

Lecture 4 - p-n Junctions: Electrostatics - Outline

- **Review**

Poisson's equation for $\phi_o(x)$ given $N_d(x)$ and $N_a(x)$:

$$\epsilon \frac{d^2\phi_o(x)}{dx^2} = q [n_i \{e^{q\phi_o(x)/kT} - e^{-q\phi_o(x)/kT}\} - N_d(x) + N_a(x)]$$

Knowing $\phi_o(x)$, we have $n_o(x) = n_i e^{q\phi_o(x)/kT}$ and $p_o(x) = n_i e^{-q\phi_o(x)/kT}$

Slowly varying profiles: quasi-neutrality holds

In n-type, for example, $n_o(x) \approx N_d(x) - N_a(x)$, $p_o(x) = n_i^2/n_o(x)$

Given n_o and/or p_o , $\phi_o(x) = (kT/q)\ln[n_o(x)/n_i] = - (kT/q)\ln[p_o(x)/n_i]$

- **Abrupt p-n junction in TE (electrostatics)**

Abrupt profile: Take as an example an abrupt p-n junction with

$$N_a(x) - N_d(x) \equiv N_{Ap} \text{ for } x < 0 \text{ and } N_d(x) - N_a(x) \equiv N_{Dn} \text{ for } x > 0$$

Observe: 1. $n_o(x)$ and $p_o(x)$ depend exponentially on $\phi_o(x)$

2. $\phi_o(x)$ is insensitive to the details of the charge profile, $\rho(x)$

Depletion approximation:

$$\text{Approximate net charge, } \rho(x) \approx \begin{cases} 0 & \text{for } x < -x_p \text{ and } x > x_n \\ -q N_{Ap} & \text{for } -x_p < x < 0 \\ q N_{Dn} & \text{for } 0 < x < x_n \end{cases}$$

Integrate once to get $E(x)$, and again to get $\phi(x)$

Find x_p and x_n by fitting $\phi_o(x)$ to known $\Delta\phi$ crossing junction

- **Applying bias to a p-n junction (what happens?)**

Non-uniform doping in thermal equilibrium

Reviewing from Lecture 3:

In a non-uniformly doped sample in TE we have: $g_L(x,t) = 0$, $J_e(x) = 0$, $J_h(x) = 0$, and $\partial/\partial t = 0$. **Also:** $n(x) = n_o(x)$ and $p(x) = p_o(x)$. Applying these conditions to the two current density equations gave:

$$0 = q\mu_e n_o(x)E(x) + qD_e \frac{dn_o(x)}{dx} \rightarrow \frac{d\phi}{dx} = \frac{D_e}{\mu_e} \frac{1}{n_o(x)} \frac{dn_o(x)}{dx}$$

and

$$0 = q\mu_h p_o(x)E(x) - qD_h \frac{dp_o(x)}{dx} \rightarrow \frac{d\phi}{dx} = -\frac{D_h}{\mu_h} \frac{1}{p_o(x)} \frac{dp_o(x)}{dx}$$

And Poisson's equation became:

$$-\frac{d^2\phi(x)}{dx^2} = \frac{dE(x)}{dx} = \frac{\rho(x)}{\epsilon} = \frac{q}{\epsilon} [p_o(x) - n_o(x) + N_d(x) - N_a(x)]$$

In the end, we had three equations in our three remaining unknowns, $n_o(x)$, $p_o(x)$, and $\phi(x)$.

Non-uniform doping in thermal equilibrium, cont.

The first two equations can be solved by integrating to get:

$$n_o(x) = n_i e^{\frac{\mu_e}{D_e} \phi(x)} \quad \text{and} \quad p_o(x) = n_i e^{-\frac{\mu_h}{D_h} \phi(x)}$$

Ref: $\phi(x) = 0$ at all x where $p_o(x) = n_o(x) = n_i$

Next use the Einstein relation:

$$\frac{\mu_h}{D_h} = \frac{\mu_e}{D_e} = \frac{q}{kT}$$

Note: @ R.T. $q/kT \approx 40 \text{ V}^{-1}$ and $kT/q \approx 25 \text{ mV}$

Using the Einstein relation we have:

$$n_o(x) = n_i e^{q\phi(x)/kT} \quad \text{and} \quad p_o(x) = n_i e^{-q\phi(x)/kT}$$

Finally, putting these in Poisson's equation, a single equation for $\phi(x)$ in a doped semiconductor in TE materializes:

$$\frac{d^2 \phi(x)}{dx^2} = -\frac{q}{\epsilon} \left[n_i \left(e^{-q\phi(x)/kT} - e^{q\phi(x)/kT} \right) + N_d(x) - N_a(x) \right]$$

Doing the numbers:

I. D to μ conversions, and visa versa

To convert between D and μ it is convenient to say $kT/q \approx 25$ mV,
in which case $q/kT \approx 40$ V⁻¹:

17°C/62°F

Example 1: $\mu_e = 1600$ cm²/V-s, $\mu_h = 600$ cm²/V-s

$$D_e = \mu_e / (q/kT) = 1600/40 = 40 \text{ cm}^2 / \text{s}$$

$$D_h = \mu_h / (q/kT) = 600/40 = 15 \text{ cm}^2 / \text{s}$$

II. Relating ϕ to n and p, and visa versa

To calculate ϕ knowing n or p it is better to say that $kT/q \approx 26$ mV,
because then $(kT/q)\ln 10 \approx 60$ mV:

28°C/83°F

Example 1: n-type, $N_D = N_a - N_a = 10^{16}$ cm⁻³

$$\phi_n = \frac{kT}{q} \ln \frac{10^{16}}{10^{10}} = \frac{kT}{q} \ln 10^6 = \frac{kT}{q} \ln 10 \cdot \log 10^6 \approx 0.06 \ln 10^6 = 0.36 \text{ eV}$$

Example 2: p-type, $N_A = N_a - N_d = 10^{17}$ cm⁻³

$$\phi_p = -\frac{kT}{q} \ln \frac{10^{17}}{10^{10}} = -\frac{kT}{q} \ln 10 \cdot \log 10^7 \approx -0.06 \cdot 7 = -0.42 \text{ eV}$$

Example 3: 60 mV rule:

For every order of magnitude the doping is above (below) n_i ,
the potential increases (decreases) by 60 meV.

