MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.013 – Electromagnetics and Applications

Problem Set 8 (two problems)

Suggested Reading: Course notes, Sections 7.4.1 - 7.4.4; 9.3.1. Material not on the next quiz, but that is covered this week and in P.S. 9 includes Sections 9.3.1 - 9.3.2; 9.4.

Quiz 2: Reminder -- Quiz 2, April 16th (Thursday) during lecture; it emphasizes material March 1 through April 8 and Problem Sets 5-8 (including this problem set, which can be delayed until Friday, just after the quiz). It is closed book with a single formula sheet provided in advance, to which formulas may be added.

Problem 8.1

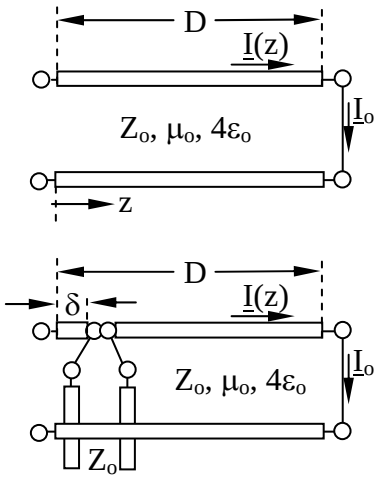
A lossless TEM resonator of length D is short-circuited at one end and open-circuit at the other, as illustrated. It is filled with insulator having $\mu = \mu_o$ and $\varepsilon = 4\varepsilon_o$.

(a) What are the resonant frequencies f_i [Hz] of this TEM resonator?

(b) Please express the complex current distribution $\underline{I}(z)$ as a function of the complex magnitude $\underline{I}_o$ of the current through the short circuit at resonant frequency f_i.

(c) What are the time-average magnetic and electric energies, w_m and w_e, stored in this resonator at frequency f_i, in terms of $\underline{I}_o$?

(d) This resonator is then coupled to an external matched circuit through a TEM line, as illustrated. Assume $Z_o = 100\Omega$. Approximately what value of δ_i yields $Q_L = 20$ at frequency f_i? (Please give the smallest value of δ_i that works.)



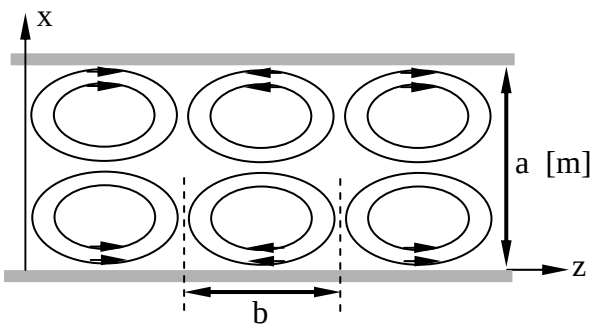
(e) Is this a series or parallel resonance? What is its half-power bandwidth Δf [Hz]?

(f) A very small resistor R_i is then placed in series with $\underline{I}_o$. What value of R_i would yield a critically matched resonator (one matched at resonance), assuming δ_i remained unchanged? What then is Q_L?

(Please turn over for Problem 8.2)

Problem 8.2

All non-zero electromagnetic fields for a certain mode (TE_m or TM_m) of an air-filled parallel-plate waveguide are sketched below at a certain instant of time. Waves propagate only in the ±z directions.



(a) Which field lines are electric and/or magnetic? What mode is this? Please briefly explain your reasoning.

(b) What are k_x and k_z for the illustrated mode?

(c) What is ω [r/s] for the illustrated wave?

(d) What is the cutoff frequency $\omega_{c.o.}$ for this mode?

(e) What is the phase velocity v_p for this mode at this frequency?

(f) What is the total time-average power flow [Watts] in the +z direction for the wave (waves) illustrated here? Assume the maximum value of $\bar{E}(t)$ is 3 volts/meter, if needed. Briefly explain your reasoning.

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