it can also cause problems, because you might inadvertently change `b` (by passing it into a procedure that changes one of its structured arguments, for example) when it is important to you to keep `a` unmodified.

Another important way to change a list is to add or delete elements. We will demonstrate adding elements to the end of a list, but see the Python documentation for more operations on lists. This statement

```
>>> a.append(9)
```

causes a new element to be added to the end of the list name `a`. The resulting memory state is:



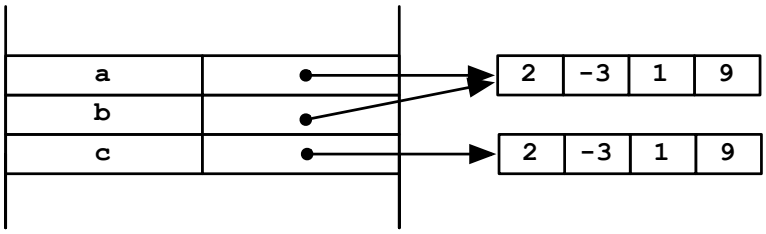
As before, because `a` and `b` are names for the same list (i.e., they point to the same memory sequence), `b` is changed too. This is a side effect of the aliasing between `a` and `b`:

```
>>> b
[2, -3, 1, 9]
```

Often, it will be important to make a fresh copy of a list so that you can change it without affecting the original one. Here are two equivalent ways to make a copy (use whichever one you can remember):

```
>>> c = list(a)
>>> c = a[:]
```

Here is a picture of the memory at this point:



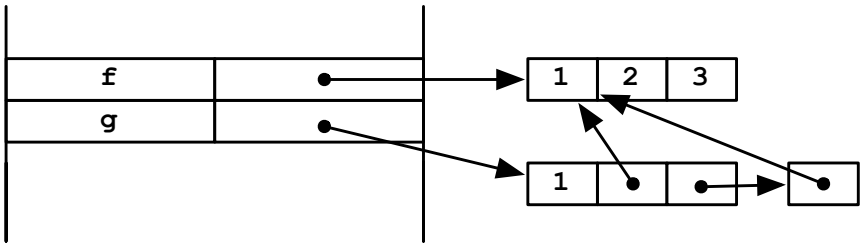
Now, if we change an element of `c`, it does not affect `a` (or `b`!):

```
>>> c[0] = 100
>>> c
[100, -3, 1, 9]
>>> a
[2, -3, 1, 9]
```

We can make crazy lists that share structure within a single list:

```
>>> f = [1, 2, 3]
>>> g = [1, f, [f]]
>>> g
[1, [1, 2, 3], [[1, 2, 3]]]
```

which results in this memory structure:



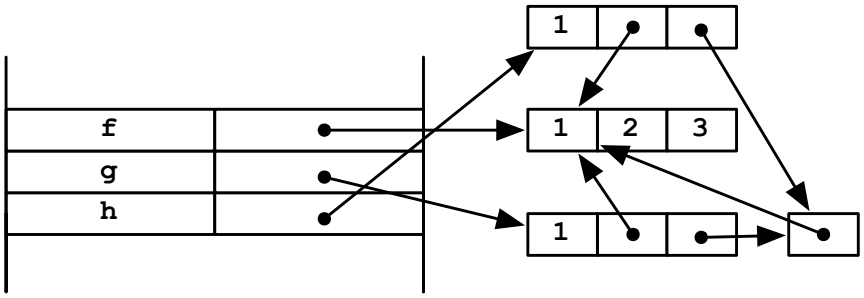
If you want to add an element to a list and get a new copy at the same time, you can do

```
>>> a + [1]
```

The + operator makes a new list that contains the elements of both of its arguments, but does not share any top-level structure. All of our methods of copying only work reliably if your lists do not contain other lists, because it only copies one level of list structure. So, for example, if we did:

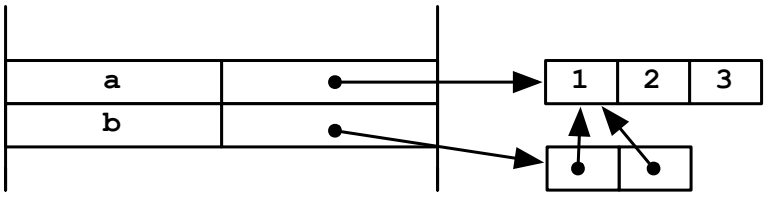
```
>>> h = list(g)
```

we would end up with this picture:

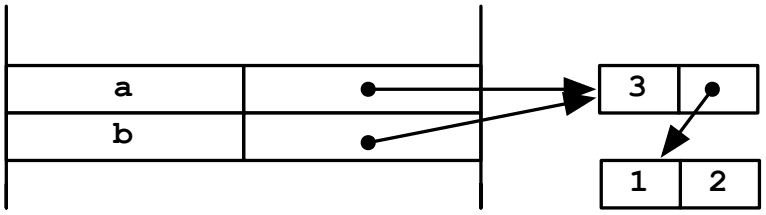


It is clear that if we were to change `f`, it would change `h`, so this is not a completely new copy. If you need to copy deep structures, that is, to make a copy not only of the top level list structure, but of the structures of any lists that list contains, and the lists those lists contain, etc., you will need to use the Python `copy.deepcopy` procedure.

Exercise 3.5. Give a sequence of Python statements which, when evaluated, would give rise to this memory structure:



Exercise 3.6. Give a sequence of Python statements which, when evaluated, would give rise to this memory structure:



Exercise 3.7. Show the memory structure after this sequence of expressions.

```
>>> a = [1, 2, 3]
>>> b = [a, a]
>>> a.append(100)
```

What will be the value of `b` at this point?

Exercise 3.8. Show the memory structure after this sequence of expressions.

```
>>> a = [5, 6]
>>> b = [1, 2]
>>> c = b + a
```

We will use this "curvy road" symbol to indicate sections of the notes or exercises that are somewhat more difficult and not crucial to understanding the rest of the notes. Feel free to skip them on first reading; but, of course, we think they are cool.[16]

[16] Thanks to Don Knuth's *Art of Computer Programming* for the idea of the curvy road sign.

Exercise 3.9.

Show the memory structure after this sequence of expressions.

```
>>> a = [1, 2, 3]
>>> a[1] = a
```

What will be the value of `a` at this point?

3.3.2 Tuples and strings

Python has two more list-like data types that are important to understand.

A *tuple* is a structure that is like a list, but is *not* mutable. You can make new tuples, but you cannot change the contents of a tuple or add elements to it. A tuple is typically written like a list, but with round parentheses instead of square ones:

```
>>> a = (1, 2, 3)
```

In fact, it is the commas and not the parentheses that matter here. So, you can write

```
>>> a = 1, 2, 3
>>> a
(1, 2, 3)
```

and still get a tuple. The only tricky thing about tuples is making a tuple with a single element. We could try

```
>>> a = (1)
>>> a
1
```

but it does not work, because in the expression `(1)` the parentheses are playing the standard grouping role (and, in fact, because parentheses do not make tuples). So, to make a tuple with a single element, we have to use a comma:

```
>>> a = 1,
>>> a
(1,)
```

This is a little inelegant, but so it goes.

Tuples will be important in contexts where we are using structured objects as 'keys', that is, to index into another data structure, and where inconsistencies would occur if those keys could be changed.

An important special kind of tuple is a *string*. A string can almost be thought of as a tuple of characters. The details of what constitutes a character and how they are encoded is complicated, because modern character sets include characters from nearly all the world's languages. We will stick to the characters we can type easily on our keyboards. In Python, you can write a string with either single or double quotes: `'abc'` or `"abc"`. You can select parts of it as you would a list:

```
>>> s = 'abc'
>>> s[0]
'a'
>>> s[-1]
'c'
```

The strange thing about this is that `s` is a string, and because Python has no special data type to represent a single character, `s[0]` is also a string.

We will frequently use + to concatenate two existing strings to make a new one:

```
>>> to = 'Jody'
>>> fromP = 'Robin'
>>> letter = 'Dear ' + to + ",\n It's over.\n" + fromP
>>> print letter
Dear Jody,
 It's over.
Robin
```

As well as using + to concatenate strings, this example illustrates several other small but important points:

- You can put a single quote inside a string that is delimited by double-quote characters (and vice versa).
- If you want a new line in your string, you can write `\n`. Or, if you delimit your string with a *triple* quote, it can go over multiple lines.
- The `print` statement can be used to print out results in your program.
- Python, like most other programming languages, has some *reserved words* that have special meaning and cannot be used as variables. In this case, we wanted to use `from`, but that has a special meaning to Python, so we used `fromP` instead.

Structured assignment

Once we have lists and tuples, we can use a nice trick in assignment statements, based on the packing and unpacking of tuples.

```
>>> a, b, c = 1, 2, 3
>>> a
1
>>> b
2
>>> c
3
```

Or, with lists,

```
>>> [a, b, c] = [1, 2, 3]
>>> a
1
>>> b
2
>>> c
3
```

When you have a list (or a tuple) on the left-hand side of an assignment statement, you have to have a list (or tuple) of matching structure on the right-hand side. Then Python will "unpack" them both, and assign to the individual components of the structure on the left hand side. You can get fancier with this method:

```
>>> thing = [8, 9, [1, 2], 'John', [33.3, 44.4]]
>>> [a, b, c, d, [e1, e2]] = thing
>>> c
[1, 2]
>>> e1
33.299999999999997
```

3.4 Procedures

Procedures are computer-program constructs that let us capture common patterns of computation by:

- Gathering together sequences of statements
- Abstracting away from particular data items on which they operate.

Here is a procedure definition,[17] and then its use:

```
def square(x):
    return x * x

>>> square(6)
36
>>> square(2 - square(2))
4
```

We will work through, in detail, what happens when the interpreter evaluates a procedure definition, and then the application of that procedure.

3.4.1 Definition

A procedure definition has the abstract form:

```
def <name>(<fp1>, ..., <fpn>):
    <statement1>
    ...
    <statementk>
```

There are essentially three parts:

[17] In the code displayed in these notes, we will show procedures being defined and then used, as if the definitions were happening in the Python shell (but without the prompts). In fact, you should not type procedure definitions into the shell, because if you make a mistake, you will have to re-type the whole thing, and because multi-line objects are not handled very well. Instead, type your procedure definitions into a file in Idle, and then test them by 'running' the file in Idle (which will actually evaluate all of the expressions in your file) and then evaluating test expressions in Idle's shell.

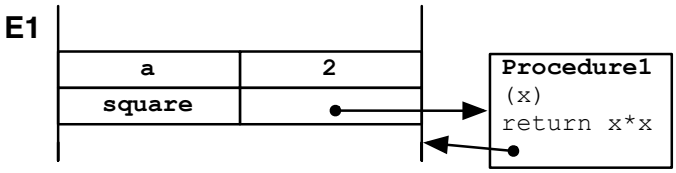
- `<name>` is a name for the procedure, with the same restrictions as a variable name;
- `<fp1>, ..., <fpn>` is a list of *formal parameters*, which will stand for the data items on which this procedure will operate; and
- `<statement1>, ..., <statementk>`, known as the *body of the procedure*, is a list of Python statements (right now, we know about assignment statements, print statements, and basic expressions, but there will be more.)

When we evaluate a procedure definition in an environment,[18] E, Python does two things:

1. Makes a procedure object[19] that contains the formal parameters, the body of the procedure, and a pointer to E; and then
2. Binds the name to have this procedure as its value.

Here is an example of an environment after we have evaluated

```
def square(x):
    return x * x
```



Note how the construct to which `square` points is a procedure object, with a list of formal parameters, a body, and a pointer to its environment.

3.4.2 Procedure calls

When you evaluate an expression of the form

```
<expr0>(<expr1>, ..., <exprn>)
```

the Python interpreter treats this as a procedure call. It will be easier to talk about a specific case of calling a procedure, so we will illustrate with the example

```
>>> square(a + 3)
```

evaluated in environment E1. Here are the steps:

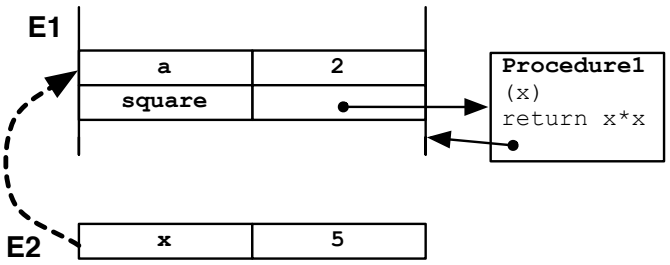
1. The expression that determines the procedure (`<expr0>`) is evaluated. In this case, we evaluate `square` in E1 and get `Procedure1`.
2. The expressions that determine the arguments (`<expr1>, ..., <exprn>`) are evaluated. In this case, we evaluate `a + 3` in E1 and get 5.

[18] Remember that every expression is always evaluated with respect to some environment.

[19] In our memory diagrams, we will show the procedure object as a box with the word `Procedure<N>`, where `N` is some integer, at the top; we give the procedure objects these numbers so we can refer to them easily in the text.

3. A *new environment* (in this case E2) is created, which:
   - binds the formal parameters of the procedure (in this case x) to the values of its arguments (in this case, 5); and
   - has as its parent the environment in which the procedure was defined (we find a pointer to this environment stored in the procedure; in this case E1 is its *parent*).

   At this point, our memory looks like this:



   The dotted line between E2 and E1 is intended to indicate that E1 is the *parent environment* of E2.
4. The statements of the procedure body are evaluated in the new environment until either a `return` statement or the end of the list of statements is reached. If a `return` statement is evaluated, the expression after the `return` is evaluated and its value is returned as the value of the procedure-call expression. Otherwise, the procedure has no return value, and the expression has the special Python value `None`.

   In our example, the only statement in the body is a return statement. So, we evaluate the expression `x * x` in E2, obtaining a value of 25. That value is returned as the value of the entire procedure-call expression `square(a + 3)`.

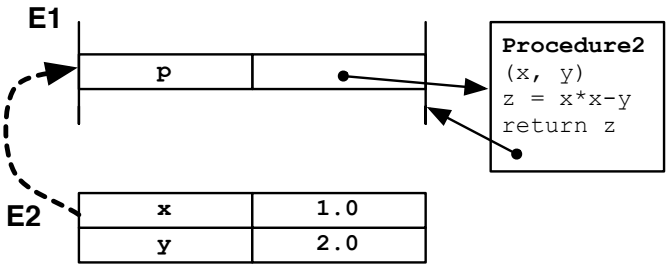
This basic mechanism can generate behavior of arbitrary complexity, when coupled with recursion or other control structures, discussed in **section 2.3**.

Worked example 1

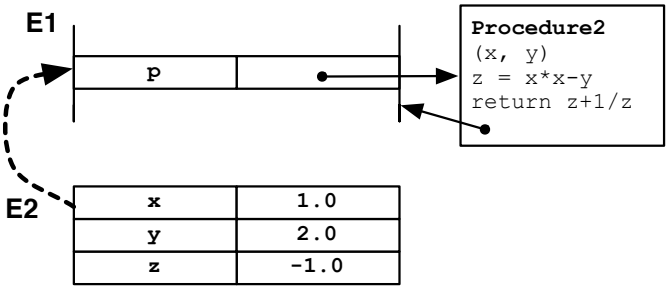
Let's examine what happens when we evaluate the following Python code:

```
def p(x, y):
    z = x*x - y
    return z + 1/z

>>> p(1.0, 2.0)
-2.0
```

Here is a picture of the calling environment (E1) and the procedure-call environment (E2) just before the body of the procedure is evaluated:



After evaluating `z = x*x - y` in E2, we have:



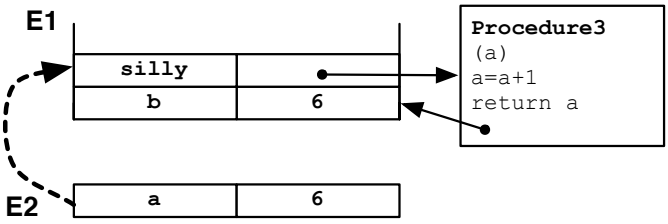
Finally, we evaluate `z + 1/z` in E2 and get -2.0, which we return.