Non-uniform doping in thermal equilibrium, cont:

We have reduced our problem to solving one equation for one unknown, in this case $\phi(x)$:

$$\frac{d^2\phi(x)}{dx^2} = -\frac{q}{\epsilon} \left[n_i \left(e^{-q\phi(x)/kT} - e^{q\phi(x)/kT} \right) + N_d(x) - N_a(x) \right]$$

Once we find $\phi(x)$ we can find n_o and p_o from:

$$n_o(x) = n_i e^{q\phi(x)/kT} \quad \text{and} \quad p_o(x) = n_i e^{-q\phi(x)/kT}$$

Solving Poisson's equation for $\phi(x)$ is in general non-trivial, and for precise answers a "Poisson Solver" program must be employed. However, in two special cases we can find very useful, insightful approximate analytical solutions:

Case I: Abrupt changes from p- to n-type (i.e., junctions)

also: surfaces (Si to air or other insulator)

interfaces (Si to metal, Si to insulator, or Si to insulator to metal)

Case II: Slowly varying doping profiles.

Non-uniform doping in thermal equilibrium, cont:

Solving Poisson's Equation in Two Special Cases:

II. Slowly varying profiles

If $|N_d(x) - N_a(x)|$ is slowly varying*, the quasineutrality holds, and the majority carrier concentration closely tracks the net doping profile.

In n-type samples:

$$n_o(x) \approx N_d(x) - N_a(x) \quad \text{and} \quad p_o(x) = n_i^2 / n_o(x)$$

In p-type samples:

$$p_o(x) \approx N_a(x) - N_d(x) \quad \text{and} \quad n_o(x) = n_i^2 / p_o(x)$$

I. Abrupt p-n junctions

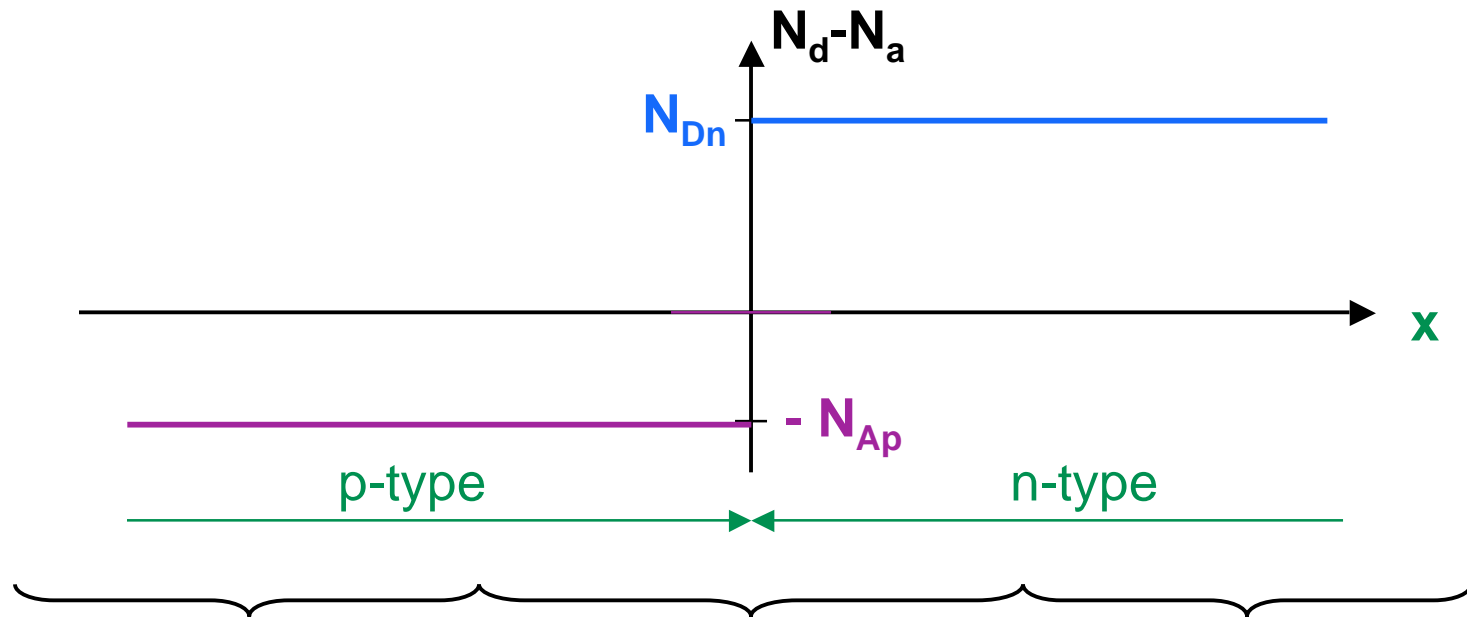
Near the junction there are very large net charge densities, and a dramatic reduction in the mobile carrier density.

The model we employ with p-n junctions (and MOS capacitors) is called the “Depletion Approximation”

Non-uniform doping in thermal equilibrium, cont.:

Case II: Abrupt p-n junctions

Consider the profile below:



$$p_o = N_{Ap}, \quad n_o = n_i^2 / N_{Ap}$$

$$\phi = -\frac{kT}{q} \ln(N_{Ap} / n_i) \equiv \phi_p$$

?

