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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.013 Electromagnetics and Applications

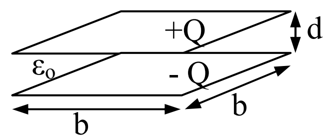
Student Name:

Final Exam Closed book, no calculators

Please note the two pages of formulas provided at the back; the laser and acoustic expressions have been revised slightly. There are 10 problems; some are on the back sides of the sheets. For full credit, please ***simplify all expressions****, present* ***numerical answers to the extent practical*** *without a calculator or tedious computation, and place your* ***final answers within the boxes provided****. You may leave natural constants and trigonometric functions in symbolic form (π, ε_o, μ_o, η_o, h, e, sin(0.9), $\sqrt{2}$, etc.). To receive partial credit, provide all related work on the same sheet of paper and give brief explanations of your answer. Spare sheets are at the back.*

Problem 1. (25/200 points)

Two square capacitor plates in air have separation d, sides of length b, and charge ±Q as illustrated. Fringing fields can be neglected.

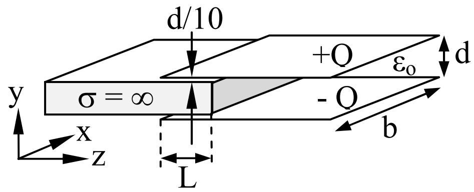


a) What is the capacitance C_a of this device?

C_a =

Please turn sheet over to answer parts (b) and (c).

b) A perfectly conducting plate is introduced between the capacitor plates, leaving parallel gaps of width d/10 above and below itself. What now is the device capacitance C_b when it is fully inserted?



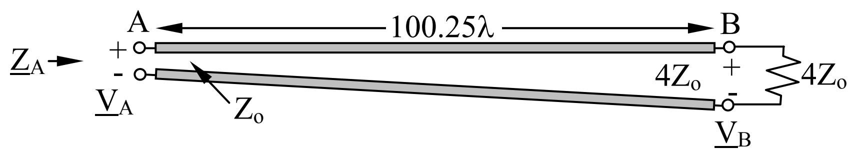
$C_b =$

c) What is the magnitude and direction of the force $\bar{f}$ on the new plate of Part (b) as a function of the insertion distance L. Please express your answer as a function of the parameters given in the figure.

$\bar{f} =$

Problem 2. (20/200 points)

The plate separation of a lossless parallel-plate TEM line many wavelengths long (length D = 100.25λ) very slowly increases from end A to end B, as illustrated. This increases the characteristic impedance of the line from Z_o at the input end A, to $4Z_o$ at the output end B. This transition from A to B is so gradual that it produces no reflections. End B is terminated with a resistor of value $4Z_o$



a) What is the input impedance $\underline{Z}_A$ seen at end A? Explain briefly.

$\underline{Z}_A$ =   Explanation:

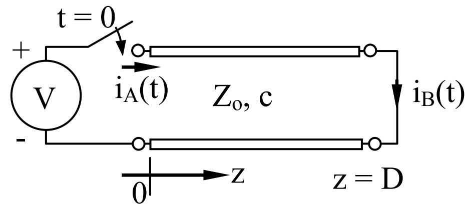
b) If the sinusoidal (complex) input input voltage is $\underline{V}_A$, what is the output voltage $\underline{V}_B$?

$\underline{V}_B$ =

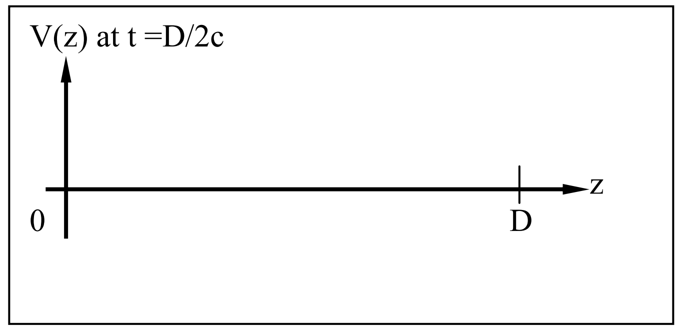
Please turn sheet over for Problem 3.