Worked example 2

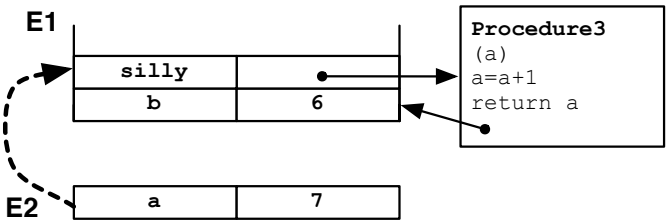
Here is another example:

```
def silly(a):
    a = a + 1
    return a

>>> b = 6
>>> silly(b)
7
```

Here is a picture of the calling environment (E1) and the procedure-call environment (E2) just before the body of the procedure is evaluated:



After evaluating `a = a + 1` in E2, we have:



Finally, we evaluate a in E2 and get 7, which we return.

Convince yourself that the execution below works fine, and notice that having a binding for a in E2 means that we leave the binding for a in E1 unmolested:

```
>>> a = 2
>>> silly(a)
3
>>> a
2
```

Exercise 3.10. Draw the environment diagram at the time the statement with the arrow is being executed:

```
def fizz(x, y):
    p = x + y
    q = p*p
    return q + x    #   <-----------

>>> fizz(2, 3)
```

Exercise 3.11. Draw the environment diagram at the time the statement with the arrow is being executed:

```
def fuzz(a, b):
    return a + b    #   <----------

def buzz(a):
    return fuzz(a, square(a))

>>> buzz(2)
```

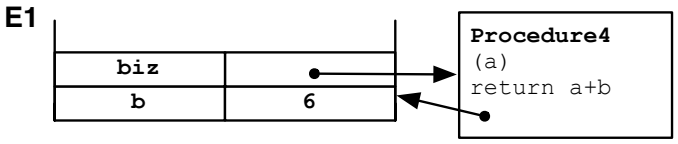
3.4.3 Non-local references

So far, whenever we needed to evaluate a variable, there was a binding for that variable in the 'local' environment (the environment in which we were evaluating the expression). But consider this example:

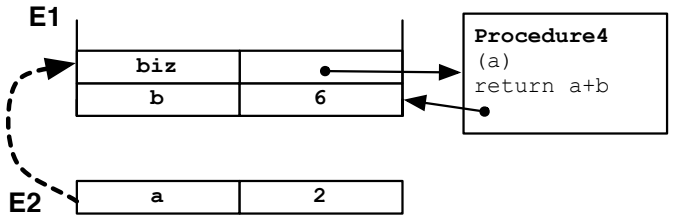
```
def biz(a):
    return a + b
```

```
>>> b = 6
>>> biz(2)
```

This actually works fine, and returns 8. Let's see why. Here is the environment, E1, in which the Python shell is working, after we execute the `b = 6` statement:



Now, when we evaluate `biz(2)` in E1, we make a new environment, E2, which binds `a` to 2 and *points to* E1 *as its parent environment.*



We need to elaborate, slightly, how it is that a variable v is evaluated in an environment E:

- We look to see if there is a binding for v in E; if so, we stop and return it.
- If not, we evaluate v in the parent environment of E. If E has no parent, we generate an error.

It is important to appreciate that this process will continue up an arbitrarily long chain of environments and their parents until either a binding for v is found or there are no more environments to examine.

So, in our case, we will evaluate the expression `a + b` in environment E2. We start by evaluating `a` and finding value 2 in E2. Then, we evaluate `b` and cannot find it in E2...but we don't panic! We follow the parent pointer to E1 and try again. We find a binding for `b` in E1 and get the value 6. So, the value of `a + b` in E2 is 8.

Exercise 3.12. Draw a picture of the relevant binding environments when the statement with the arrow is being executed. What value does the procedure return?

```
def f(a):
    return a + g(a)

def g(b):
    return a + b    # <--------------

>>> f(3)
```

Exercise 3.13. Draw a picture of the relevant binding environments when the statement with the arrow is being executed. What value does the procedure return?

```
def f(a):
    def g(b):
        return a + b    # <-------------
    return a + g(a)

>>> f(3)
```

Exercise 3.14. Draw a picture of the relevant binding environments when the statement with the arrow is being executed. What value does the procedure return? Does it cause an error?

```
def a(a):
    return a * a

>>> a(2)
```

3.4.4 Environments in Python

Generally, Python establishes the following binding environments:

1. `__builtin__`: the mother of all environments: it contains the definitions of all sorts of basic symbols, like `list` and `sum`. It is the parent of all module environments.
2. Module: each separate file that contains Python code is called a module and establishes its own environment, whose parent is `__builtin__`.
3. Procedure calls: as described in this section. A procedure that is defined at the 'top level' of a module (that is, not nested in the definition of another procedure) has the module's environment as its parent, and has its name defined in the module's environment. Procedures that are defined inside other procedures have the procedure-call environment of the containing procedure as their parent.

We have seen two operations that cause bindings to be created: assignments and procedure calls. Bindings are also created when you evaluate an `import` statement. If you evaluate

```
import math
```

then a file associated with the `math` module is evaluated and the name `math` is bound, in the current environment, to the math module, which is an environment. No other names are added to the current environment, and if you want to refer to names in that module, you have to qualify them, as in `math.sqrt`. (As we will see in a subsequent section, the evaluation of this expression first evaluates `math`, which returns an environment. We can then evaluate `sqrt` with respect to that environment, thus returning a pointer to the procedure stored there.) If you execute

```
from math import sqrt
```

then the `math` file is evaluated, and the name `sqrt` is bound, in the current environment, to whatever the name `sqrt` is bound in the `math` module. But note that if you do this, the name `math` is not bound to anything, and you cannot access any other procedures in the `math` module unless you import them explicitly, as well.

3.4.5 Non-local references in procedures

There is an important subtlety in the way names are handled in the environment created by a procedure call. When a name that is not bound in the local environment is referenced, then it is looked up in the chain of parent environments. So, as we have seen, it is fine to have

```
a = 2
def b():
    return a
```

When a name is assigned inside a procedure, a new binding is created for it in the environment associated with the current call of that procedure. So, it is fine to have

```
a = 2
def b():
    a = 3
    c = 4
    return a + c
```

Both assignments cause new bindings to be made in the local environment, and it is those bindings that are used to supply values in the return expression. It will not change `a` in the global environment.

But here is a code fragment that causes trouble:

```
a = 3
def b():
    a = a + 1
    print a
```

It seems completely reasonable, and you might expect `b()` to return 4. But, instead, it generates an error. What is going on?? It all has to do with when Python decides to add a binding to the local environment. When it sees this procedure definition, it sees that the name `a` occurs on the left-hand-side of an assignment statement, and so, at the very beginning, it puts a new entry for `a` in the local environment, but without any value bound to it. Now, when it is time to evaluate the statement

```
a = a + 1
```

Python starts by evaluating the expression on the right hand side: `a + 1`. When it tries to look up the name `a` in the procedure-call environment, it finds that `a` has been added to the environment, but has not yet had a value specified. So it generates an error.

In Python, we can write code to increment a number named in the global environment, by using the `global` declaration:

```
a = 3
def b():
    global a
    a = a + 1
    print a
>>> b()
4
>>> b()
5
>>> a
5
```

The statement `global a` asks that a new binding for `a` *not* be made in the procedure-call environment. Now, all references to `a` are to the binding in the module's environment, and so this procedure actually changes `a`.

In Python, we can only make assignments to names in the procedure-call environment or to the module environment, but not to names in intermediate environments. So, for example,

```
def outer():
    def inner():
        a = a + 1
    a = 0
    inner()
```

In this example, we get an error, because Python has made a new binding for `a` in the environment for the call to `inner`. We would really like `inner` to be able to see and modify the `a` that belongs to the environment for `outer`, but there is no way to arrange this. Some other programming languages, such as Scheme, offer more fine-grained control over how the scoping of variables is handled.

3.4.6 Procedures as first-class objects

In Python, unlike many other languages, procedures are treated in much the same way as numbers: they can be stored as values of variables, can be passed as arguments to procedures, and can be returned as results of procedure calls. Because of this, we say that procedures are treated as "first-class" objects. We will explore this treatment of "higher-order" procedures (procedures that manipulated procedures) throughout this section.

First of all, it is useful to see that we can construct (some) procedures without naming them using the `lambda` constructor:

```
lambda <var1>, ..., <varn> : <expr>
```

The formal parameters are `<var1>, ..., <varn>` and the body is `<expr>`. There is no need for an explicit `return` statement; the value of the expression is always returned. A single expression can only be one line of Python, such as you could put on the right hand side of an assignment statement. Here are some examples:

```
>>> f = lambda x: x*x
>>> f
<function <lambda> at 0x4ecf0>
```

```
>>> f(4)
16
```

Here is a procedure of two arguments defined with `lambda`:

```
>>> g = lambda x,y : x * y
>>> g(3, 4)
12
```

Using the expression-evaluation rules we have already established, we can do some fancy things with procedures, which we will illustrate throughout the rest of this section.

```
>>> procs = [lambda x: x, lambda x: x + 1, lambda x: x + 2]
>>> procs[0]
<function <lambda> at 0x83d70>
>>> procs[1](6)
7
```

Here, we have defined a list of three procedures. We can see that an individual element of the list (e.g., `procs[0]`) is a procedure.[20] So, then `procs[1](6)` applies the second procedure in the list to the argument 6. Since the second procedure returns its argument plus 1, the result is 7.

Here is a demonstration that procedures can be assigned to names in just the way other values can.

```
>>> fuzz = procs[2]
>>> fuzz(3)
5
```

```
def thing(a):
    return a * a
```

```
>>> thing(3)
9
>>> thang = thing
>>> thang(3)
9
```

Passing procedures as arguments

Just as we can pass numbers or lists into procedures as arguments, we can pass in procedures as arguments, as well.

What if we find that we are often wanting to perform the same procedure twice on an argument? That is, we seem to keep writing `square(square(x))`. If it were always the same procedure we were applying twice, we could just write a new procedure

```
def squaretwice(x):
    return square(square(x))
```

[20] In the programming world, people often use the words "function" and "procedure" either interchangeable or with minor subtle distinctions. The Python interpreter refers to the objects we are calling procedures as "functions."

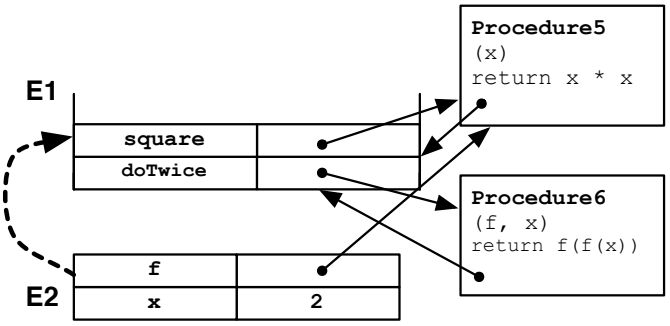
But what if it is different procedures? We can write a new procedure that takes a procedure `f` as an argument and applies it twice:

```
def doTwice(f, x):
    return f(f(x))
```

So, if we wanted to square twice, we could do:

```
>>> doTwice(square,2)
16
```

Here is a picture of the environment structure in place when we are evaluating the `return f(f(x))` statement in `doTwice`:



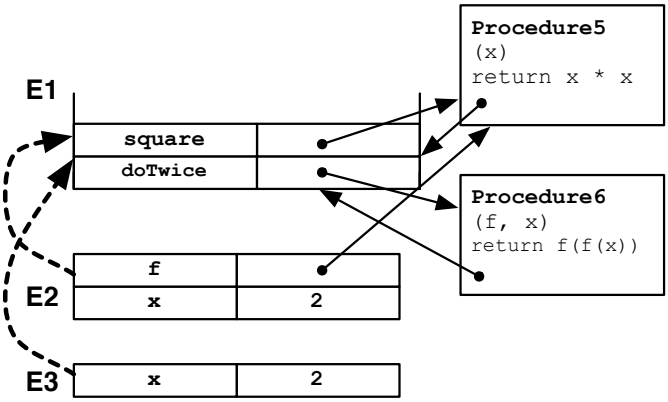
Environment E1 is the module environment, where procedures `square` and `doTwice` were defined and where the expression `doTwice(square, 2)` is being evaluated. The interpreter:

- Evaluates `doTwice` in E1 and gets `Procedure6`.
- Evaluates `square` in E1 and gets `Procedure5`.
- Evaluates 2 in E1 and gets 2.
- Makes the new binding environment E2, binding the formal parameters, `f` and `x`, of `Procedure 6`, to actual arguments `Procedure5` and 2.
- Evaluates the body of `Procedure6` in E2.

Now, let's peek one level deeper into the process. To evaluate `f(f(x))` in E2, the interpreter starts by evaluating the inner `f(x)` expression. It

- Evaluates `f` in E2 and gets `Procedure5`.
- Evaluates `x` in E2 and gets 2.
- Makes the new binding environment E3, binding the formal parameter `x`, of `Procedure5`, to 2. (Remember that the parent of E3 is E1 because `Procedure5` has E1 as a parent.)
- Evaluates the body of `Procedure5` in E3, getting 4.

The environments at the time of this last evaluation step are:



A similar thing happens when we evaluate the outer application of `f`, but now with argument 4, and a return value of 16.

Exercise 3.15. Here is the definition of a procedure `sumOfProcs` that takes two procedures, `f` and `g`, as well as another value `x`, as arguments, and returns `f(x) + g(x)`. The `sumOfProcs` procedure is then applied to two little test procedures:

```
def sumOfProcs(f, g, x):
    return f(x) + g(x)

def thing1(a):
    return a*a*a
def thing2(b):
    return b+1    #   <--------------------

>>> sumOfProcs(thing1, thing2, 2)
```

Draw a picture of all of the relevant environments at the moment the statement with the arrow is being evaluated. What is the return value of the call to `sumOfProcs`?

Returning procedures as values

Another way to apply a procedure multiple times is this:

```
def doTwiceMaker(f):
    return lambda x: f(f(x))
```

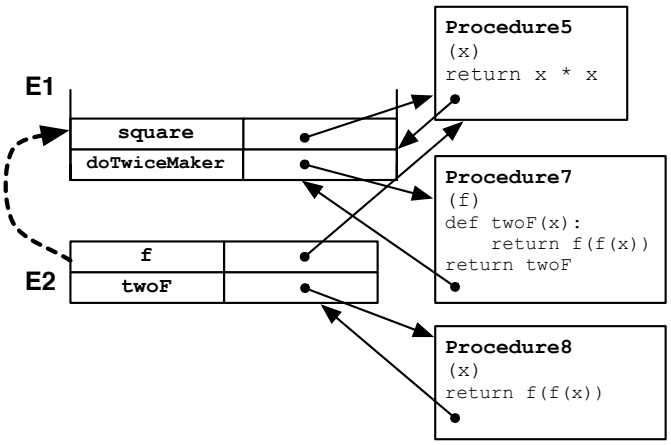
This is a procedure that *returns a procedure*! If you would rather not use `lambda`, you could write it this way:

```
def doTwiceMaker(f):
    def twoF(x):
        return f(f(x))
    return twoF
```

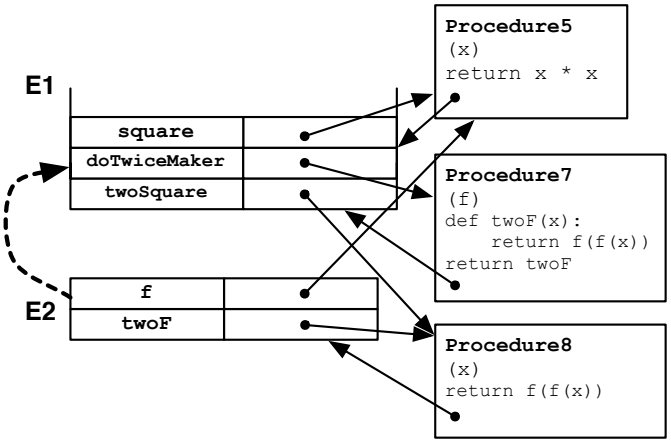
Now, to use `doTwiceMaker`, we might start by calling it with a procedure, such as `square`, as an argument and naming the resulting procedure.

```
>>> twoSquare = doTwiceMaker(square)
```

Here is a picture of the environments just before the `return twoF` statement in `doTwiceMaker` is evaluated.