$$n_o(x) = ?$$

$$p_o(x) = ?$$

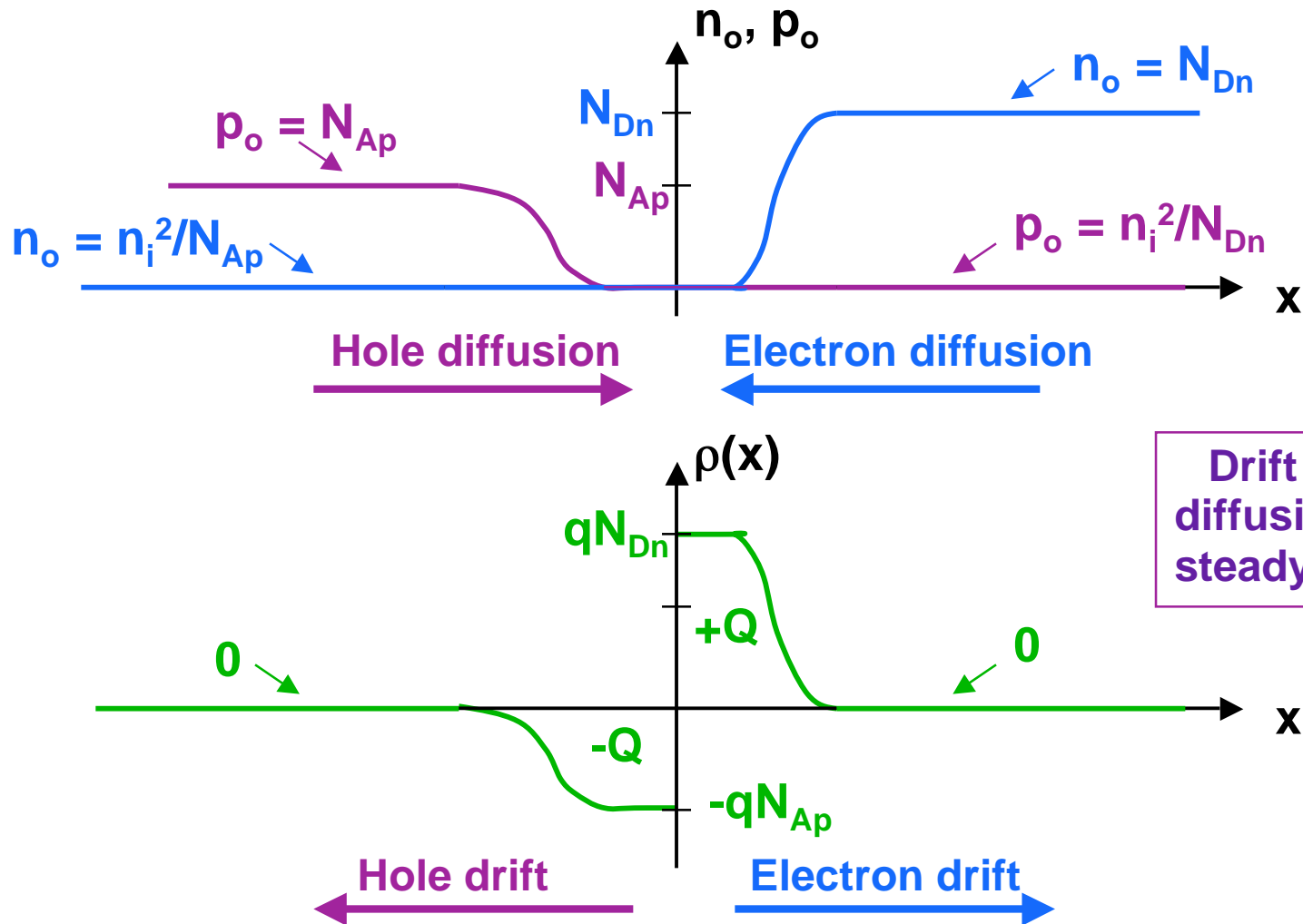
$$\phi(x) = ?$$

$$n_o = N_{Dn}, \quad p_o = n_i^2 / N_{Dn}$$

$$\phi = \frac{kT}{q} \ln(N_{Dn} / n_i) \equiv \phi_n$$

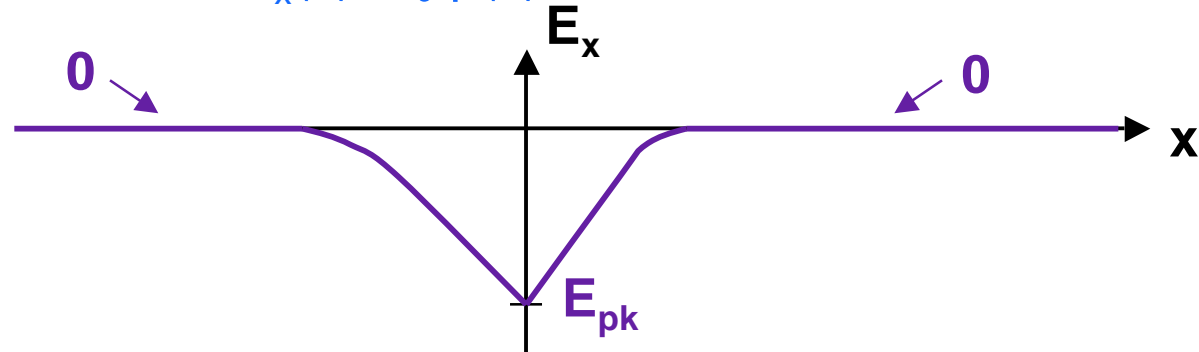
Abrupt p-n junctions, cont:

First look why there is a dipole layer in the vicinity of the junction, and a "built-in" electric field.