Problem 3. (25/200 points)

At $t = 0$ a switch connects a voltage V to a passive air-filled short-circuited TEM line of length D and characteristic impedance Z_o, as illustrated. Please sketch and quantify dimension:



a) The line voltage v(z) at $t = D/2c$.

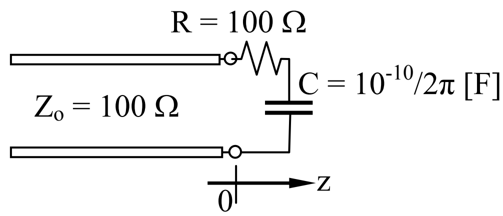


b) The current $i_B(t)$ through the short circuit for $0 < t < 2D/c$.

c) The current $i_A(t)$ from the voltage source ($z = 0$) for $0 < t < 3D/c$.

Problem 4. (30/200 points)

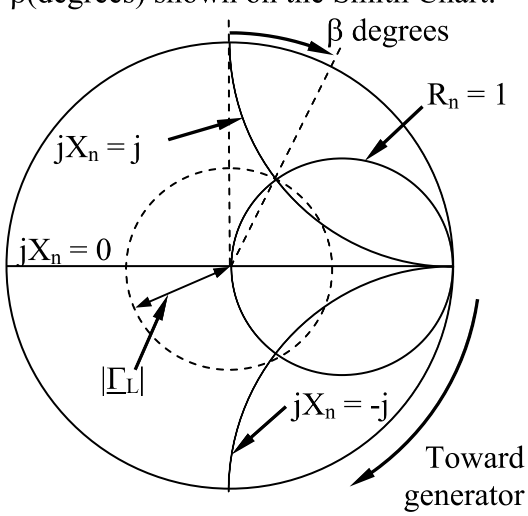
A 100-ohm air-filled lossless TEM line is terminated with a 100-ohm resistor and a $10^{-10}/2\pi$ Farad capacitor in series, as illustrated. It is driven at 100 MHz.



a) What fraction $F = |\underline{\Gamma}_L|^2$ of the incident power is reflected from this load?

F =

b) What is the minimum distance D(meters) from the load at which the line current $|\underline{I}(z)|$ is maximum? You may express your answer in terms of the angle β(degrees) shown on the Smith Chart.



D_{min} =

Please turn sheet over to answer part (c).

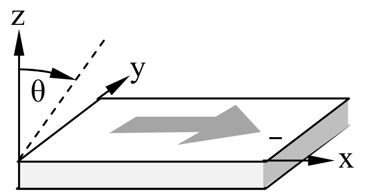
c) Can we match this load by adding another capacitor in series somewhere and, if so, at what distance D and with what value C_m?

Can we match? YES NO

D =

C_m =

Problem 5. (20/200 points)



A flat perfect conductor has a surface current in the xy plane at z = 0 of:

$$\overline{J}_s = \hat{x}\, J_o e^{-jbx} \ \ [A/m].$$

a) Approximately what is $\overline{H}$ in the xy plane at z = 0+?

$\overline{H}$(z = 0+) =

b) How might one easily induce this current sheet at frequency f [Hz] on the surface of a good conductor? Please be reasonably specific and quantitative.

To induce this current one might:

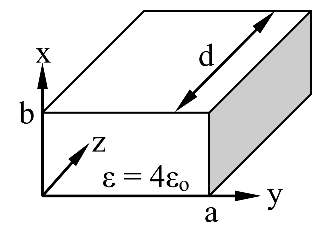
Please turn sheet over for Problem 6.

Problem 6. (10/200 points)

A certain evansescent wave at angular frequency ω in a slightly lossy medium has $\underline{\bar{E}} = \hat{y}\, E_o e^{\alpha(x-0.01z) - jbz}$; assume $\mu = \mu_o$. What is the distance D between phase fronts for this wave?