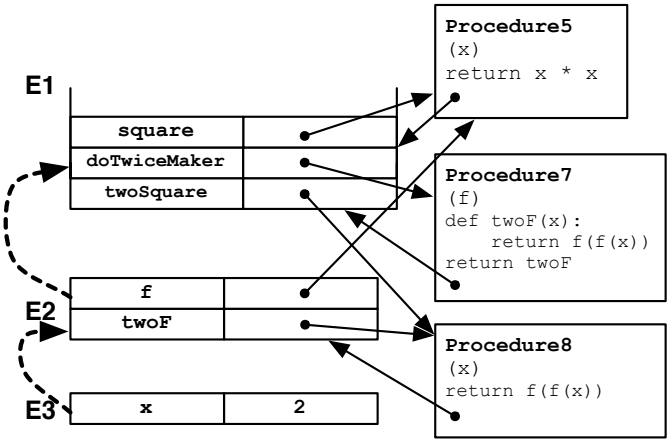
Here is a picture of the environments after the `doTwiceMaker` returns its value and it is assigned to `twoSquare` in E1. It is important to see that `Procedure8` is the return value of the call to `doTwiceMaker` and that, because `Procedure8` retains a pointer to the environment in which it was defined, we need to keep E2 around. And it is E2 that remembers which procedure (via its binding for `f`) is going to be applied twice.



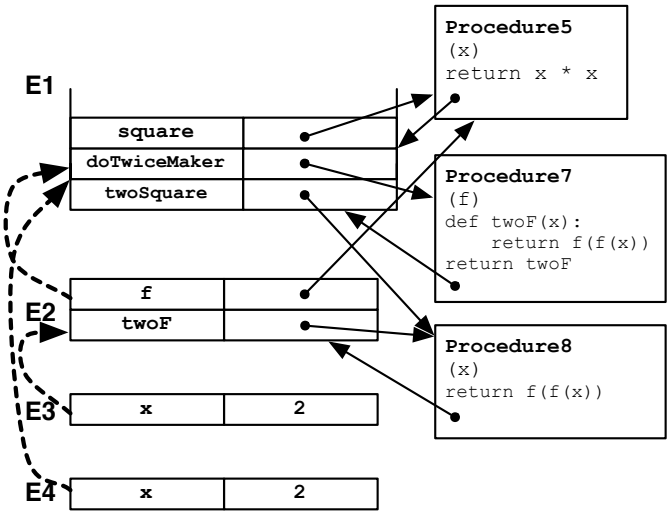
Now, when we evaluate this expression in E1

```
>>> twoSquare(2)
```

we start by making a new binding environment, E3, for the procedure call. Note that, because the procedure we are calling, `Procedure8`, has E2 stored in it, we set the parent of E3 to be E2.



Next, we evaluate the body of `Procedure8`, which is `return f(f(x))` in E3. Let's just consider evaluating the inner expression `f(x)` in E3. We evaluate `f` in E3 and get `Procedure5`, and evaluate `x` and get 2. Now, we make a new binding environment, E4, to bind the formal parameter of `Procedure5` to 2. Because `Procedure5` has a stored pointer to E1, E4's parent is E1, as shown here:



Evaluating the body of `Procedure5` in E4 yields 4. We will repeat this process to evaluate the outer application of `f`, in `f(f(x))`, now with argument 4, and end with result 16.

Essentially the same process would happen when we evaluate

```
>>> doTwiceMaker(square)(2)
16
```

except the procedure that is created by the expression `doTwiceMaker(square)` is not assigned a name; it is simply used as an intermediate result in the expression evaluation.

3.5 Object-oriented programming

We have seen structured data and interesting procedures that can operate on that data. It will often be useful to make a close association between collections of data and the operations that apply

to them. The style of programming that takes this point of view is *object-oriented programming* (OOP). It requires adding some simple mechanisms to our interpreter, but is not a big conceptual departure from the things we have already seen. It is, however, a different style of organizing large programs.

3.5.1 Classes and instances

In OOP, we introduce the idea of *classes* and *instances*. An *instance* is a collection of data that describe a single entity in our domain of interest, such as a person or a car or a point in 3D space. If we have many instances that share some data values, or upon which we would want to perform similar operations, then we can represent them as being members of a *class* and store the shared information once with the class, rather than replicating it in the instances.

Consider the following staff database for a large undergraduate course:

name	role	age	building	room	course
Pat	Prof	60	34	501	6.01
Kelly	TA	31	34	501	6.01
Lynn	TA	29	34	501	6.01
Dana	LA	19	34	501	6.01
Chris	LA	20	34	501	6.01

There are lots of shared values here, so we might want to define a class. A class definition has the form:

```
class <name>:
    <statement1>
    ...
    <statementn>
```

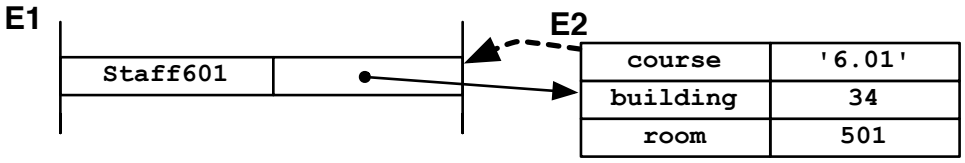
Here is the definition of simple class in our example domain:

```
class Staff601:
    course = '6.01'
    building = 34
    room = 501
```

From the implementation perspective, the most important thing to know is that ***classes and instances are environments***.

When we define a new class, we make a new environment. In this case, the act of defining `class Staff601` in an environment `E1` results in a binding from `Staff601` to `E2`, an empty environment whose parent is `E1`, the environment in which the class definition was evaluated. Now, the statements inside the class definition are evaluated in the new environment, resulting in a memory state like this:[21]

[21] In fact, the string `'6.01'` should be shown as an external memory structure, with a pointer stored in the binding environment; for compactness in the following diagrams we will sometimes show the strings themselves as if they were stored directly in the environment.



Note how the common values and names have been captured in a separate environment.

Caveat: when we discuss methods in **section 3.5.2**, we will see that the rules for evaluating procedure definitions inside a class definition are slightly different from those for evaluating procedure definitions that are not embedded in a class definition.

We will often call names that are bound in a class's environment *attributes* of the class. We can access these attributes of the class after we have defined them, using the *dot* notation:

```
<envExpr>.<var>
```

When the interpreter evaluates such an expression, it first evaluates `<envExpr>`; if the result is not an environment, then an error is signaled. If it is an environment, E, then the name `<var>` is looked up in E (using the general process of looking in the parent environment if it is not found directly in E) and the associated value returned.

So, for example, we could do:

```
>>> Staff601.room
501
```

We can also use the dot notation on the left-hand side of an assignment statement, if we wish to modify an environment. An assignment statement of the form

```
<envExpr>.<var> = <valExpr>
```

causes the name `<var>` in the environment that is the result of evaluating `<envExpr>` to be bound to the result of evaluating `<valExpr>`.

So, we might change the room in which 6.01 meets with:

```
Staff601.room = Staff601.room - 100
```

or add a new attribute with

```
Staff601.coolness = 11  # out of 10, of course...
```

Now, we can make an *instance* of a class with an expression of the form:

```
<classExpr>()
```

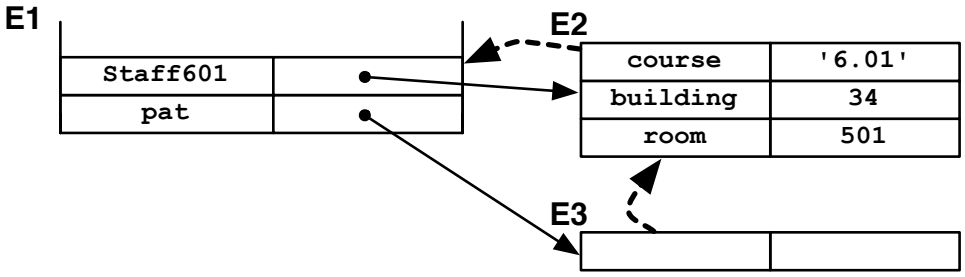
This expression has as its value a new empty environment whose parent pointer is the environment obtained as a result of evaluating `<classExpr>`.[22]

So, if we do:

```
>>> pat = Staff601()
```

we will end up with an environment state like this:

[22] Another way of thinking of this is that whenever you define a class, Python defines a procedure with the same name, which is used to create a instance of that class.



At this point, given our standard rules of evaluation and the dot notation, we can say:

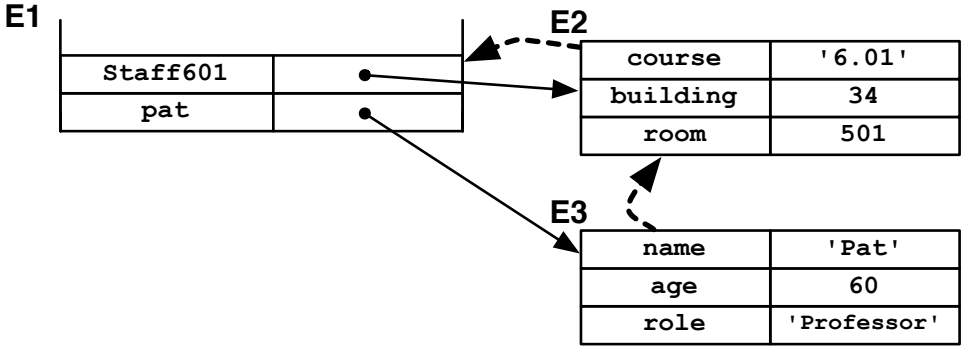
```
>>> pat.course
'6.01'
```

The interpreter evaluates `pat` in E1 to get the environment E3 and then looks up the name `course`. It does not find it in E3, so it follows the parent pointer to E2, and finds there that it is bound to `'6.01'`.

Similarly, we can set attribute values in the instance. So, if we were to do:

```
pat.name = 'Pat'
pat.age = 60
pat.role = 'Professor'
```

we would get this environment structure.

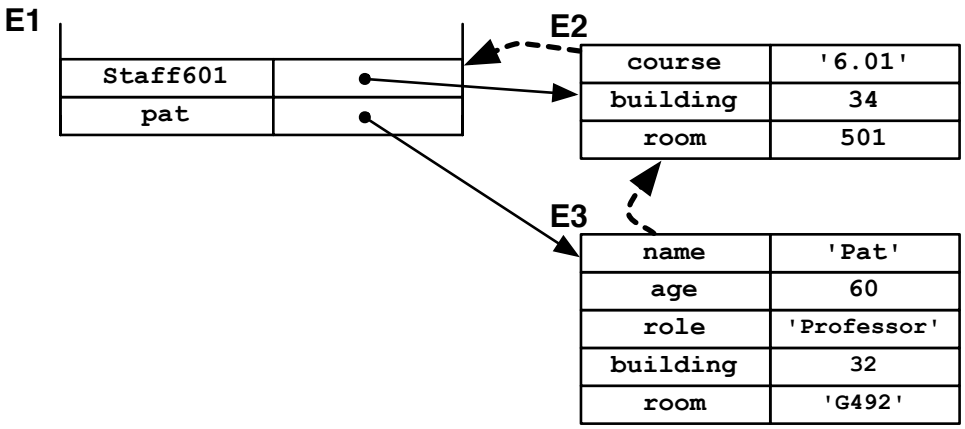


Note that these names are bound in the instance environment, not the class.

These structures are quite flexible. If we wanted to say that Professor Pat is an exception, holding office hours in a different place from the rest of the 6.01 staff, we could say:

```
pat.building = 32
pat.office = 'G492'
```

Here is the new environment state. Nothing is changed in the `Staff601` class: these assignments just make new bindings in `pat`'s environment, which 'shadow' the bindings in the class, so that when we ask for `pat`'s building, we get 32.



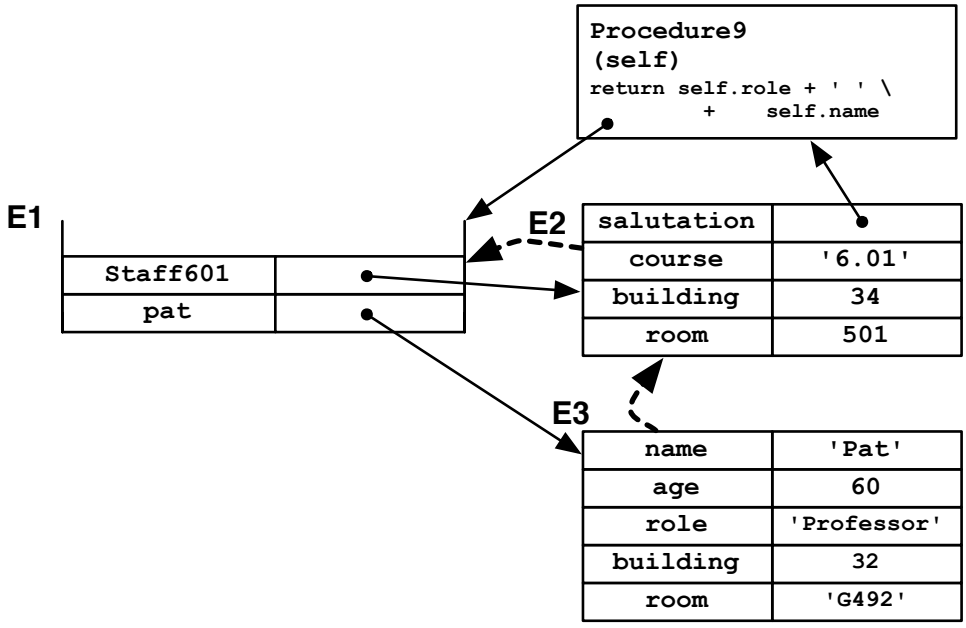
3.5.2 Methods

Objects and classes are a good way to organize procedures, as well as data. When we define a procedure that is associated with a particular class, we call it a *method* of that class. Method definition requires only a small variation on our existing evaluation rules.