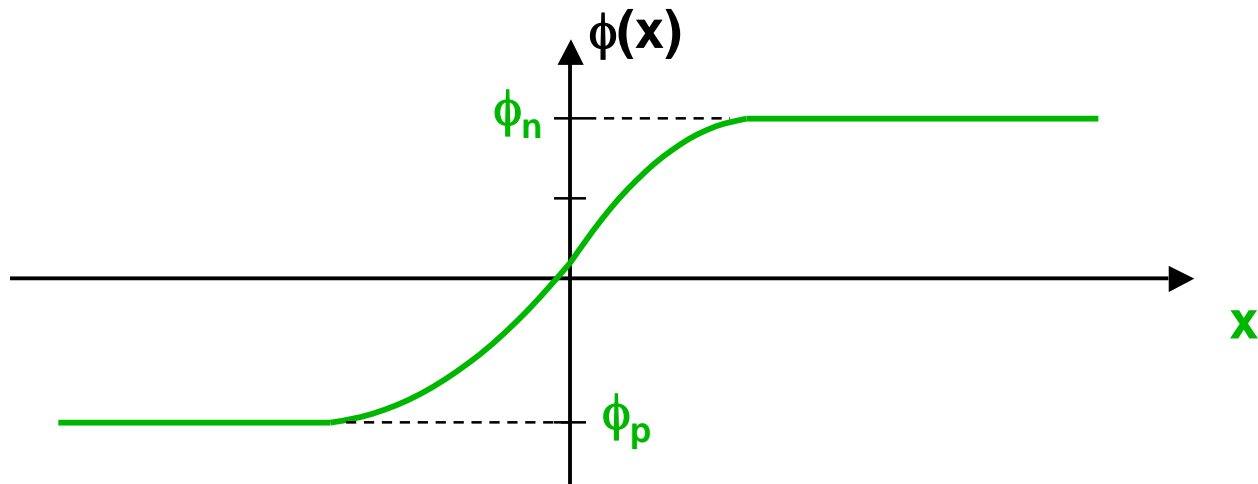


Abrupt p-n junctions, cont:

If the charge density is no longer zero there must be an electric field: $\epsilon E_x(x) = \int \rho(x) dx$

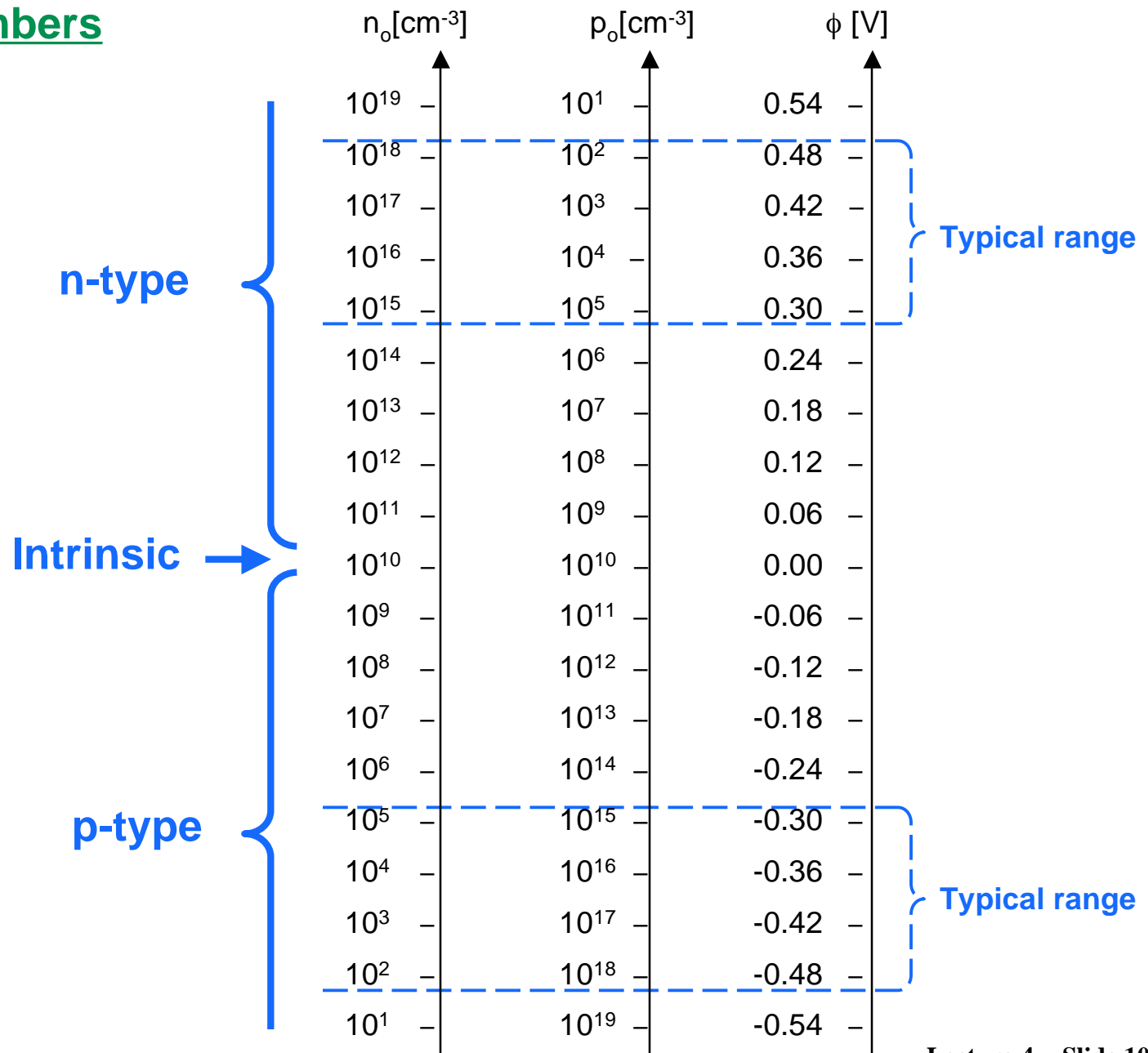


and an electrostatic potential step: $\phi(x) = -\int E_x(x) dx$



Ok, but how do we find $\phi(x)$?

More numbers



Abrupt p-n Junctions, cont.: *the general strategy*

We have to solve a non-linear, 2nd. order DiffyQ for ϕ :

$$\frac{d^2\phi(x)}{dx^2} = -\frac{\rho(x)}{\epsilon} = -\frac{q}{\epsilon} \left[n_i \left(e^{-q\phi(x)/kT} - e^{q\phi(x)/kT} \right) + N_d(x) - N_a(x) \right]$$

To see how to proceed write this relationship as an integral:

$$\phi(x) = -\iint \frac{\rho(x)}{\epsilon} dx + Ax^2 + Bx$$

After integrating $\rho(x)$ twice many of details of its shape will be lost, so if we have a good general idea of what $\rho(x)$ looks like, we might be able to make an iteration strategy work:

Guess a starting $\rho(x)$.

