Recitation 12: CMOS Noise Margin

Yesterday we talked about a CMOS Inverter (Figure 1 part a). Compared with a NMOS inverter(Figure 1 part b), the resistor R is replaced with a PMOS:

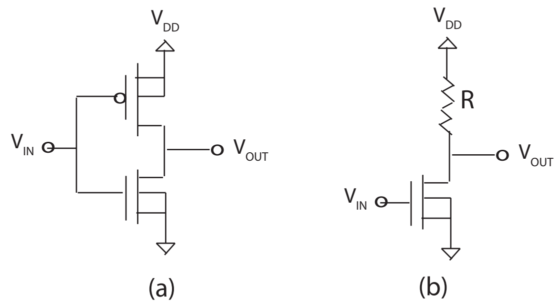


Figure 1: CMOS vs. N-MOS inverter

Today we will focus on the noise margin of a CMOS inverter. To consider the noise margin, we first need the transfer characteristic (i.e. $V_{\text{out}} - V_{\text{in}}$)

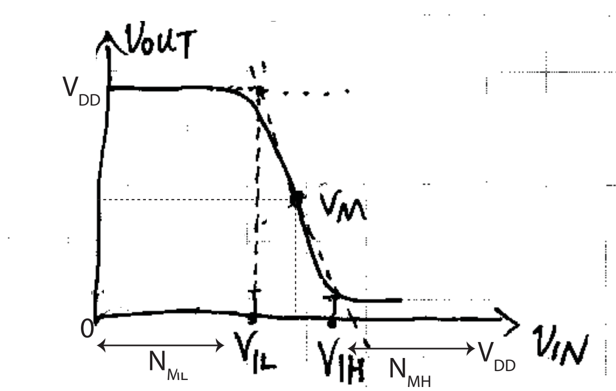


Figure 2:

$$
\begin{aligned}
V_{\text{o,max}} &= V_{\text{DD}} \quad \text{this is our } V_{\text{OH}} \\
V_{\text{o,min}} &= 0 \quad \text{this is our } V_{\text{OL}} \\
\therefore, N_{\text{MH}} &= V_{\text{OH}} - V_{\text{IH}} \\
&= V_{\text{DD}} - V_{\text{IH}} \quad \text{can be seen on Figure 2} \\
N_{\text{ML}} &= V_{\text{IL}} - V_{\text{OL}} \quad \text{can be seen on Figure 2}
\end{aligned}
$$

To find noise margin, or V_{IH} or V_{IL}, we will need voltage V_{M} and the slope (gain) at V_{M}.

From simple geometry, one can derive:

$$\begin{aligned} N_{\mathrm{ML}} &= V_{\mathrm{M}} - \frac{V_{\mathrm{DD}} - V_{\mathrm{M}}}{|A_{\mathrm{V}}|} \\ N_{\mathrm{MH}} &= V_{\mathrm{DD}} - V_{\mathrm{M}} - \frac{V_{\mathrm{M}}}{|A_{\mathrm{V}}|} \end{aligned}$$

Note: A_{V} at V_{M} is negative, and $|A_{\mathrm{V}}|$ is absolute value.

How to find V_{M}?

V_{M} is the point when both NMOS and PMOS are in saturation:

$$\begin{aligned} I_{\mathrm{D_n}} &= -I_{\mathrm{D_p}} \\ \frac{w_{\mathrm{n}}}{2L_{\mathrm{n}}}\mu_{\mathrm{n}} C_{\mathrm{ox}}(V_{\mathrm{GS_n}} - V_{\mathrm{T_n}})^2 &= \frac{w_{\mathrm{p}}}{2L_{\mathrm{p}}}\mu_{\mathrm{p}} C_{\mathrm{ox}}(V_{\mathrm{SG_p}} + V_{\mathrm{T_p}})^2 \end{aligned}$$

We let

$$\begin{aligned} k_{\mathrm{n}} &= \frac{w_{\mathrm{n}}}{L_{\mathrm{n}}}\mu_{\mathrm{n}} C_{\mathrm{ox}} \\ k_{\mathrm{p}} &= \frac{w_{\mathrm{p}}}{L_{\mathrm{p}}}\mu_{\mathrm{p}} C_{\mathrm{ox}} \end{aligned}$$