So, imagine we want to be able to greet 6.01 staff members appropriately on the staff web site. We might add the definition of a `salutation` method:

```
class Staff601:
    course = '6.01'
    building = 34
    room = 501

    def salutation(self):
        return self.role + ' ' + self.name
```

This procedure definition, made inside the class definition, is evaluated in *almost* the standard way, resulting in a binding of the name `salutation` in the class environment to the new procedure. The way in which this process deviates from the standard procedure evaluation process is that the environment stored in the procedure is the module (file) environment, no matter how deeply nested the class and method definitions are:



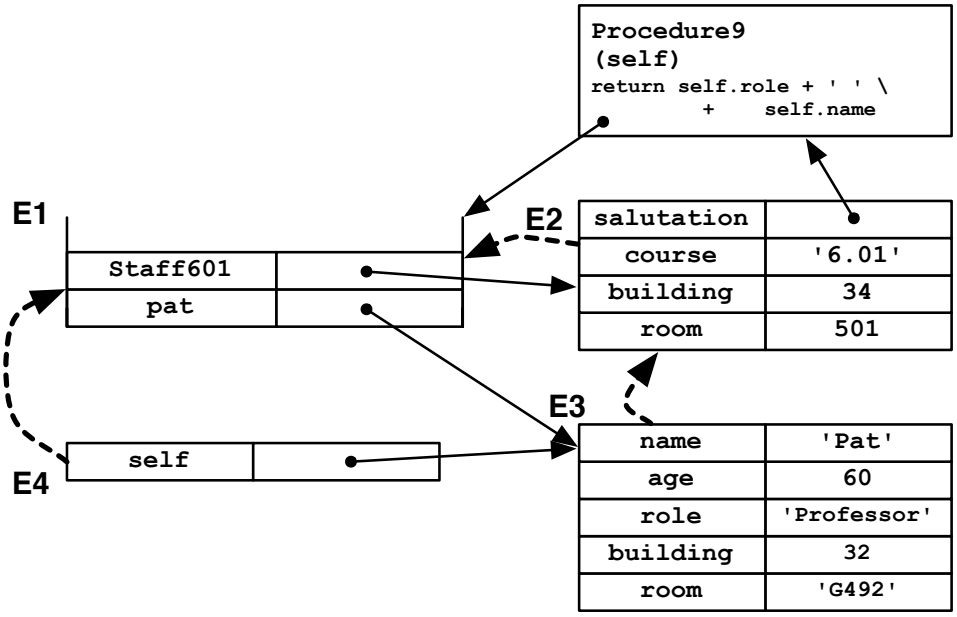
Now, for example, we could do:

```
Staff601.saluation(pat)
```

The interpreter finds that `Staff601` is an environment, in which it looks up the name `saluation` and finds `Procedure9`. To call that procedure we follow the same steps as in **section 3.4.2** :

- Evaluate `pat` to get the instance E3.
- Make a new environment, E4, binding `self` to E3. The parent of E4 is E1, because we are evaluating this procedure call in E1.
- Evaluate `self.role + ' ' + self.name` in E4.
- In E4, we look up `self` and get E3, look up `role` in E3 and get `'Professor'`, etc.
- Ultimately, we return `'Professor Pat'`.

Here is a picture of the binding environments while we are evaluating `self.role + ' ' + self.name`.



Note (especially for Java programmers!) that the way the body of `salutation` has access to the attributes of the instance on which it is operating and to attributes of the class is via the instance we passed to it as the parameter `self`. The parent environment is `E1`, which means that methods cannot simply use the names of class attributes without accessing them through the instance.

The notation

```
Staff601.salutation(pat)
```

is a little clumsy; it requires that we remember to what class `pat` belongs, in order to get the appropriate `salutation` method. We ought, instead, to be able to write

```
pat.salutation(pat)    #### Danger:  not legal Python
```

This should have exactly the same result. (Verify this for yourself if you do not see it, by tracing through the environment diagrams. Even though the `pat` object has no binding for `saluation`, the environment-lookup process will proceed to its parent environment and find a binding in the class environment.)

But this is a bit redundant, having to mention `pat` twice. So, Python added a special rule that says: *If you access a class method by looking in an **instance**, then that instance is automatically passed in as the first argument of the method.*

So, we can write

```
pat.salutation()
```

This is **exactly** equivalent to

```
Staff601.salutation(pat)
```

A consequence of this decision is that every method must have an initial argument that is an instance of the class to which the method belongs. That initial argument is traditionally called `self`, but it is not necessary to do so.

Here is a new method. Imagine that `Staff601` instances have a numeric `salary` attribute. So, we might say

```
pat.salary = 100000
```

Now, we want to write a method that will give a 6.01 staff member a k-percent raise:

```
class Staff601:
    course = '6.01'
    building = 34
    room = 501

    def salutation(self):
        return self.role + ' ' + self.name

    def giveRaise(self, percentage):
        self.salary = self.salary + self.salary * percentage
```

As before, we could call this method as

```
Staff601.giveRaise(pat, 0.5)
```

or we could use the short-cut notation and write:

```
pat.giveRaise(0.5)
```

This will change the `salary` attribute of the `pat` instance to 150000.[23]

3.5.3 Initialization

When we made the `pat` instance, we first made an empty instance, and then added attribute values to it. Repeating this process for every new instance can get tedious, and we might wish to guarantee that every new instance we create has some set of attributes defined. Python has a mechanism to streamline the process of initializing new instances. If we define a class method with the special name `__init__`, Python promises to call that method whenever a new instance of that class is created.

We might add an initialization method to our `Staff601` class:

```
class Staff601:
    course = '6.01'
    building = 34
    room = 501
```

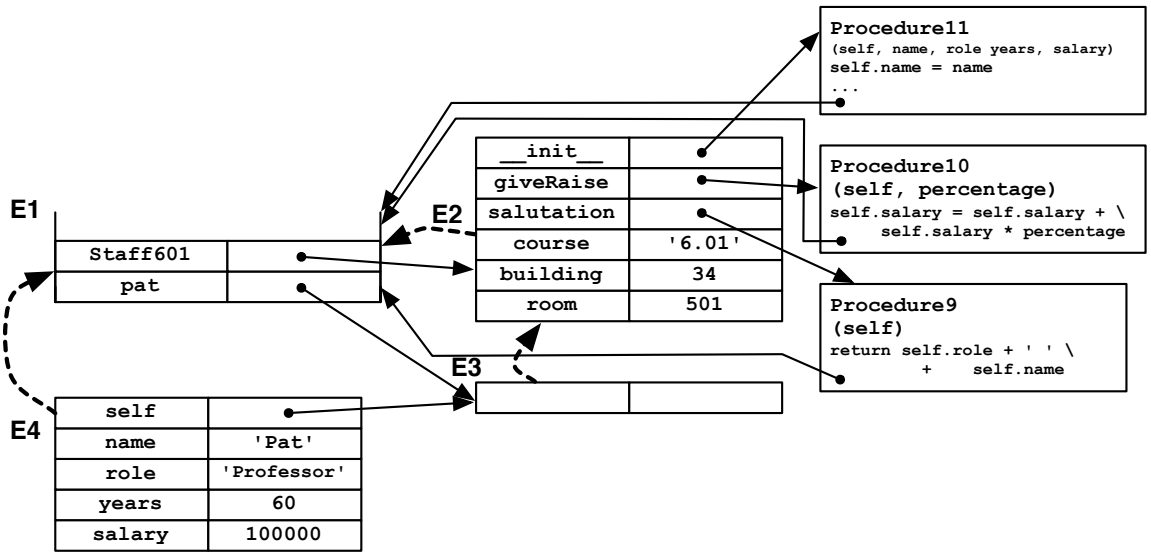
[23] Something to watch out for!!! A common debugging error happens when you: make an instance of a class (such as our `pat` above); then change the class definition and re-evaluate the file; then try to test your changes using your old instance, `pat`. Instances remember the definitions of all the methods in their class *when they were created*. So if you change the class definition, you need to make a new instance (it could still be called `pat`) in order to get the new definitions.

```
def __init__(self, name, role, years, salary):
    self.name = name
    self.role = role
    self.age = years
    self.salary = salary

def salutation(self):
    return self.role + ' ' + self.name

def giveRaise(self, percentage):
    self.salary = self.salary + self.salary * percentage
```

Now, to create an instance, we would do:[24]

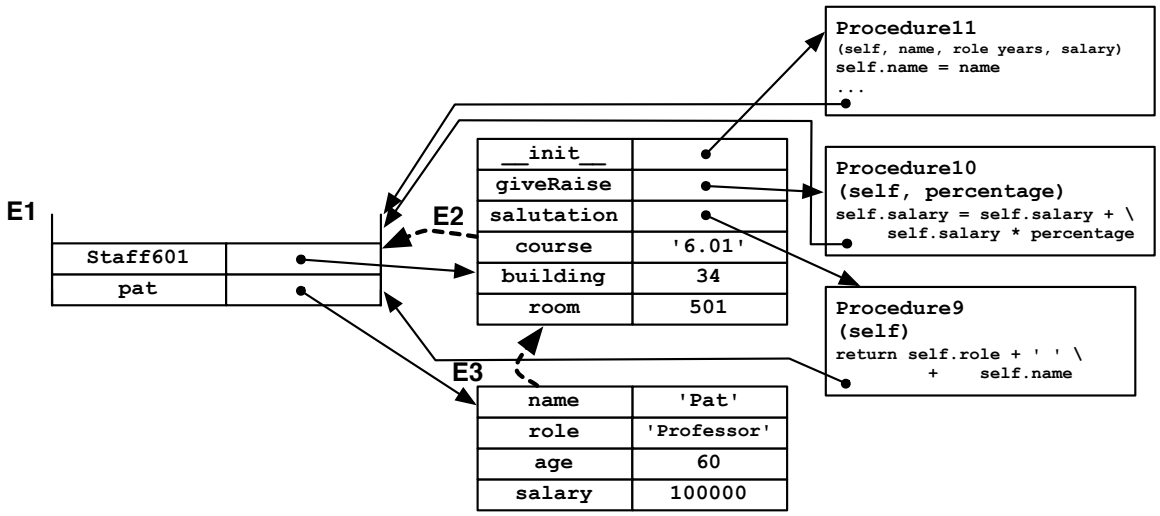
```
pat = Staff601('Pat', 'Professor', 60, 100000)
```

Here is a diagram of the environments when the body of the `__init__` procedure is about to be executed:



Note that the formal parameter `self` has been bound to the newly-created instance. Here is the situation after the initialization method has finished executing:

[24] We called the fourth formal parameter `years`, when `age` would have been clearer, just to illustrate that the names of formal parameters do not have to match the attributes to which they are bound inside the object.



This method seems very formulaic, but it is frequently all we need to do. To see how initialization methods may vary, we might instead do something like this, which sets the salary attribute based on the role and age arguments passed into the initializer.

```
class Staff601:
    def __init__(self, name, role, age):
        self.name = name
        self.role = role
        if self.role == 'Professor':
            self.salary = 100000
        elif self.role = 'TA':
            self.salary = 30000
        else:
            self.salary = 10000
        self.salary = self.giveRaise((age - 18) * 0.03)
```

3.5.4 Inheritance

We see that we are differentiating among different groups of 6.01 staff members. We can gain clarity in our code by building that differentiation into our object-oriented representation using *subclasses* and *inheritance*.

At the mechanism level, the notion of a subclass is very simple: if we define a class in the following way:

```
def class <className>(<superclassName):
    <body>
```

then, when the interpreter makes the environment for this new class, it sets the parent pointer of the class environment to be the environment named by `<superclassName>`.

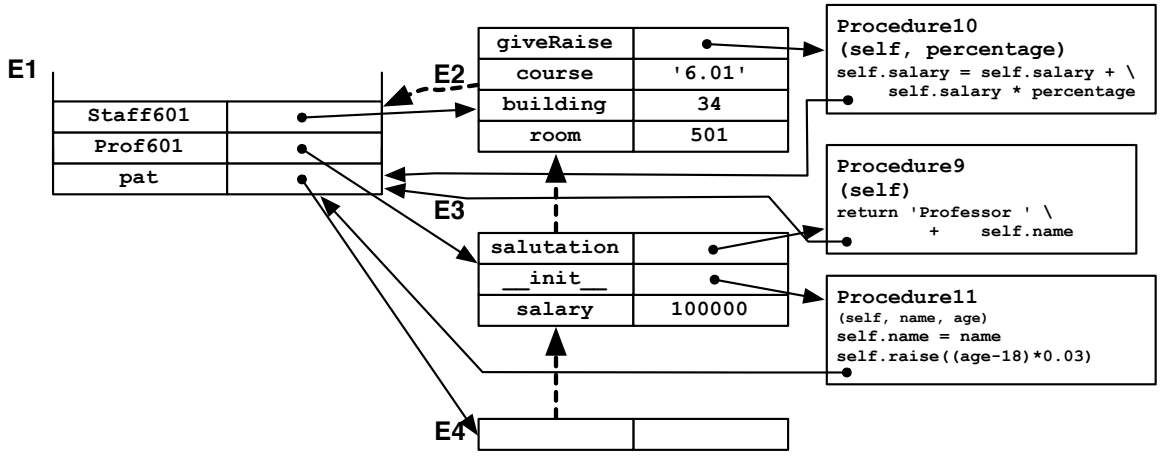
This mechanism allows us to factor class definitions into related, interdependent aspects. For example, in the 6.01 staff case, we might have a base class where aspects that are common to all kinds of staff are stored, and then subclasses for different roles, such as professors:

```
class Staff601:
    course = '6.01'
    building = 34
    room = 501

    def giveRaise(self, percentage):
        self.salary = self.salary + self.salary * percentage

class Prof601(Staff601):
    salary = 100000

    def __init__(self, name, age):
        self.name = name
        self.giveRaise((age - 18) * 0.03)

    def salutation(self):
        return 'Professor' + self.name
```

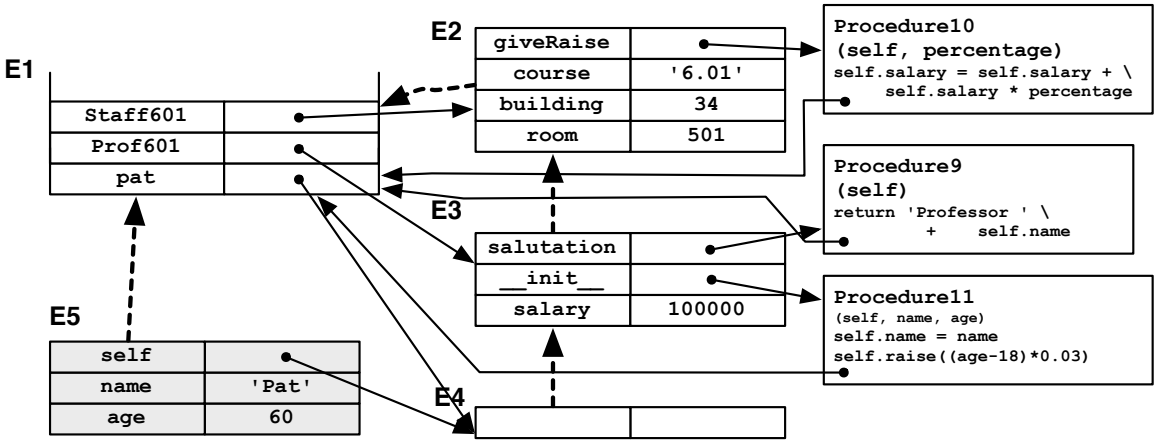
Let's trace what happens when we make a new instance of `Prof601` with the expression

```
Prof601('Pat', 60)
```

First a new environment, E4, is constructed, with E3 (the environment associated with the class `Prof601` as its parent). Here is the memory picture now:



As soon as it is created, the `__init__` method is called, with the new environment E4 as its first parameter and the rest of its parameters obtained by evaluating the expressions that were passed into to `Prof601`, in this case, `'Pat'` and 60. Now, we need to make the procedure-call environment, binding the formal parameters of `Procedure11`; it is E5 in this figure:



We evaluate the body of `Procedure11` in E5. It starts straightforwardly by evaluating