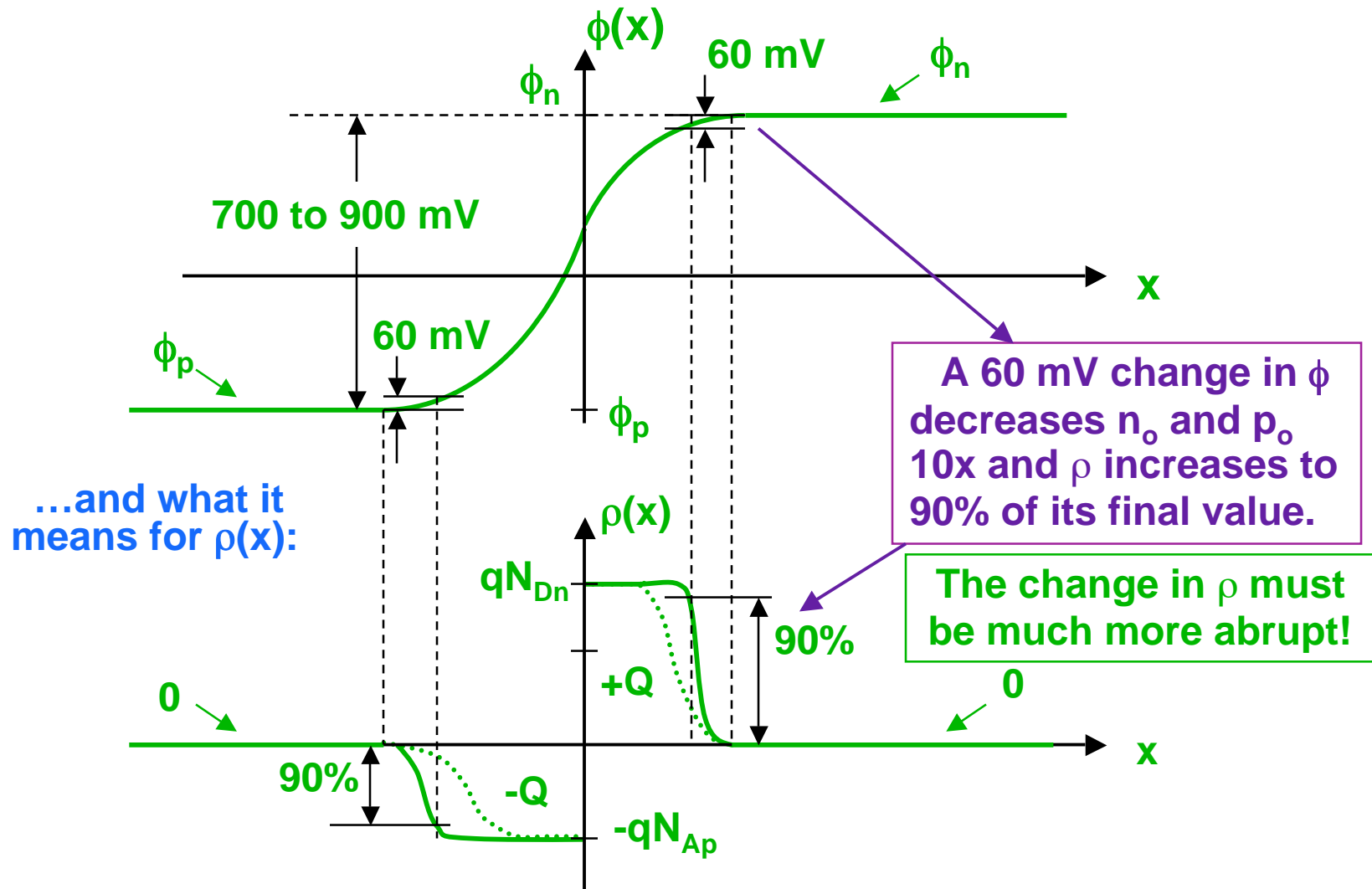
Integrated once to get $E(x)$, and again to get $\phi(x)$.

Use $\phi(x)$ to find $p_0(x)$, $n_0(x)$, and, ultimately, a new $\rho(x)$.

Compare the new $\rho(x)$ to the starting $\rho(x)$.

- If it is not close enough, use the new $\rho(x)$ to iterate again.
- If it is close enough, quit.

To figure out a good first guess for $\rho(x)$, look at how quickly n_o and p_o must change by looking first at how ϕ changes:

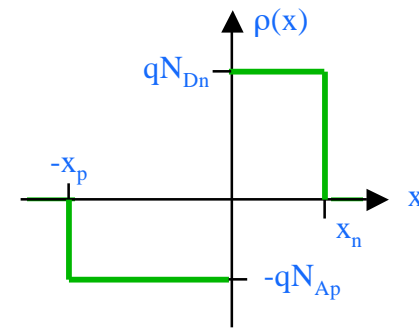


The observation that ρ changes a lot, when ϕ changes a little, is the key to the depletion approximation.

Abrupt p-n Junctions, cont.: *The Depletion Approximation - an informed first estimate of $\rho(x)$*

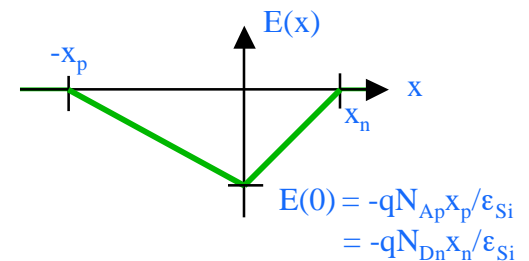
Assume full depletion for $-x_p < x < x_n$, where x_p and x_n are two unknowns yet to be determined. This leads to:

$$\rho(x) = \begin{cases} 0 & \text{for } x < -x_p \\ -qN_{Ap} & \text{for } -x_p < x < 0 \\ qN_{Dn} & \text{for } 0 < x < x_n \\ 0 & \text{for } x_n < x \end{cases}$$