D =

Problem 7. (25/200 points)



A resonator is filled with a dielectric having $\varepsilon = 4\varepsilon_o$ and has dimensions b, a, and d along the x, y, and z directions, respectively, where d > a > b.

a) What is the lowest resonant frequency $f_{m,n,q}$ [Hz] for this resonator?

$f_{m,n,q}$ [Hz] =

b) What is the polarization of the electric vector $\overline{E}$ at the center of the resonator for this lowest frequency mode?

Polarization of $\overline{E}$ is:

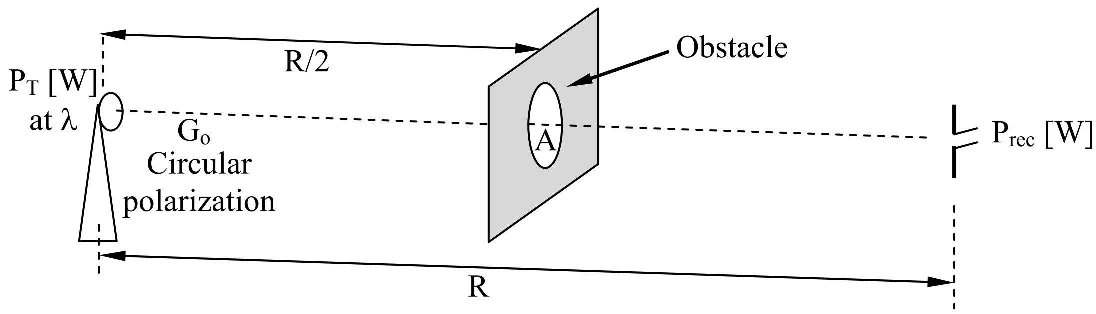
Please turn sheet over to answer part (c).

c) What is the Q of this resonance if the dielectric has a slight conductivity σ? Hint: a ratio of integrals may suffice, so the integrals might not need to be computed.

$Q =$

Problem 8. (20/200 points)

A certain transmitter transmits P_T watts of circularly polarized radiation with antenna gain G_o (in circular polarization) toward an optimally oriented matched short-dipole receiving antenna (gain = 1.5) located a distance R away. The wavelength is λ.



a) In the absence of any obstacles or reflections, what power P_R is received?

P_R =

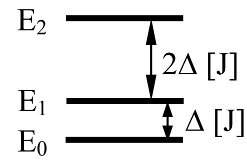
Please turn sheet over to answer part (b).

b) A large metal fence is then erected half way between the transmitter and receiver, and perpendicular to the line of sight. Fortunately it has a round hole of area A centered on that line of sight. Assume the hole is sufficiently small that the electrical phase of the incident wave is constant over its entirety. What power is received now?

P_R =

Problem 9. (15/200 points)

An ideal lossless three-level laser has the illustrated energy level structure. Level 1 is Δ Joules above the ground state, and Level 2 is 3Δ Joules above the ground state. All rates of spontaneous emission A_{ij} have the same finite value except for A_{21}, which is infinite.



a) What should be the laser frequency f_L [Hz]?

f_L [Hz] =

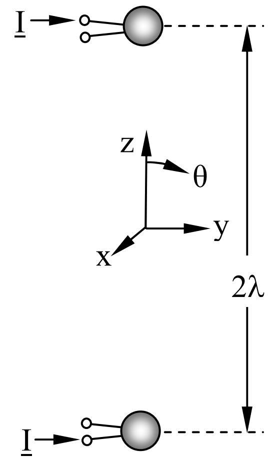
b) What is this laser's maximum possible efficiency η = (laser power)/(pump power)?

η =

Please turn sheet over for Problem 10.

Problem 10. (10/200 points)

Two monopole (isotropic) acoustic antennas lying on the z axis are aligned in the z direction and separated by 2λ, as illustrated. They are fed 180° out of phase. In what directions θ does this acoustic array have maximum gain G(θ)? Simple expressions suffice. If more than one direction has the same maximum gain, please describe all such directions.



θ =

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