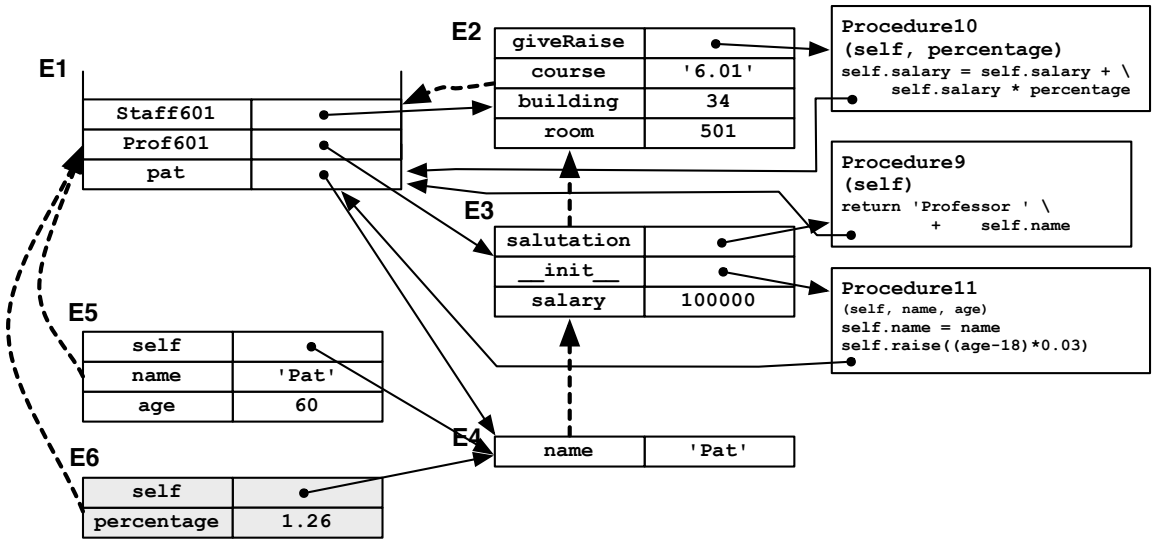
```
self.name = name
```

which creates a binding from `name` to `'Pat'` in the object named `self`, which is E4. Now, it evaluates

```
self.giveRaise((age - 18) * 0.03)
```

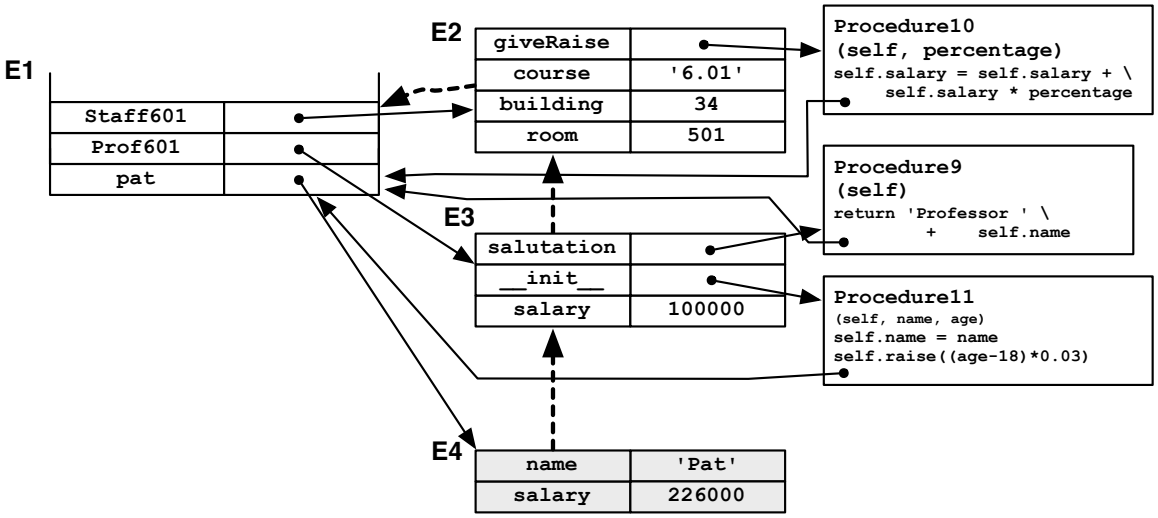
in E5. It starts by evaluating `self.giveRaise`. `self` is E4, so we look for a binding of `giveRaise`. It is not bound in E4, so we look in the parent E3; it is not bound in E3 so we look in E2 and find that it is bound to `Procedure10`. We are taking advantage of the fact that raises are not handled specially for this individual or for the subclass of 6.01 professors, and use the definition in the general class of 6.01 staff. The interpreter evaluates the argument to `Procedure10`, `(60 - 18) * 0.03`, getting 1.26.

It is time, now, to call `Procedure10`. We have to remember that the first argument will be the object through which the method was accessed: E4. So, we make a binding environment for the parameters of `Procedure10`, called E6:



Now the fun really starts! We evaluate `self.salary = self.salary + self.salary * percentage` in E6. We start by evaluating the right hand side: `self` is E4, `self.salary` is 100000, and `percentage` is 1.26, so the right-hand side is 226000. Now, we assign `self.salary`, which means we make a binding in E4 for `salary`, to 22600. It is important to see that, within a method call, all access to attributes of the object, class, or superclass goes through `self`. It is not possible to 'see' the definition of the `building` attribute directly from the `giveRaise` method: if `giveRaise` needed to depend on the `building` attribute, it would need to access it via the object, with `self.building`. This guarantees that we always get the definition of any attribute or method that is appropriate for that particular object, which may have overridden its definition in the class.

When the procedure calls are all done, the environments are finally like this:



It is useful to back up and see the structure here:

- The instance `pat` is an environment, E4.
- The parent of the instance is an environment,E3, which is the class `Prof601`.
- The parent of the class is an environment, E2, which the superclass `Staff601`.

3.5.5 Using Inheritance

Frequently when we're using inheritance, we override some methods of a parent class, and use some of its methods unchanged. Occasionally, we will want to modify or augment an existing method from the parent class. In such a situation, we can 'wrap' that method by writing our own version of that method in the child class, but calling the method of the parent to do some of the work. To call a method `bar` from a parent class named `Foo`, you can do something like the following.

```
def SubclassOfFoo(Foo):
    def bar(self, arg):
        Foo.bar(self, arg)
```

When you call a method using the class name, rather than an object name, you need to pass the object in explicitly to the method call.

Here is a concrete example. Imagine that you have a class for managing calendars. It has these methods, plus possibly some others.

```
class Calendar:
    def __init__(self):
        self.appointments = []
    def makeAppointment(self, name, dateTime):
        self.appointments.append((name, dateTime))
    def printCalendar(self, start, end):
        # complicated stuff
```

Now, you'd like to make a calendar that can handle appointments that are made in different time zones. It should store all of the appointments in a single time zone, so that it can sort and display appropriately.

```
class CalendarTimeZone(Calendar):
    def __init__(self, zone):
        Calendar.__init__(self)
        self.zone = zone

    def makeAppointment(self, name, dateTime):
        Calendar.makeAppointment(self, name, dateTime + zone)
```

We make a subclass of `Calendar` which takes a time zone `zone` as an argument to its initialization method. It starts by calling the `Calendar` initialization method, to set up whatever internal data structures are necessary, and then sets the `zone` attribute of the new object. Now, when it's time to make an appointment, we add the time zone offset to the appointed time, and then call the `makeAppointment` method of `Calendar`.

What is nice about this example is that we were able to take advantage of the `Calendar` class, and even add to some of its methods, without knowing anything about its internal representation of calendars.

3.6 Recursion

There are many control structures in Python, and other modern languages, which allow you write short programs that do a lot of work. In this section, we discuss recursion, which is also a way to write programs of arbitrary complexity. It is of particular importance here, because the structure of a language interpreter is recursive, and in the next section we will explore, in detail, how to construct an interpreter.

We have seen how we can define a procedure, and then can use it without remembering or caring about the details of how it is implemented. We sometimes say that we can treat it as a *black box*, meaning that it is unnecessary to look inside it to use it. This is crucial for maintaining sanity when building complex pieces of software. An even more interesting case is when we can think of the procedure that we are in the middle of defining as a black box. That is what we do when we write a recursive procedure.

Recursive procedures are ways of doing a lot of work. The amount of work to be done is controlled by one or more arguments to the procedure. The way we are going to do a lot of work is by calling the procedure, over and over again, from inside itself! The way we make sure this process actually terminates is by being sure that the argument that controls how much work we do gets smaller every time we call the procedure again. The argument might be a number that counts down to zero, or a string or list that gets shorter.

There are two parts to writing a recursive procedure: the base case(s) and the recursive case. The *base case* happens when the thing that is controlling how much work you do has gotten to its smallest value; usually this is 0 or the empty string or list, but it can be anything, as long as you know it is sure to happen. In the base case, you just compute the answer directly (no more calls to the recursive procedure!) and return it. Otherwise, you are in the *recursive* case. In the recursive case, you try to be as lazy as possible, and foist most of the work off on another call to this procedure, but with one of its arguments getting smaller. Then, when you get the answer back from the recursive call, you do some additional work and return the result.

Another way to think about it is this: when you are given a problem of size n, assume someone already gave you a way to solve problems of size $n-1$. So, now, your job is only to figure out

- To which problem of size $n-1$ you would like to know the answer, and
- How to do some simple operations to make that answer into the answer to your problem of size n.

What if you wanted to add two positive numbers, but your computer only had the ability to increment and decrement (add and subtract 1)? You could do this recursively, by thinking about it like this. I need to add `m` and `n`:

- Presupposing the ability to solve simpler problems, I could get the answer to the problem `m` plus `n-1`.
- Now, I just need to add 1 to that answer, to get the answer to my problem.

We further need to reason that when `n` is 0, then the answer is just `m`. This leads to the following Python definition:

```
def slowAdd(m, n):
    if n == 0:
        return m
    else:
        return 1 + slowAdd(m, n-1)
```

Note how in the final `return` expression, we have reduced the answer to a problem of size `n` to a simpler version of the same problem (of size `n-1` plus some simple operations.

Here is an example recursive procedure that returns a string of n 1's:

```
def bunchaOnes(n):
    if n == 0:
        return ''
    else:
        return bunchaOnes(n-1) + '1'
```

The thing that is getting smaller is `n`. In the base case, we just return the empty string. In the recursive case, we get someone else to figure out the answer to the question of `n-1` ones, and then we just do a little additional work (adding one more `'1'` to the end of the string) and return it.

Exercise 3.16. What is the result of evaluating

```
bunchaOnes(-5)
```

Here is another example. It is kind of a crazy way to do multiplication, but logicians love it.

```
def mult(a,b):
    if a==0:
        return 0
    else:
        return b + mult(a-1,b)
```

Trace through an example of what happens when you call `mult(3, 4)`, by adding a print statement that prints arguments `a` and `b` as the first line of the procedure, and seeing what happens.

Here is a more interesting example of recursion. Imagine we wanted to compute the binary representation of an integer. For example, the binary representation of 145 is '10010001'. Our procedure will take an integer as input, and return a string of 1's and 0's.

```
def bin(n):
    if n == 0:
        return '0'
    elif n == 1:
        return '1'
    else:
        return bin(n/2) + bin(n%2)
```

The easy cases (base cases) are when we are down to a 1 or a 0, in which case the answer is obvious. If we do not have an easy case, we divide up our problem into two that are easier. So, if we convert `n/2` (the integer result of dividing `n` by 2, which by Python's definition will be a smaller number since it truncates the result, throwing away any remainder), into a string of digits, we will have all but the last digit. And `n%2` (`n` modulo 2) is 1 or 0 depending on whether the number is even or odd, so one more call of `bin` will return a string of '0' or '1'. The other thing that is important to remember is that the `+` operation here is being used for string concatenation, not addition of numbers.

How do we know that this procedure is going to terminate? We know that the number on which it is operating is a positive integer that is getting smaller and smaller, and will eventually be either a 1 or a 0, which can be handled by the base case.

You can also do recursion on lists. Here a way to add up the values of a list of numbers:

```
def addList(elts):
    if elts == []:
        return 0
    else:
        return elts[0] + addList(elts[1:])
```

The `addList` procedure consumed a list and produced a number. The `incrementElements` procedure below shows how to use recursion to do something to every element of a list and make a new list containing the results.

```
def incrementElements(elts):
    if elts == []:
        return []
    else:
        return [elts[0]+1] + incrementElements(elts[1:])
```

If the list of elements is empty, then there is no work to be done, and the result is just the empty list. Otherwise, the result is a new list: the first element of the new list is the first element of the old list, plus 1; the rest of the new list is the result of calling `incrementElement` recursively on the rest of the input list. Because the list we are operating on is getting shorter on every recursive call, we know we will reach the base case, and all will be well.

3.7 Implementing an interpreter

This section can be skipped unless you are interested. It's very cool but a bit tricky.

From the preceding sections, you should have an informal understanding of what happens when a Python program is evaluated. Now, we want to understand it formally enough to actually implement an interpreter in Python.

3.7.1 Spy

We will study a simplified language, which we call Spy, because it is a mixture of Scheme and Python. From Scheme, an elegant interpreted programming language with very simple syntax and semantics, we take the syntax of the language, and from Python, we take the basic object-oriented programming system. Spy has a small subset of the features of these languages, but it is powerful enough to implement any computer program that can be implemented in any other language (though it could be *mighty* tedious).

To learn more: Interestingly, very minimal versions of several different models of computation (imperative, functional/recursive, rewrite systems) have been shown to be formally equivalent. Some have theorized them to be complete in the sense that there are no computations that cannot be expressed this way. For more information, see:
http://plato.stanford.edu/entries/church-turing/

The syntax of Spy, like Scheme, is *fully-parenthesized prefix* syntax. *Prefix* means that the name of the function or operation comes before its arguments, and *fully-parenthesized* means that there are parentheses around every sub-expression. So, to write `1 + num * 4`, we would write

```
(+ 1 (* num 4))    .
```

The reason for adopting this syntax is that it is very easy to manipulate with a computer program (although some humans find it hard to read). In the following, we will assume that someone has already written a *tokenizer*, which is a program that can consume a stream of characters and break it into "words" for us. In particular, we will assume that the tokenizer can break an input Spy program down into a list of lists (of lists...of elements that are strings or integers). So the expression `(+ 1 (* num 4))` would be converted by the tokenizer into a representation such as: `('+', 1, ('*', 'num', 4))`.

Spy has the following features:

- **Integer constants:**

  `0, 1, -1, 2, -2, ...`
- **Basic built-in functions:**

  `+`, `-`, `*`, `/`, `=`, with meanings you would expect; note that in this language `=` is a test for equality on two integers, which returns a Boolean.
- **Assignment:**

  `(set a 7)` will set (or assign) the value of variable `a` to be `7`
- **Function application:**

  A list of expressions, the first of which is not a special word in Spy, is treated as function application, or function call. So, `(+ 3 a)` would return 3 plus the value of variable `a`, and `(f)` is a call of a function `f` with no arguments. Note that the first element can, itself, be an expression, so

  ```
  ((myFunctionMaker 3) (+ 4 5))
  ```

  is also a valid expression, as long as the value of `(myFunctionMaker 3)` is a function that will consume a single argument (which, in this case, will be the value of the function application `(+ 4 5)`).
- **Function definition:**

  New functions can be defined, much as in Python or Scheme. So,

  ```
  (def myFun (x y) (* (+ x y) y))
  ```

  defines a new function named `myFun` of two arguments, which can be applied with an expression like `(myFun 4 9)`.
- **If:**

  `(if a b c)` will evaluate and return the value of expression `b` if the value of expression `a` is equal to `True`, and otherwise will evaluate and return the value of expression `c`.
- **Compound expression:**

  In Spy, the body of a function definition, and the branches of an `if` expression, are expected to be a single expression; in order to evaluate multiple expressions in one of those locations, we need to group them together with `begin`, like this:

```
(begin (set a 7) (set b 8) (+ a b))  .
```

(The reason we need `begin` to be a special word is so that we can distinguish this list of expressions from a function application, which is also, syntactically, a list of expressions.) The value of a compound expression is the value of its last component. So, in our example, the value of the whole expression would be 15.