Integrating the charge once gives the electric field

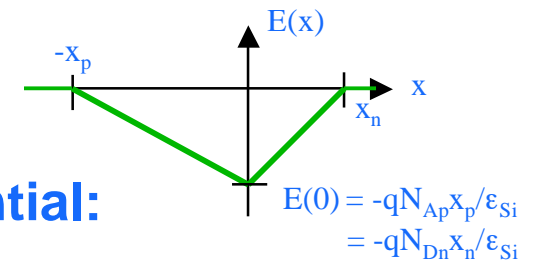
$$E(x) = \begin{cases} 0 & \text{for } x < -x_p \\ -\frac{qN_{Ap}}{\epsilon_{Si}}(x + x_p) & \text{for } -x_p < x < 0 \\ \frac{qN_{Dn}}{\epsilon_{Si}}(x - x_n) & \text{for } 0 < x < x_n \\ 0 & \text{for } x_n < x \end{cases}$$



The Depletion Approximation, cont.:

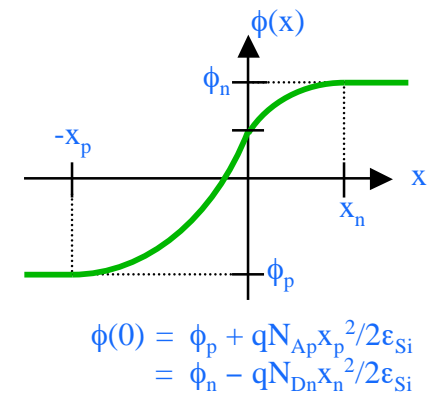
Insisting $E(x)$ is continuous at $x = 0$ yields our first equation relating our unknowns, x_n and x_p :

$$N_{Ap}x_p = N_{Dn}x_n \quad \mathbf{1}$$



Integrating again gives the electrostatic potential:

$$\phi(x) = \begin{cases} \phi_p & \text{for } x < -x_p \\ \phi_p + \frac{qN_{Ap}}{2\epsilon_{Si}}(x + x_p)^2 & \text{for } -x_p < x < 0 \\ \phi_n - \frac{qN_{Dn}}{2\epsilon_{Si}}(x - x_n)^2 & \text{for } 0 < x < x_n \\ \phi_n & \text{for } x_n < x \end{cases}$$



Requiring that the potential be continuous at $x = 0$ gives us our second relationship between x_n and x_p :

$$\phi_p + \frac{qN_{Ap}}{2\epsilon_{Si}}x_p^2 = \phi_n - \frac{qN_{Dn}}{2\epsilon_{Si}}x_n^2 \quad \mathbf{2}$$

The Depletion Approximation, cont.:

Combining our two equations and solving for x_p and x_n gives:

$$x_p = \sqrt{\frac{2\epsilon_{Si} \phi_b}{q} \frac{N_{Dn}}{N_{Ap}(N_{Ap} + N_{Dn})}}, \quad x_n = \sqrt{\frac{2\epsilon_{Si} \phi_b}{q} \frac{N_{Ap}}{N_{Dn}(N_{Ap} + N_{Dn})}}$$

where we have introduced the built-in potential, ϕ_b :

$$\phi_b \equiv \phi_n - \phi_p = \frac{kT}{q} \ln \frac{N_{Dn}}{n_i} - \left(-\frac{kT}{q} \ln \frac{N_{Ap}}{n_i} \right) = \frac{kT}{q} \ln \frac{N_{Dn} N_{Ap}}{n_i^2}$$

We also care about the total width of the depletion region, w :

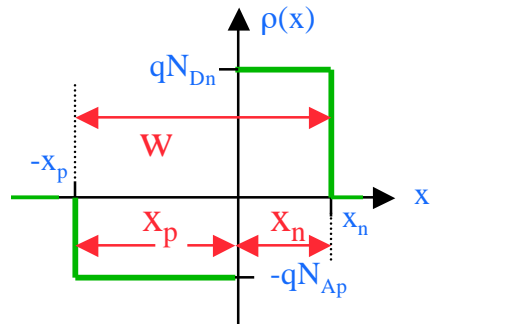
$$w = x_p + x_n = \sqrt{\frac{2\epsilon_{Si} \phi_b}{q} \frac{(N_{Ap} + N_{Dn})}{N_{Ap} N_{Dn}}}$$

And we want to know the peak electric field, $|E_{pk}|$:

$$|E_{pk}| = E(0) = \frac{qN_{Ap}x_p}{\epsilon_{Si}} = \frac{qN_{Dn}x_n}{\epsilon_{Si}} = \sqrt{\frac{2q\phi_b}{\epsilon_{Si}} \frac{N_{Ap}N_{Dn}}{(N_{Ap} + N_{Dn})}}$$

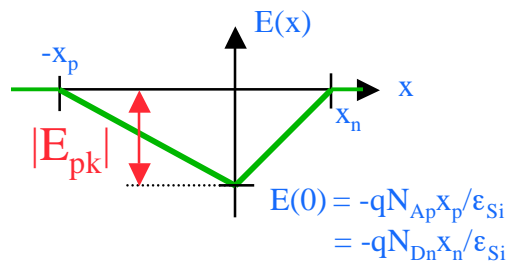
The Depletion Approximation, cont.:

Beating on these results a bit more:

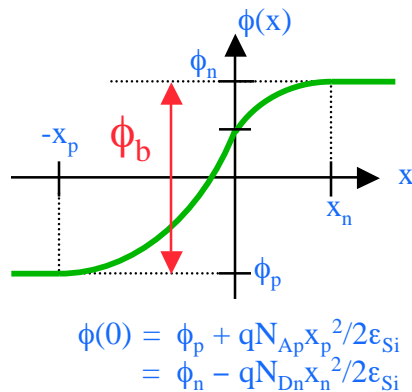


$$w = \sqrt{\frac{2\epsilon_{Si} \phi_b (N_{Ap} + N_{Dn})}{q N_{Ap} N_{Dn}}}$$

$$x_p = \frac{N_{Dn} w}{(N_{Ap} + N_{Dn})}, \quad x_n = \frac{N_{Ap} w}{(N_{Ap} + N_{Dn})}$$