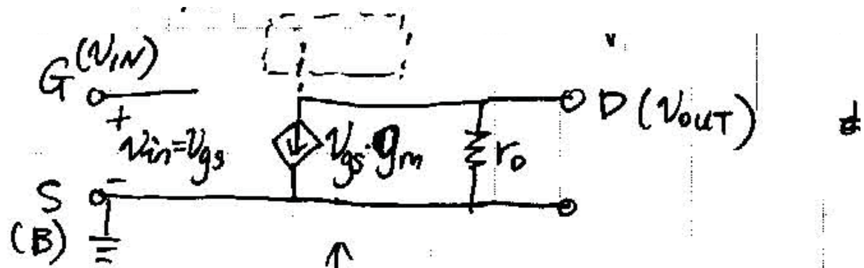
Note: Very useful for MOSFET circuit designer: $\frac{w}{L}$ and for the process engineer: $\mu_{\mathrm{n}}, C_{\mathrm{ox}}$ and μ_{p}

$$\begin{aligned} \frac{1}{2}k_{\mathrm{n}}(V_{\mathrm{M}} - V_{\mathrm{T_n}})^2 &= \frac{1}{2}k_{\mathrm{p}}(V_{\mathrm{DD}} - V_{\mathrm{M}} + V_{\mathrm{T_p}})^2 \\ V_{\mathrm{M}} &= \frac{V_{\mathrm{T_n}} + \sqrt{\frac{k_{\mathrm{p}}}{k_{\mathrm{n}}}}(V_{\mathrm{DD}} + V_{\mathrm{T_p}})}{1 + \sqrt{\frac{k_{\mathrm{p}}}{k_{\mathrm{n}}}}} \end{aligned}$$

Usually, $V_{\mathrm{T_n}}, V_{\mathrm{T_p}}$ are known, if we have V_{M}, we can find $\frac{k_{\mathrm{p}}}{k_{\mathrm{n}}}$, or vice versa.

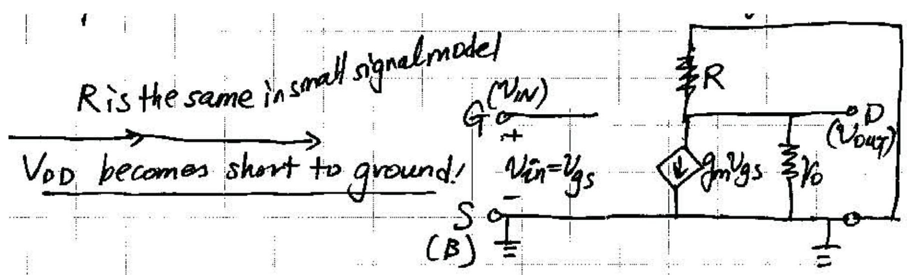
How to find $|A_{\mathrm{V}}(V_{\mathrm{M}})|$?

Last time, for n-MOS inverter, we did not have much time to look at the gain either. We can look at the gain of the NMOS inverter, which is easier.
Take the circuit of Figure 1 part(b), first replace the N-MOS with its small-signal model:

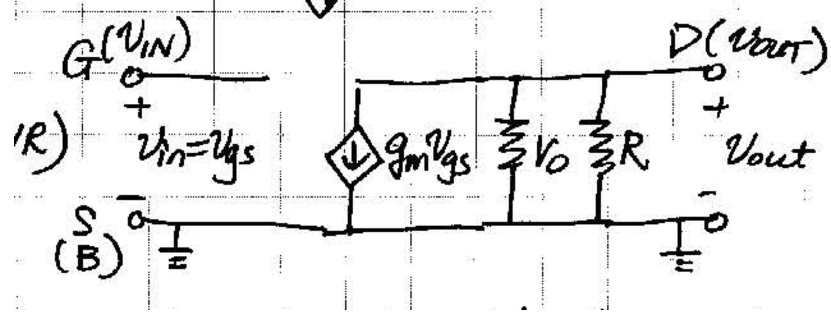


∵ Bulk tied to source, $v_{bs}=0$

⇒ no $g_{mb}\cdot v_{bs}$ part



flip R to make it better

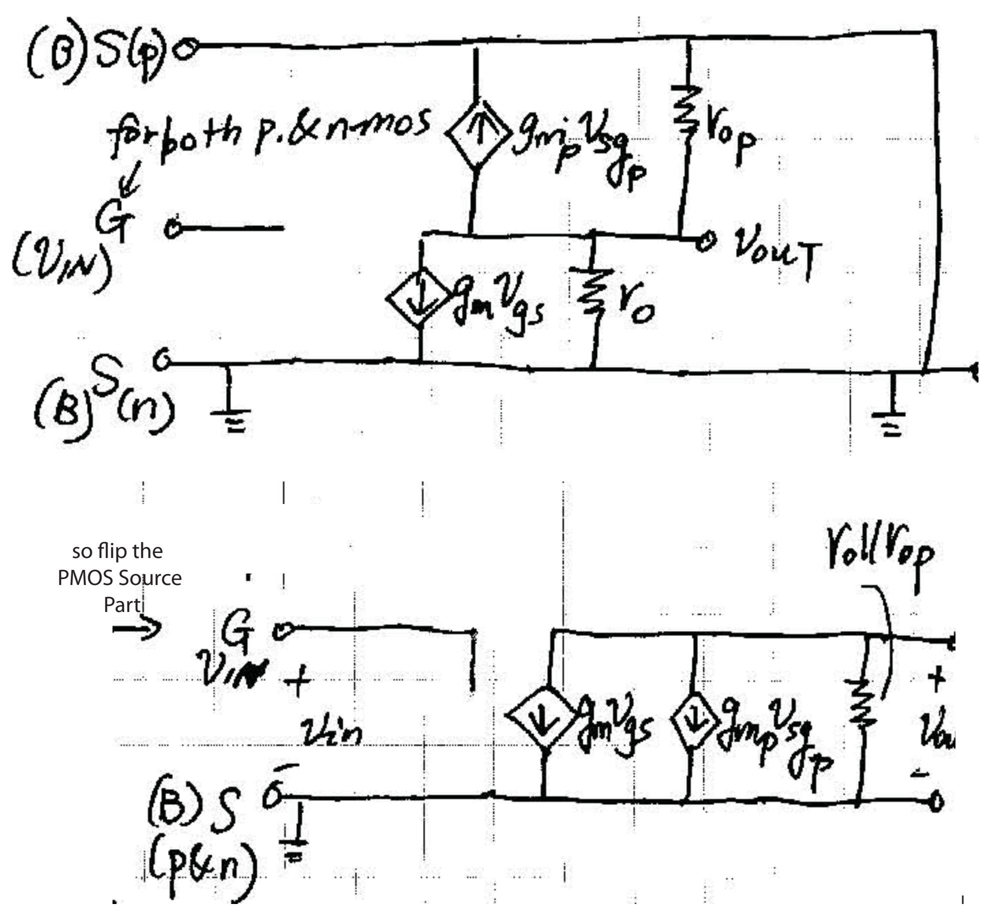


Then:

$$
\begin{aligned}
A_{\mathrm{V}} &= \left.\frac{V_{\mathrm{out}}}{V_{\mathrm{in}}}\right|_{V_{\mathrm{M}}} \\
&= -\frac{\overbrace{g_{\mathrm{m}}\cancel{V_{\mathrm{gs}}}}^{\text{current}}\cdot(r_{\mathrm{o}}||R)}{\cancel{v_{\mathrm{gs}}}} = -g_{\mathrm{m}}(r_{\mathrm{o}}||R)
\end{aligned}
$$

Small Signal Model of CMOS Inverter

To get the small signal model of CMOS inverter, take the circuit of Figure 1 part (a),



$$\begin{aligned}
V_{\text{in}} &= V_{\text{gs}} = V_{\text{sg}_\text{p}} \\
A_\text{v}\Big|_{V_\text{M}} &= \frac{V_{\text{out}}}{V_{\text{in}}}\Big|_{V_\text{M}} = -\frac{(g_{\text{mn}}+g_{\text{mp}})\cancel{V_{\text{in}}}\cdot(r_\text{o}||r_{\text{op}})}{\cancel{V_{\text{in}}}} \\
&= -(g_{\text{mn}}+g_{\text{mp}})\cdot(r_{\text{on}}||r_{\text{op}}) = -\frac{g_{\text{mn}}+g_{\text{mp}}}{g_{\text{on}}+g_{\text{op}}} \\
g_{\text{mn}} &= \frac{\delta i_{\text{D}_\text{n}}}{\delta V_{\text{GS}}}\Big|_{V_\text{M}} = \frac{w_\text{n}}{L_\text{n}}\mu_\text{n} C_{\text{ox}}(V_\text{M}-V_{\text{T}_\text{n}}) = \sqrt{\frac{2w_\text{n}}{L_\text{n}}\mu_\text{n} C_{\text{ox}} I_{\text{D}_\text{n}}} = \sqrt{2k_\text{n} I_\text{D}} \\
g_{\text{mp}} &= \frac{\delta i_{\text{D}_\text{p}}}{\delta V_{\text{SG}}}\Big|_{V_\text{M}} = \frac{w_\text{p}}{L_\text{p}}\mu_\text{p} C_{\text{ox}}(V_{\text{DD}}-V_\text{M}+V_{\text{T}_\text{p}}) = \sqrt{\frac{2w_\text{p}}{L_\text{p}}\mu_\text{p} C_{\text{ox}} I_{\text{D}_\text{p}}} = \sqrt{2k_\text{p}(-I_{\text{D}_\text{p}})} \\
g_{\text{on}} &= \frac{\delta i_{\text{D}_\text{n}}}{\delta V_{\text{DS}}}\Big|_{V_\text{M}} = \lambda_\text{n} I_{\text{D}_\text{n}} \\
g_{\text{op}} &= \frac{-\delta i_{\text{D}_\text{p}}}{\delta V_{\text{SD}}}\Big|_{V_\text{M}} = \lambda_\text{p}(-I_{\text{D}_\text{p}})
\end{aligned}$$