Note that the names `begin`, `set`, `def`, and `if` have special meaning in Spy and cannot be used as the names of user-defined functions.

Spy also has some object-oriented features, but we will introduce those in **section 3.7.3**.

Exercise 3.17. Write a Spy procedure that takes two positive integers as input and returns `True` if the first is greater than the second. Use only the primitive functions `=` and `+` and recursion.

Exercise 3.18. Write a Spy procedure that takes `n` as input and returns the nth number in the Fibonacci series.

3.7.2 Evaluating Spy expressions

In this section, we will describe, in complete detail, how it is that a program in Spy can be executed. The basic operations are evaluating expressions, and assigning values to names. In the following sections, we will develop a recursive definition that allows us to compute the value resulting from any Spy program. We will start with simple expressions, and work up to complex ones.

The `spyEval` function will consume an expression and some additional information, and return the value of the expression. It will have the structure of a long set of if-elif-else clauses, each of which tests to see whether the expression has a particular form and performs the appropriate evaluation.

To learn more: An *interpreter* is a program that takes in the text of a computer program and executes it directly. We are going to build an interpreter for Spy in this section, and when we use Python, we use an interpreter. A *compiler* is a program that takes in the text of a computer program and turns it into low-level instructions for a particular computer, which can later be executed. A gross characterization of the difference is that it is easier to debug when working with an interpreter, but that your program runs faster when compiled. For more information start with Wikipedia articles on **Compiler** and **Interpreter_(computing)**.

3.7.2.1 Numbers

What is the value of the program `'7'`? It's 7, of course. So, we can begin defining a function that will consume a Spy expression (described as a list of lists of strings) and return a value.

Spy Eval

```
def spyEval(form, env):
    if isinstance(form, int):
        return form
```

Here, we need to be able to examine an expression to see if it is an integer (the `isinstance` procedure in Python takes any Python object and a type or class, and returns `True` if the object is of that type or class, and `False` otherwise). If it is an integer, then we just return that integer value. (You might have imagined that `form` would be a string, and that we'd have to convert that string to an integer; in fact, the tokenizer, when it finds a token that is a number, converts it for us in advance.)

For now, ignore the `env` argument; we'll explain it soon.

3.7.2.2 Compound expressions

As we discussed above, a list of expressions, where the first one is the word `'begin'` is a compound expression, whose value is the value of the last component. But, for reasons we will see in section 3.7.2.3, it is important to evaluate all of the expressions, not just the last one. So, we can extend our definition of `spyEval` by adding another clause (the ellipsis below is meant to include the text from the previous `spyEval` code):

Spy Eval

```
...
elif form[0] == 'begin':
    val = None
    for entry in form[1:]:
        val = spyEval(entry, env)
    return val
```

3.7.2.3 Symbols

We'll use the term *symbol* to refer to any syntactic item that isn't a number or parenthesis.

What is the value of the program `'a'`? All by itself, it is an error, because `'a'` is undefined. So, let's consider a more complicated program:

```
(begin (set a 6)
       a)
```

Why would we ever want to evaluate expressions and throw their values away? In a *pure functional* language, the answer is that we wouldn't. But in Spy, we have assignment expressions.

They don't even have a value, but they make a change to the environment in which they are evaluated. Before we go any further, we'll have to take a slight detour, in order to understand the idea of environments.

Environments

An *environment* consists of two parts:

- a dictionary
- a parent environment (can be `None`)

The dictionary is a data structure that lets us associate values (numbers, functions, objects, etc.) with symbols; a Python dictionary works perfectly for this purpose.

We will say that a symbol is *bound* in an environment if it exists as a key in the environment's dictionary. We'll sometimes call the relationship between a key and a value in an environment's dictionary a *binding*.

There are three operations we can do on an environment: look up a symbol, add a binding, and extend an environment. We'll define the first two here, and return to environment extension later.

The *lookup* operation takes a symbol and an environment, and performs a slightly generalized version of a dictionary lookup, looking in a parent environment if the symbol is not in the dictionary:

- If the symbol is a key in the environment's dictionary, then it returns the associated value;
- Otherwise, if this environment has a parent, then it returns the result of looking the symbol up in the parent environment;
- Otherwise, it generates an error.

The *add binding* operation takes a symbol, an environment, and a value; it adds the value to the dictionary associated with the environment, using the symbol as the key. If there was already a value associated with that symbol in the dictionary, the old value is overwritten by the new value.

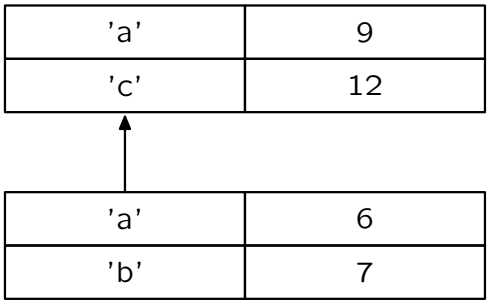


Figure 3.1 A simple environment.

Exercise 3.19. If `e` is the environment in **Figure 3.1**, what are the results of:

- `e.lookup('a')`
- `e.lookup('c')`
- `e.lookup('d')`
- `e.add('f', 1)`
- `e.add('c', 2)`
- `e.add('a', 2)`

Where do environments come from? There is one environment that is made by the interpreter, called `global`. It contains bindings for the primitive procedures, and is the environment in which our main program is evaluated.

Assigning values

So, let's go back to the expression

```
(set a 6)
```

We'll refer to the second and third elements of this expression (the `'a'` and the 6) as the left-hand-side (lhs) and right-hand-side (rhs) respectively.[25] For now, the lhs will always be a symbol. Now, to evaluate this expression in an environment, we evaluate the expression that is the rhs to get a value, and then bind the symbol that is the lhs to that value in the environment.

So, here's our augmented definition of `eval`. Note that it we have added the `env` argument, which is the environment in which we're evaluating the expression:

Spy Eval

```
elif form[0] == 'set':
    (tag, lhs, rhs) = form
    env.add(lhs, spyEval(rhs, env))
```

So, in our simple example, `lhs` is the symbol `'a'`; `expr` is 6, which does not need further evaluation (the tokenizer has already turned the string `'6'`) into an internal Python integer 6. More generally, though, the `expr` can be an arbitrarily complicated expression which might take a great deal of work to evaluate.

After we have evaluated that expression, our global environment will look like this:

global

'a'	6
'+'	<func>
⋮	

Evaluating symbols

Now, finally, we can evaluate the expression `'a'`. All we do to evaluate a symbol is look it up in the environment. So, the return value of this whole program

```
(begin (set a 6)
       a)
```

is the value 6.

[25] That usage comes because a typical assignment statement has the form `lhs = rhs`.

So, now, we can add another clause to `spyEval`:

Spy Eval

```
...
elif isinstance(form, str):
    return env.lookup(form)
```

3.7.2.4 Functions

In the Spy language, any list of expressions, the first element of which is not a special symbol is treated as a function call.[26] Each of the expressions in the list is evaluated, yielding a function and some number of values.

Before we go any further, we have to talk about functions in some detail. There are two types of functions: *primitive functions* and *user defined functions*. Primitive functions come built into the interpreter and are, ultimately, implemented as part of the interpreter. Spy provides several primitive functions that are bound to names in the global environment. So, to call a primitive function, all we have to do is pass the values we obtained by evaluating the rest of the expressions in the function call into the primitive function.

Function definitions

Before we can talk about calling user-defined functions, we have to talk about how to define them. Here is an example definition:

```
(def fizz (x y)
    (+ x y))
```

It is a list of four elements:

1. `def`, which is a special symbol, indicating that this expression is not a function call, and therefore requires special handling;
2. the *name* of the function being defined (`'fizz'` in this case);
3. a list of symbols, called the *formal parameters* (in this example, `('x', 'y')`; and
4. a function *body*, which is a Spy expression, in this case, `'(+ x y)'`

Not very much happens at function definition time. We construct a data structure (typically an instance of a `Function` class) that simply stores three components:

- the formal parameters
- the function body
- the environment in which the function definition was evaluated

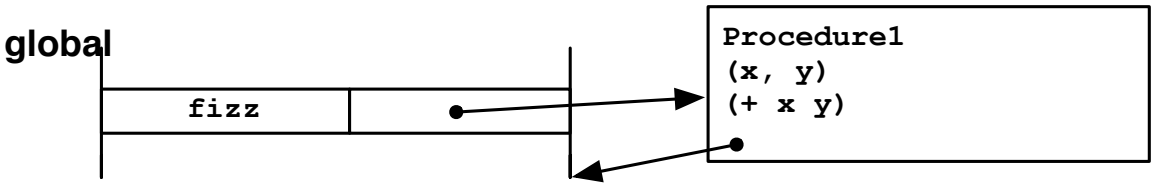
Then, we add a binding in the environment in which the function was defined from the function name to this function structure. With this understanding of function definition, we can add another clause to `spyEval`:

[26] So far, our special symbols are `begin` and `set`, and we'll add `def` and `if`.

Spy Eval

```
...
elif form[0] == 'def':
    (tag, name, params, body) = form
    env.add(name, Function(params, body, env))
```

Here is a picture of the global environment after the definition of `fizz` is made. Note the binding from the name `'fizz'` to a `Function` instance which, itself, contains a reference to the environment.



Function calls

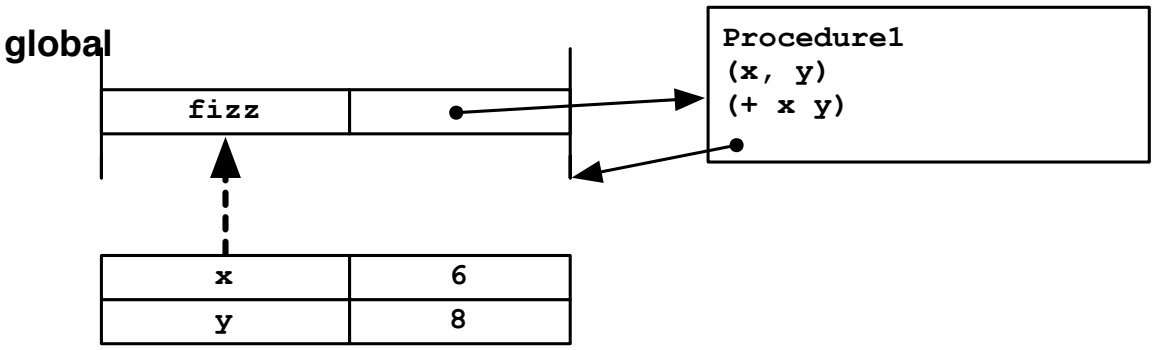
Now, we can see what happens when we actually call a function. Let's assume that the function `fizz` from the previous section has been defined, and now we want to evaluate

```
(fizz 6 8)
```

in the environment `env`. This is the trickiest part of the whole interpreter. First, we evaluate each of these expressions, getting: a `Function` instance (which we created when we defined the `fizz` function), the number 6 and the number 8.

Now, we make a new environment. Let's call it (imaginatively) `newEnv`. The dictionary part of `newEnv` is used to associate the symbols that are the formal parameters of the function (in the case of the `fizz` function, `'x'` and `'y'`) with the actual values being passed into the function (in this case, the numbers 6 and 8). The parent environment of `newEnv` is the environment that we stored with the function when it was defined (not the environment in which the function is being called!!). In this case, `fizz` was defined in the global environment. And so, the parent pointer is also the global environment. (Later, we may see some examples where this pointer is different, in a way that matters.)

Here is a diagram of the entire environment, at this point:



Now that we have this new environment, we evaluate the expression that is the body of the function, in this case, `'(+ x y)'` in `newEnv`. This will all work out roughly as you would expect: during the process of evaluating the body, we will have to evaluate the symbols `'+'`, `'x'` and `'y'`. We will find `'x'` and `'y'` in the dictionary of `newEnv`; and we will find `'+'` in its parent environment, which is `global`. So, we'll have the function object that is bound to `'+'`, and the number 6 and the number 8; then, we'll call the primitive function on these values, and end up with the result 14.

Here is our augmented definition of spyEval:

Spy Eval

```
...
elif isinstance(form, list):
    f = spyEval(form[0], env)
    return spyEval(f.body,
                   Environment(f.formal,
                               [spyEval(x, env) for x in form[1:]],
                               f.environment))
```

As required, this process evaluates the first expression in the list to get a function. Then it evaluates the body of that function in a new environment that maps the function's formal parameters to the results of evaluating all the rest of the expressions in the list, and whose parent is the environment stored in the function.

A good way to understand what is going on with `spyEval` is to show a *trace* of its behavior. So, when we evaluate the following expression:

```
(begin (def fizz (a b)
         (+ a b))
       (fizz 3 4))
```

we can print out the arguments that are passed into `spyEval` (though we have left out the environment, to keep things from being too cluttered), and the result of each call. Each time it is called recursively, we print out the arguments and results with one level more of indentation:

```
args: ['begin', ['def', 'fizz', ['a', 'b'], ['+', 'a', 'b']], ['fizz', 3, 4]]
    args: ['def', 'fizz', ['a', 'b'], ['+', 'a', 'b']]
    result: None
    args: ['fizz', 3, 4]
```

```
        args: fizz
        result: <__main__.Function instance at 0x7b1e18>
        args: 3
        result: 3
        args: 4
        result: 4
        args: ['+', 'a', 'b']
            args: +
            result: Primitive <built-in function add>
            args: a
            result: 3
            args: b
            result: 4
        result: 7
    result: 7
result: 7
```

Here is another version, showing the basic part of the environment, but not the parent environment.

```
args: ['begin', ['def', 'fizz', ['a', 'b'], ['+', 'a', 'b']], ['fizz', 3, 4]] Env: global
    args: ['def', 'fizz', ['a', 'b'], ['+', 'a', 'b']] Env: global
    result: None
    args: ['fizz', 3, 4] Env: global
        args: fizz Env: global
        result: <__main__.Function instance at 0x7b1df0>
        args: 3 Env: global
        result: 3
        args: 4 Env: global
        result: 4
        args: ['+', 'a', 'b'] Env:{'a': 3, 'b': 4}
            args: + Env:{'a': 3, 'b': 4}
            result: Primitive <built-in function add>
            args: a Env:{'a': 3, 'b': 4}
            result: 3
            args: b Env:{'a': 3, 'b': 4}
            result: 4
        result: 7
    result: 7
result: 7
```

3.7.2.5 If

The last special type of expression in Spy is a conditional, of the form:

```
(if (= x 3)
    (fizz x 10)
    (+ x 4))
```

It might seem, at first, that it would be okay to implement `if` as a primitive function, similar to `+`. But there is an important reason not to do so. Consider the definition of the factorial function, in Spy:

```
(def factorial (x)
  (if (= x 1)
      1
      (* x (factorial (- x 1)))))
```

The most straightforward application of this function, `(factorial 1)` will get us into trouble. Why? We would start by evaluating `'factorial'` and `'1'`, and getting a function and 1. So far so good. Now we make a new environment, and evaluate the body in that environment. That requires evaluating all of the elements of the body in that environment. So, we'd find that `'if'` evaluates to a primitive function, that `(= x 1)` evaluates to `True`, and that `1` evaluates to 1. Then, we'd need to evaluate that last expression, which is itself a function call. That, in itself, is no problem. But we can see that eventually, we'll have to evaluate factorial in an environment where its argument is -1. And that will require evaluating factorial of -2, which will require evaluating factorial of -3, and so on.

The most important thing about `'if'` is that *it evaluates only one of its branches, depending on whether its condition is True or False*. Without this property, we cannot stop recursion.

So, now we can see what to do with an `'if'` expression. We'll call the second, third, and fourth elements the *condition*, *then* and *else* parts. We start by evaluating the condition part. If it is `True`, we evaluate the *then* part, otherwise we evaluate the *else* part.

It is pretty straightforward to add this to our `eval` function; but note that we had to put it before the function application clause, so that we can keep `if` expressions from being evaluated as if they were function applications.

Here is the whole interpreter.

Spy Eval

```
def spyEval(form, env=globalEnv):
    if isinstance(form, int):
        return form
    elif isinstance(form, str):
        return env.lookup(form)
    elif form[0] == 'begin':
        val = None
        for entry in form[1:]:
            val = spyEval(entry, env)
        return val
    elif form[0] == 'set':
        (tag, lhs, rhs) = form
        env.add(lhs, spyEval(rhs, env))
    elif form[0] == 'def':
        (tag, name, params, body) = form
        env.add(name, Function(params, body, env))
    elif form[0] == 'if':
        (tag, condition, ifBody, elseBody) = form
        if spyEval(condition, env)
            return spyEval(ifBody, env)
        else:
            return spyEval(elseBody, env)
    elif isinstance(form, list):
        f = spyEval(form[0], env)
        return spyEval(f.body,
                       Environment(f.formal,
                                   [spyEval(x, env) for x in form[1:]],
                                   f.environment))
    else:
        Error("Illegal expression: "+str(form))
```

Yay! Now we have a complete Spy interpreter. Well, we still need implementations of the `Environment` and `Function` classes, but that's not very much more work. We can implement those classes as exercises.

3.7.3 Object-Oriented Spy

We can add a simple object-oriented facility to Spy, which is modeled on Python's OOP facility, but is somewhat simpler. The crucial idea here is that *classes and instances are both environments*, of exactly the same kind that we have been using to support binding and function calls in basic Spy. The dictionary part of the environment supports the binding of attribute names to values within the instance or class; and the parent environment part allows an instance to be connected to its class, or a class to its superclass, so that attributes that are not defined within the instance may be found in the class or superclass.

We only have to add two new syntactic features to the language: attribute lookup and class definition.

3.7.3.1 Attribute lookup

In Python, when you want to get the value of an attribute `a` from an instance `obj`, you say `obj.a`. In OOSpy, we'll say `(attr obj a)`. Remembering that an instance is an environment, all we have to do to evaluate such an expression is to look up the symbol `'a'` in the environment `'obj'`.

Referring to the second part of this form as the `object` part and the third as the `name` part, we can add a clause for `attr` expressions to our interpreter. The name will always be a single symbol; but the object part can be a general expression (we might, for example, call a function `bigFun` that returns an object, yielding an expression like `(attr (bigFun 3) x)` to get the attribute named `x` from the object returned by `bigFun`). This means that we have to evaluate the object part, using our standard evaluation function. Now, we can add a clause for `attr` expressions to our `spyEval`:

Spy Eval

```
...
elif form[0] == 'attr':
    (tag, objectExpr, name) = form
    return spyEval(objectExpr, env).lookup(name)
...
```

3.7.3.2 Class definition

All the rest of our work happens at class definition time. Here is a simple OOSpy class definition:

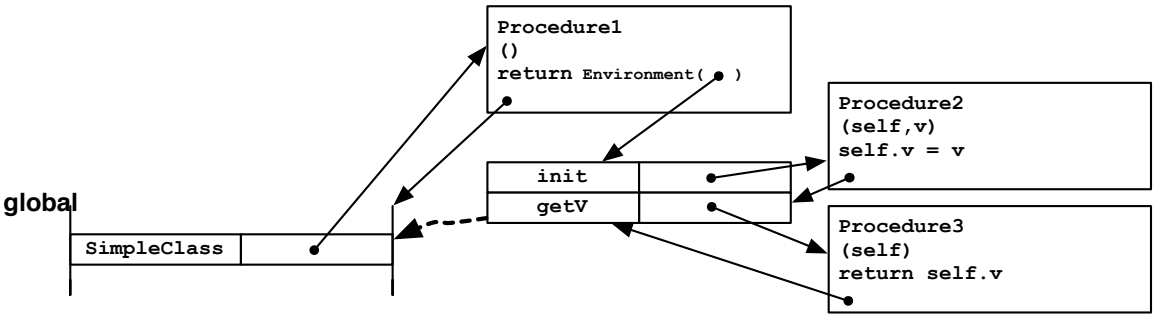
```
(class SimpleClass None
    (begin (def init (self v)
              (set (attr self v) v))
           (def getV (self)
               (attr self v))))
```

For reference, it is roughly equivalent to the Python:

```
class SimpleClass:
    def init (self, v):
        self.v = v
    def getV (self):
        return self.v
```

It has four parts: the special symbol `'class'`, the class name `SimpleClass`, the name of a superclass (in this case, there is no superclass, so we write `None`), and a class body, which is a Spy expression. Our simple Spy implementation doesn't have an equivalent to Python's `__init__` facility, in which the specially-named procedure is called automatically. So we call the method `init` without underscores to make that clear.

Here is a picture of the environment that we would like to have result from this definition:



Note that Spy differs from Python here in an important way: in Python, the environment stored in a method is always the module environment; in Spy, to avoid having a special case, it is the class environment in which it is defined.

Also, rather than binding `SimpleClass` directly to the environment for the class, we will bind it to a procedure that makes a new environment whose parent is the class environment. This procedure can be called directly to create a new instance of the class.

There are three steps involved in processing this class definition.

1. *Make an environment for the class* In this step, we just make a new empty environment, and set its parent to be the class specified as the superclass of this class. If the superclass is `None`, then we use `global` as the parent environment, so that global variables and primitive function definitions will be accessible from the class body.
2. *Evaluate the class body in the class environment* In this step, we use our regular Spy evaluation mechanism, to evaluate the class body, in the new environment. In our `SimpleClass` body, we define two methods; these definitions will appear in the class environment but will not, for example, be accessible directly from the global environment.
3. *Make a constructor function* Finally, we define a new function, with the same name as the class, in the environment in which the class definition is being evaluated (not the class environment; if we did that, nobody would every be able to find the constructor!). The constructor function has the job of making a new instance of this class. What is an instance? An environment. And a brand new instance of any class is simply an empty environment, whose parent environment is the class environment.

After defining `SimpleClass`, we can use it as follows:

```
(begin
    (set a (SimpleClass))
    ((attr a init) a 37)
    ((attr a getV) a))
```

This is roughly equivalent to the Python

```
a = SimpleClass()
a.init(37)
a.getV()
```

This may look a little bit strange. Let's go step by step. First, we make an instance by calling the constructor function. That's pretty straightforward. Now, the value of variable `a` is an instance of the `SimpleClass` class, which means that it is an environment.

Now, what about this expression?

```
((attr a init) a 37)
```

Let's try to work through it the way the interpreter would. It is a list, and the first element is not a special symbol, so it must be a function call. That means we should evaluate each of the forms in the list.

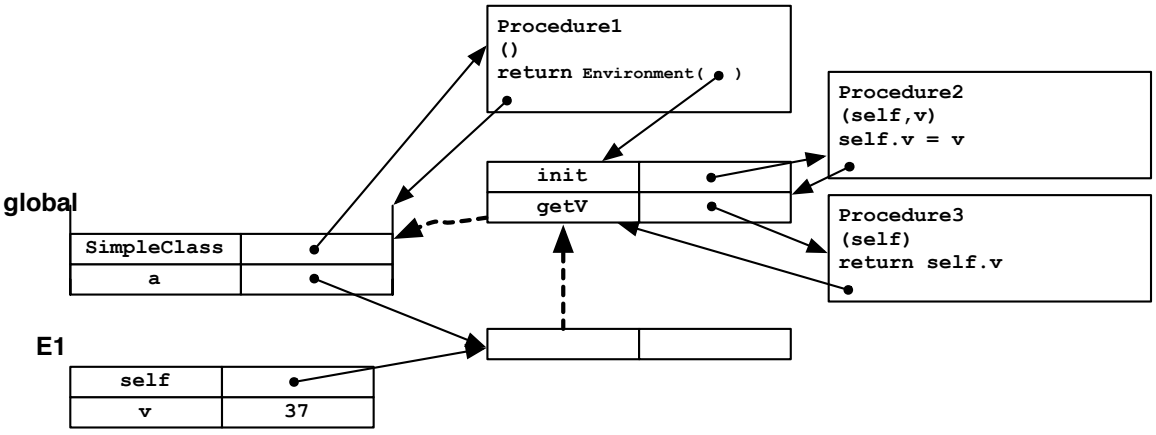
First we evaluate `(attr a init)`, which is a list starting with the special symbol `attr`, so we know it's looking up an attribute in a class. It evaluates the expression `a`, getting the `SimpleClass` instance, then looks for the `init` attribute, which yields the `init` function that we defined inside the class body.

Evaluating the second and third elements of `((attr a init) a 37)` yield our `SimpleClass` instance and the number 37.

Now, we're ready to do a function call. Let's do it carefully. The formal parameters of the function we're calling are `('self', 'v')`. So, we make a new environment, in which those parameters are bound to the `SimpleClass` instance and to 37, respectively, and whose parent environment is the environment in which `((attr a init) a 37)` is being evaluated. Now, we evaluate the body of that function, which is

```
(set (attr self v) v)
```

in this new environment. Let's call this environment `E1`. In order to understand what's going on here, we have to go slowly and carefully. Here's `E1`:

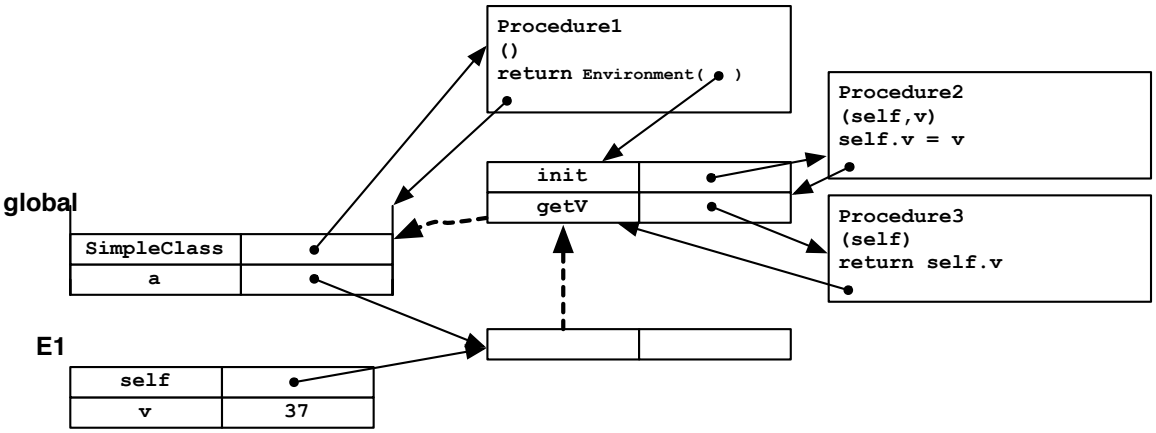


First, we recognize this as a `set` expression. But, so far, our `set` expressions have always had a single symbol as their second element. Now, we will have to generalize it, so the second element of a `set` expression can also be an `attr` expression. Let's consider this kind of expression, in general:

```
(set (attr x y) z)
```

In this form, `x` can be any expression that, when evaluated, yields an instance; `y` must be a single symbol; and `z` can be any expression that, when evaluated, yields a value. The result of this form is to set attribute `y` in the instance resulting from evaluating expression `x` to have the value resulting from evaluating the expression `z`.

So, we need to evaluate `(set (attr self v) v)` in the environment `E1`. We start by evaluating `self`, and getting the `SimpleClass` instance; we look for an attribute named `v` there and, not finding one, make a new one. Now, we evaluate the third part of the `set` expression, which is `v`, in the `E1`, and get the value 37. Finally, we set attribute `v` of our instance to 37. Yay. Here's the new state of the environment:



One thing to notice is that Spy, as we have designed it, doesn't have Python's special facility that automatically uses an object as the first argument of a method, when that method is accessed via that object. So, we have to pass the object in again, explicitly, as the first argument.

Exercise 3.20.

Add this syntactic facility to OOSpy.
That is, make it so that we can write `((attr a init) 37)` and `((attr a getV))`. *This is a little bit tricky.*

Evaluating the expression `((attr a getV) a)` works similarly (you should be sure you understand how), and returns the value 37.

So, after all that, we can add the last clause to our OOSpy interpreter, for handling class definitions. Note that we also have to extend the code for handling `set`, so it can deal with setting attributes of instances.

Spy Eval

```
            ...
            elif form[0] == 'set':
                (tag, lhs, expr) = form
                (targetEnv, name) = lhsEval(lhs, env)
                targetEnv.add(name, spyEval(expr, env))
            elif form[0] == 'class':
                (tag, name, super, body) = form
                if super == 'None':
                    super = globalEnv
                classEnv = Environment(parent = super)
                env.add(name, Primitive(lambda : Environment(parent = classEnv)))
                spyEval(body, classEnv)
            ...

        def lhsEval(lhs, env):
            if isinstance(lhs, list):
                (tag, objectExpr, name) = lhs
                return (spyEval(objectExpr, env), name)
            else:
                return (env, lhs)
```

`Primitive` is a class that represents a primitive function in Spy. Its initializer just takes a procedure as an argument: in this case, it is a procedure that makes a new environment whose parent is the the class environment.

We're done with a whole object-oriented language!

Exercise 3.21. Write an OOSpy class definition for an `Averager` class. It should have a method `addValue` that allows you to "add" a numeric value to the set of data it as seen and method `getAverage` that returns the average of all of the values that have been added so far.

3.8 Object-Oriented Programming Examples

Here are some examples of object-oriented programming in Python.

3.8.1 A simple method example

Here's an example to illustrate the definition and use of classes, instances, and methods.

```
class Square:
    def __init__(self, initialDim):
        self.dim = initialDim

    def getArea (self):
        return self.dim * self.dim
```

```
    def setArea (self, area):
        self.dim = area**0.5

    def __str__(self):
        return "Square of dim " + str(self.dim)
```

This class is meant to represent a square. Squares need to store, or remember, their dimension, so we make an attribute for it, and assign it in the `__init__` method. Now, we define a method `getArea` that is intended to return the area of the square. There are a couple of interesting things going on here.

Like all methods, `getArea` has an argument, `self`, which will stand for the instance that this method is supposed to operate on. Now, the way we can find the dimension of the square is by finding the value of the `dim` attribute of `self`.

We define another method, `setArea`, which will set the area of the square to a given value. In order to change the square's area, we have to compute a new dimension and store it in the `dim` attribute of the square instance.[27]

Now, we can experiment with instances of class `Square`.

```
>>> s = Square(6)
>>> s.getArea()
36
>>> Square.getArea(s)
36
>>> s.dim
6
>>> s.setArea(100)
>>> s.dim
10.0
>>> s1 = Square(10)
>>> s1.dim
10
>>> s1.getArea()
100
>>> s2 = Square(100)
>>> s2.getArea()
10000
>>> print s1
Square of dim 10
```

Our class `Square` has the `__str__` method defined, so it prints out nicely.