Exercise

CMOS inverter design specification: $V_\text{M} = 2.5\,\text{V}$, $\text{V}_{\text{DD}} = 5\,\text{V}$.

$$\begin{aligned}
I_{\text{D}_\text{n}} &= -I_{\text{D}_\text{p}} = 200\,\mu\text{A at V}_{\text{IN}} = \text{V}_\text{M} \\
N_{\text{ML}} &= N_{\text{MH}} \geq 2.25\,\text{V}
\end{aligned}$$

Find specific maximum λ that can be tolerated to meet design specifications (in terms of NM or noise margin). Assume $\lambda_\text{n} = \lambda_\text{p}$.
Device data:

$$\begin{aligned}
\mu_\text{n} C_{\text{ox}} &= 2\mu_\text{p} C_{\text{ox}} = 50\,\mu\text{A/V}^2 \\
V_{\text{T}_\text{n}} &= -V_{\text{T}_\text{p}} = 1\,\text{V}
\end{aligned}$$

Because noise margin $\geq 2.25\,\text{V}$,

$$\begin{aligned}
N_{\text{ML}} &= V_\text{M} - \frac{V_{\text{DD}}-V_\text{M}}{|A_\text{V}|} \geq 2.25 \\
\frac{V_{\text{DD}}-V_\text{M}}{|A_\text{V}|} &\leq V_\text{M} - 2.25 - 0.25\,\text{V} \\
|A_\text{V}| &\geq \frac{2.5\,\text{V}}{0.25\,\text{V}} = 10 \\
|A_\text{V}| &= \frac{g_{\text{mn}}+g_{\text{mp}}}{g_{\text{op}}+g_{\text{on}}} \\
g_{\text{mn}} &= \sqrt{2k_\text{n} I_\text{D}} \quad g_{\text{mp}} = \sqrt{2k_\text{p} I_\text{D}} \\
g_{\text{on}} &= \lambda_\text{n} I_\text{D} \\
g_{\text{op}} &= \lambda_\text{p} I_\text{D} \\
I_\text{D} &= |-I_{\text{D}_\text{p}}| = |I_{\text{D}_\text{n}}|
\end{aligned}$$

We have information of I_{D}, we need to find $k_{\mathrm{n}}, k_{\mathrm{p}}$ then we find $\lambda_{\mathrm{n}}, \lambda_{\mathrm{p}}$.

$$\begin{aligned}
\because V_{\mathrm{T_n}} &= -V_{\mathrm{T_p}}, V_{\mathrm{M}} = \frac{V_{\mathrm{DD}}}{2}, \text{ symmetric : } \sqrt{\frac{k_{\mathrm{n}}}{k_{\mathrm{p}}}} = 1 \,\mathrm{or}\, k_{\mathrm{n}} = k_{\mathrm{p}} \\
\frac{1}{2}k_{\mathrm{n}}(V_{\mathrm{M}} - V_{\mathrm{T_n}})^2 &= I_{\mathrm{D_n}} = 200\,\mu\mathrm{A} \\
k_{\mathrm{n}} &= \frac{400\,\mu\mathrm{A} \times 2}{(2.5-1)^2, \mathrm{V}^2} = 355\,\mu\mathrm{A/V}^2 = \mathrm{k_p} \\
g_{\mathrm{mn}} &= g_{\mathrm{mp}} = \sqrt{2k_{\mathrm{n}}I_{\mathrm{D}}} = \sqrt{2 \times 355\,\mu\mathrm{A/V}^2 \times 200\,\mu\mathrm{A}} = 376.8\,\mu\mathrm{A/V} = 0.376\,\mathrm{ms} \\
|A_{\mathrm{V}}| &= \frac{g_{\mathrm{mn}} + g_{\mathrm{mp}}}{\lambda_{\mathrm{n}}I_{\mathrm{D}} + \lambda_{\mathrm{p}}I_{\mathrm{D}}} \geq 10 \implies \lambda_{\mathrm{n}} \,\mathrm{or}\, \lambda_{\mathrm{p}} \leq \frac{2 \times 0.376\,\mathrm{ms}}{10 \times 200\,\mu\mathrm{A}} = 0.376\,\mathrm{V}^{-1}
\end{aligned}$$

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