3.8.2 Superclasses and inheritance

What if we wanted to make a set of classes and instances that would help us to run a bank? We could start like this:

[27] We compute the square root of a value by raising to the power 0.5.

```
class Account:
    def __init__(self, initialBalance):
        self.currentBalance = initialBalance
    def balance(self):
        return self.currentBalance
    def deposit(self, amount):
        self.currentBalance = self.currentBalance + amount
    def creditLimit(self):
        return min(self.currentBalance * 0.5, 10000000)

>>> a = Account(100)
>>> b = Account(1000000)

>>> Account.balance(a)
100
>>> a.balance()
100
>>> Account.deposit(a, 100)
>>> a.deposit(100)
>>> a.balance()
300
>>> b.balance()
1000000
```

We've made an `Account` class[28] that maintains a balance as an attribute. There are methods for returning the balance, for making a deposit, and for returning the credit limit.

The `Account` class contains the procedures that are common to all bank accounts; the individual instances contain state in the values associated with the names in their environments. That state is *persistent*, in the sense that it exists for the lifetime of the program that is running, and doesn't disappear when a particular method call is over.

Now, imagine that the bank we're running is getting bigger, and we want to have several different kinds of accounts, with the credit limit depending on the account type. If we wanted to define another type of account as a Python class, we could do it this way:

```
class PremierAccount:
    def __init__(self, initialBalance):
        self.currentBalance = initialBalance
    def balance(self):
        return self.currentBalance
    def deposit(self, amount):
        self.currentBalance = self.currentBalance + amount
    def creditLimit(self):
        return min(self.currentBalance * 1.5, 10000000)
>>> c = PremierAccount(1000)
>>> c.creditLimit()
1500.0
```

[28] A note on style. It is useful to adopt some conventions for naming things, just to help your programs be more readable. We've used the convention that variables and procedure names start with lower case letters and that class names start with upper case letters. And we try to be consistent about using something called "camel caps" for writing compound words, which is to write a compound name with the successiveWordsCapitalized. An alternative is_to_use_underscores.

This will let people with premier accounts have larger credit limits. And, the nice thing is that we can ask for its credit limit without knowing what kind of an account it is, so we see that objects support *generic functions*, which operate on different kinds of objects by doing different, but type-appropriate operations.

However, this solution is still not satisfactory. In order to make a premier account, we had to repeat a lot of the definitions from the basic account class, violating our fundamental principle of laziness: never do twice what you could do once; instead, abstract and reuse.

Inheritance lets us make a new class that's like an old class, but with some parts overridden or new parts added. When defining a class, you can actually specify an argument, which is another class. You are saying that this new class should be exactly like the *parent class* or *superclass*, but with certain definitions added or overridden. So, for example, we can say

```
class PremierAccount(Account):
    def creditLimit(self):
        return min(self.currentBalance * 1.5, 10000000)

class EconomyAccount(Account):
    def creditLimit(self):
        return min(self.currentBalance*0.5, 20.00)
```

```
>>> a = Account(100)
>>> b = PremierAccount(100)
>>> c = EconomyAccount(100)
>>> a.creditLimit()
100.0
>>> b.creditLimit()
150.0
>>> c.creditLimit()
20.0
```

We automatically inherit the methods of our superclass (including `__init__`). So we still know how to make deposits into a premier account:

```
>>> b.deposit(100)
>>> b.balance()
200
```

The fact that instances know what class they were derived from allows us to ask each instance to do the operations appropriate to it, without having to take specific notice of what class it is. Procedures that can operate on instances of different types or classes without explicitly taking their types or classes into account are called *polymorphic*. Polymorphism is a very powerful method for capturing common patterns.

3.8.3 A data type for times

Object-oriented programming is particularly useful when we want to define a new compositional data type: that is, a representation for the data defining a class of objects and one or more operations for creating new instances of the class from old instances. We might do this with complex numbers or points in a high-dimensional space.

Here we demonstrate the basic ideas with a very simple class for representing times.

We'll start by defining an initialization method, with default values specified for the parameters.

```
class Time:
    def __init__(self, hours = 0, minutes = 0):
        self.hours = hours
        self. minutes = minutes
```

Here is a method for adding two time objects. If the summed minutes are greater than 60, it carries into the hours. If the summed hours are greater than 24, then the excess is simply thrown away. It is important that adding two `Time`s returns a new `Time` object, and that it *does not change any aspect of either of the arguments.*

```
    def add(self, other):
        newMinutes = (self.minutes + other.minutes) % 60
        carryHours = (self.minutes + other.minutes) / 60
        newHours = (self.hours + other.hours + carryHours) % 24
        return Time(newHours, newMinutes)
```

Python has a special facility for making user-defined types especially cool: if, for instance, you define a method called `__add__` for your class `Foo`, then whenever it finds an expression of the form `<obj1> + <obj2>`, and the expression `<obj1>` evaluates to an object of class `Foo`, it will `Foo`'s `__add__` method. So, here we set that up for times:

```
    def __add__(self, other):
        return self.add(other)
```

Additionally, defining methods called `__str__` and `__repr__` that convert elements of your class into strings will mean that they can be printed out nicely by the Python shell.

```
    def __str__(self):
        return str(self.hours) + ':' + str(self.minutes)
    def __repr__(self):
        return str(self)
```

Here, we make some times. In the second case, we specify only the minutes, and so the hours default to 0.

```
>>> t1 = Time(6, 30)
>>> t2 = Time(minutes = 45)
>>> t3 = Time(23, 59)
```

And now, we can do some operations on them. Note how our `__str__` method allows them to be printed nicely.

```
>>> t1
6:30
>>> t1 + t2
7:15
>>> t3 + t2
0:44
>>> t1 + t2 + t3
```

```
7:14
>>> (t1 + t2 + t3).minutes
14
```

3.8.4 Practice problem: argmax

Write a Python procedure `argmax` that takes two arguments: the first is a list of elements, and the second is a function from elements to numerical values. It should return the element of the list with the highest numerical score.

```
def argmax(elements, f):
    bestScore = None
    bestElement = None
    for e in elements:
        score = f(e)
        if bestScore == None or score > bestScore:
            bestScore = score
            bestElement = e
    return bestElement

def argmax(elements, f):
    bestElement = elements[0]
    for e in elements:
        if f(e) > f(bestElement):
           bestElement = e
    return bestElement

def argmax(elements, f):
    vals = [f(e) for e in elements]
    return elements[vals.index(max(vals))]

def argmax(elements, f):
    return max(elements, key=f])
```

There are many ways of writing this. Here are a few. It's important to keep straight in your head which variables contain scores and which contain elements. Any of these solutions would have been fine; we wouldn't expect most people to get the last one.

3.8.5 Practice problem: OOP

The following definitions have been entered into a Python shell:

```
class A:
    yours = 0
    def __init__(self, inp):
        self.yours = inp
    def give(self, amt):
        self.yours = self.yours + amt
        return self.yours
    def howmuch(self):
        return (A.yours, self.yours)
```

```
class B(A):
    yours = 0
    def give(self, amt):
        B.yours = B.yours + amt
        return A.howmuch(self)
    def take(self, amt):
        self.yours = self.yours - amt
        return self.yours
    def howmuch(self):
        return (B.yours, A.yours, self.yours)
```

Write the values of the following expressions. Write `None` when there is no value; write `Error` when an error results and explain briefly why it is an error. Assume that these expressions are evaluated one after another (all of the left column first, then right column).

```
test = A(5)

test.take(2)

test.give(6)

test.howmuch()
```

```
test = B(5)

test.take(2)

test.give(6)

test.howmuch()
```

- `None`: This makes `test` be equal to a new instance of `A`, with attribute `yours` equal to 5.
- `Error`: Instances of `A` don't have a `take` method.
- `11`: This adds 6 to the value of `test.yours` and returns its value.
- `(0, 11)`: At this point, we haven't changed the class attribute `A.yours`, so it still has value 0.
- `None`: This makes `test` be equal to a new instance of `B`; it inherits the init method from `A`, so at this point `test.yours` is equal to 5.
- `3`: Now, since `test` is an instance of `B`, the method `test.take` is defined; it changes `test.yours` to be 3 and returns its value.
- `(0, 3)`: Note that this refers to the `give` method defined in class `B`. So, first, the amount is used to increment `B.yours` to have value 6. Then, we return `A.yours` and `test.yours`, which are both unchanged.
- `(6, 0, 3)`: This just returns the current values of all three `yours` attributes, one in each class definition as well as `test.yours`.

3.8.6 Practice problem: The Best and the Brightest

Here are some class definitions, meant to represent a collection of students making up the student body of a prestigious university.

```
class Student:
    def __init__(self, name, iq, salary, height):
        self.name = name
        self.iq = iq
        self.salary = salary
        self.height = height
```

```
class StudentBody:
    def __init__(self):
        self.students = []
    def addStudent(self, student):
        self.students.append(student)
    def nameOfSmartest(self):
        # code will go here
    def funOfBest(self, fun, feature):
        # code will go here
```

The `StudentBody` class is missing the code for two methods:

- `nameOfSmartest`: returns the name attribute of the student with the highest IQ
- `funOfBest`: takes a procedure `fun` and a procedure `feature` as input, and returns the result of applying procedure `fun` to the student for whom the procedure `feature` yields the highest value

For the first two problems below, assume that these methods have been implemented.

Here is a student body:

```
jody = Student('Jody', 100, 100000, 80)
chris = Student('Chris', 150, 40000, 62)
dana = Student('Dana', 120, 2000, 70)
aardvarkU = StudentBody()
aardvarkU.addStudent(jody)
aardvarkU.addStudent(chris)
aardvarkU.addStudent(dana)
```

1. What is the result of evaluating `aardvarkU.nameOfSmartest()`?

'Chris'

2. Write a Python expression that will compute the name of the person who has the greatest value of IQ + height in aardvarkU (not just for the example student body above). You can define additional procedures if you need them.

```
aardvarkU.funOfBest(lambda x: x.name, lambda x: x.iq + x.height)
```

The second `lambda` expression specifies how to evaluate an element (that is, to sum its `iq` and `height` attributes) and the first `lambda` expression says what function to apply to the element with the highest score.

3. Implement the `nameOfSmartest` method. For full credit, use `util.argmax` (defined below) or the `funOfBest` method.

If `l` is a list of items and `f` is a procedure that maps an item into a numeric score, then `util.argmax(l, f)` returns the element of `l` that has the highest score.

```
def nameOfSmartest(self):
    return util.argmax(self.students, lambda x: x.iq).name

# or

def nameOfSmartest(self):
    return funOfBest(lambda x: x.name, lambda x: x.iq)
```

Note that in the first solution, the expression `util.argmax(self.students, lambda x: x.iq)` returns the object with the highest `iq` attribute; then we return the `name` attribute of that object.

4. Implement the `funOfBest` method. For full credit, use `util.argmax`.

```
def funOfBest(self, fun, feature):
    return fun(util.argmax(self.students, feature))
```

3.8.7 Practice Problem: A Library with Class

Let's build a class to represent a library; let's call it `Library`. In this problem, we'll deal with some standard types of objects:

- A book is represented as a string – its title.
- A patron (person who uses the library) is represented as a string – his/her name.
- A date is represented by an integer – the number of days since the library opened.

The class should have an attribute called `dailyFine` that starts out as `0.25`. The class should have the following methods:

- `__init__`: takes a list of books and initializes the library.
- `checkOut`: is given a book, a patron and a date on which the book is being checked out and it records this. Each book can be kept for 7 days before it becomes overdue, i.e. if checked out on day x, it becomes due on day $x + 7$ and it will be considered overdue by one day on day $x + 8$. It returns `None`.
- `checkIn`: is given a book and a date on which the book is being returned and it updates the records. It returns a number representing the fine due if the book is overdue and 0.0 otherwise. The fine is the number of days overdue times the value of the `dailyFine` attribute.
- `overdueBooks`: is given a patron and a date and returns the list of books which that patron has checked out which are overdue at the given date.

Here is an example of the operation of the library:

```
>>> lib = Library(['a', 'b', 'c', 'd', 'e', 'f'])
>>> lib.checkOut('a', 'T', 1)
>>> lib.checkOut('c', 'T', 1)
>>> lib.checkOut('e', 'T', 10)
>>> lib.overdueBooks('T', 13)
['a', 'c']
>>> lib.checkIn('a', 13)
1.25
>>> lib.checkIn('c', 18)
2.50
>>> lib.checkIn('e', 18)
0.25
```

In the boxes below, define the `Library` class as described above. Above each answer box we repeat the specification for each of the attributes and methods given above. Make sure that you enter complete definitions in the boxes, including complete `class` and `def` statements.

Use a dictionary to store the contents of the library. Do not repeat code if at all possible. You can assume that all the operations are legal, for example, all books checked out are in the library and books checked in have been previously checked out.

Class definition:

Include both the start of the class definition and the method definition for `__init__` in this first answer box.

The class should have an attribute called `dailyFine` that starts out as `0.25`.

`__init__`: takes a list of books and initializes the library.

```
class Library:
    dailyFine = 0.25
    def __init__(self, books):
        self.shelf = {}
        for book in books:
            self.shelf[book] = (None, None)  # (patron, dueDate)
```

The crucial part here is deciding what information to store. Here, for each book, we store a tuple of who has checked it out, and when it is due.

`checkOut`: is given a book, a patron and a date on which the book is being checked out and it records this. Each book can be kept for 7 days before it becomes overdue, i.e. if checked out on day x, it becomes due on day $x + 7$ and it will be considered overdue by one day on day $x + 8$. It returns `None`.

```
    def checkOut(self, book, patron, date):
        self.shelf[book] = (patron, date+7)
```

`checkIn`: is given a book and a date on which the book is being returned and it updates the records. It returns a number representing the fine due if the book is overdue and 0.0 otherwise. The fine is the number of days overdue times the value of the `dailyFine` attribute.

```
    def checkIn(self, book, date):
        patron, due = self.shelf[book]
        self.shelf[book] = (None, None)
        return max(0.0, (date - due))*self.dailyFine
```

Note the destructuring assignment in the first line. It's not crucial, but it's nice style, and keeps us from having to refer to components like `self.shelf[book][1]`, which are ugly, long, and hard to get right. Instead of using a `max`, you could use an `if` statement to handle the case of the book being overdue or not, but this is more compact and pretty clear.

`overdueBooks`: is given a patron and a date and returns the list of books which that patron has checked out which are overdue at the given date.

```
    def overdueBooks(self, patron, date):
        overdue = []
        for book in self.shelf:
            p, d = self.shelf[book]
            if p and d and p == patron and date > d:
                overdue.append(book)
        return overdue
```

It's not really necessary to check to be sure that `p` and `d` are not `None`, because `p == patron` will only be true for a real patron, and if the patron is not `None` then `d` won't be either. But this code makes it clear that we're only interested in books that have been checked out.

Define a new class called `LibraryGrace` that behaves just like the `Library` class except that it provides a grace period (some number of days after the actual due date) before fines start being accumulated. The number of days in the grace period is specified when an instance is created. See the example below.

```
>>> lib = LibraryGrace(2, ['a', 'b', 'c', 'd', 'e', 'f'])
>>> lib.checkOut('a', 'T', 1)
>>> lib.checkIn('a', 13)
0.75
```

Write the complete class definition for `LibraryGrace`. To get full credit you should not repeat any code that is already in the implementation of `Library`, in particular, you should not need to repeat the computation of the fine.

Class definition:

```
class LibraryGrace(Library):
    def __init__(self, grace, books):
        self.grace = grace
        Library.__init__(self, books)
    def checkIn(self, book, date):
        return Library.checkIn(self, book, date - self.grace)
```

The crucial things here are: remembering to call the `__init__` method of the parent class and doing something to handle the grace period. In this example, when we check a book back in, we pretend we're actually checking it in on an earlier date. Alternatively, we could have changed the checkout method to pretend that the checkout was actually happening in the future.

6.01SC Introduction to Electrical Engineering and Computer Science

Spring 2011