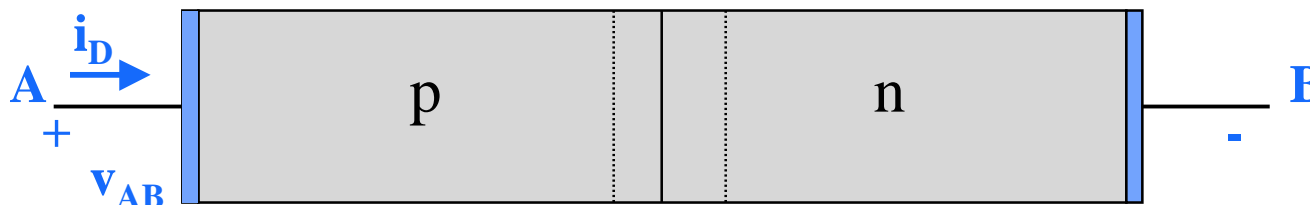
$$|E_{pk}| = \sqrt{\frac{2q \phi_b N_{Ap} N_{Dn}}{\epsilon_{Si} (N_{Ap} + N_{Dn})}}$$



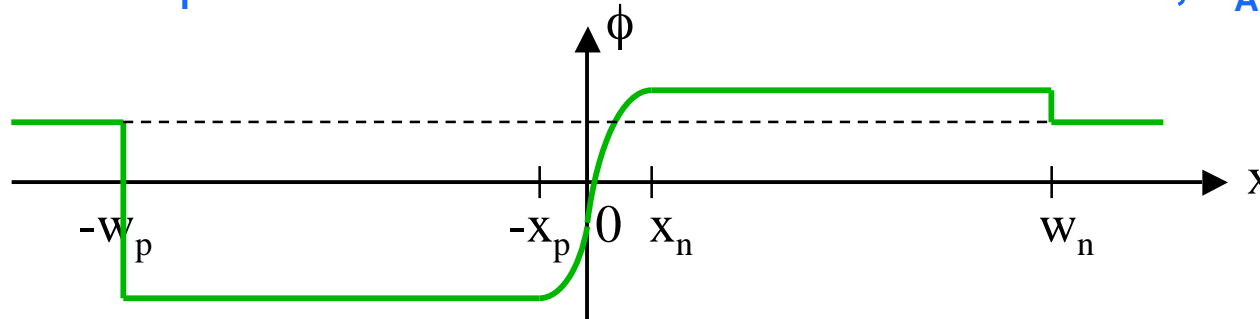
$$\phi_b \equiv \phi_n - \phi_p = \frac{kT}{q} \ln \frac{N_{Dn} N_{Ap}}{n_i^2}$$

Abrupt p-n junctions, cont: Applying bias a p-n diode

Next the question is, "What happens to our $\rho(x)$, $E(x)$, and $\phi(x)$ pictures when a voltage is applied at the device terminals?"



First look at ϕ from contact to contact with zero bias, $v_{AB} = 0$:



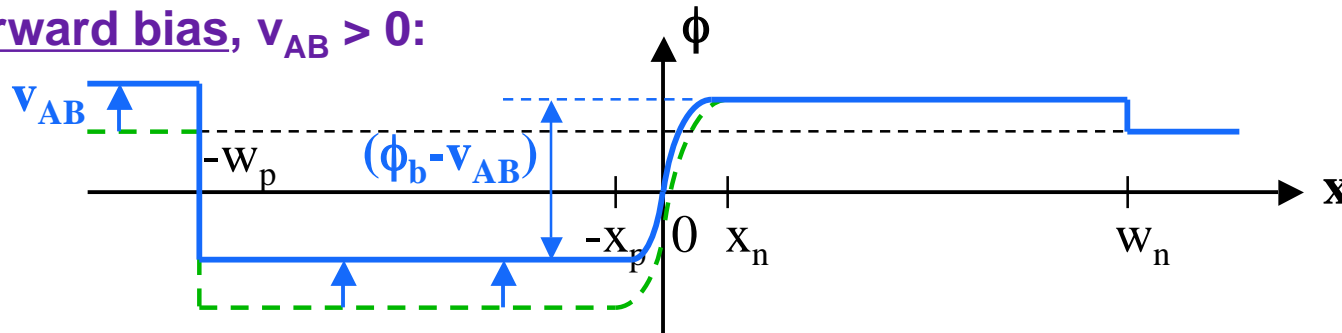
With good contacts and low resistance wires and semiconductor bulk, the only impediment to current flow is the junction, and all of any applied voltage "falls" there.*

Abrupt p-n junctions, cont:

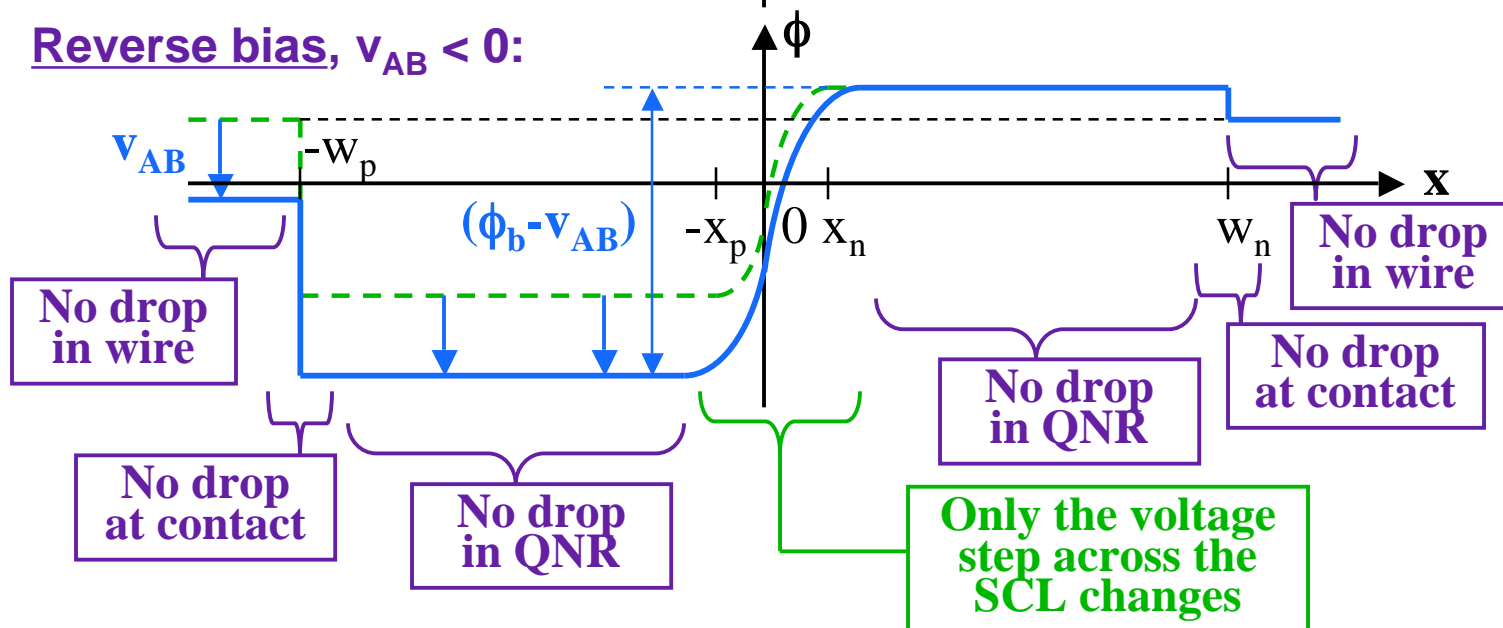
Bias applied, cont.:

All of the applied voltage "appears" across the junction, and no other voltage drops occur.*

Forward bias, $v_{AB} > 0$:



Reverse bias, $v_{AB} < 0$:



Abrupt p-n junctions, cont:

Reverse bias applied: *no current flows, but the potential step changes*

Assume the applied bias, v_{AB} , is negative. This means that the potential on the p-side is reduced relative to that on the n-side, and thus that the change in potential going across the junction is increased from $\phi_n - \phi_p = \phi_b$, to $(\phi_b - v_{AB})$.

We use the Depletion Approximation model as before, now with the new potential step height, obtaining:

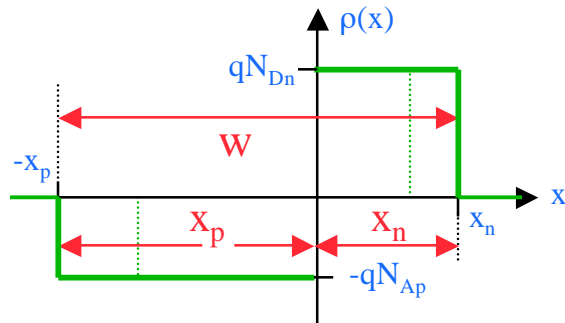
$$x_p = \sqrt{\frac{2\varepsilon_{Si} (\phi_b - v_{AB}) N_{Dn}}{q N_{Ap} (N_{Ap} + N_{Dn})}}, \quad x_n = \sqrt{\frac{2\varepsilon_{Si} (\phi_b - v_{AB}) N_{Ap}}{q N_{Dn} (N_{Ap} + N_{Dn})}}$$

$$w = \sqrt{\frac{2\varepsilon_{Si} (\phi_b - v_{AB}) (N_{Ap} + N_{Dn})}{q N_{Ap} N_{Dn}}}$$

$$|E_{pk}| = \sqrt{\frac{2q (\phi_b - v_{AB}) N_{Ap} N_{Dn}}{\varepsilon_{Si} (N_{Ap} + N_{Dn})}}$$

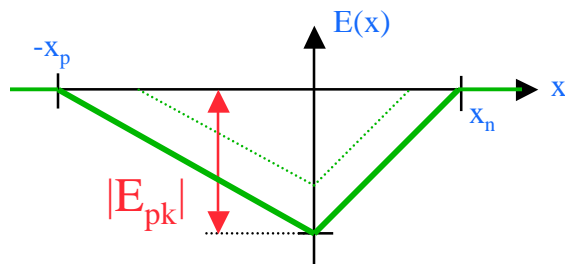
The Depletion Approximation, cont.:

Adding v_{AB} to our earlier sketches: *assume reverse bias, $v_{AB} < 0$*

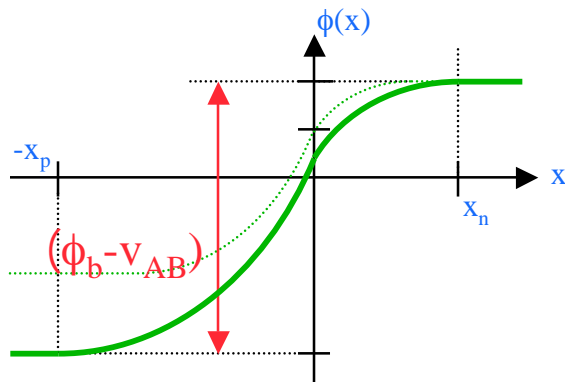


$$w = \sqrt{\frac{2\epsilon_{Si} (\phi_b - v_{AB}) (N_{Ap} + N_{Dn})}{q N_{Ap} N_{Dn}}}$$

$$x_p = \frac{N_{Dn} w}{(N_{Ap} + N_{Dn})}, \quad x_n = \frac{N_{Ap} w}{(N_{Ap} + N_{Dn})}$$



$$|E_{pk}| = \sqrt{\frac{2q (\phi_b - v_{AB}) N_{Ap} N_{Dn}}{\epsilon_{Si} (N_{Ap} + N_{Dn})}}$$



$$\Delta\phi = \phi_b - v_{AB}$$

$$\text{and } \phi_b = \frac{kT}{q} \ln \frac{N_{Dn} N_{Ap}}{n_i^2}$$

Reverse bias, cont: *Is it really that easy? Reverse bias FAQs:*

1. How come we can still use the Depletion Approximation?

No current → no mobile charge → electrostatics unchanged

2. What happens in forward bias?

This same modeling applies, until the current turns on at $v_{AB} \approx 0.5$ V.

By the time v_{AB} approaches ϕ_b , the model no longer holds; the current is too large.

3. What happens at very large reverse bias?

When E_{pk} gets too big, the junction "breaks down" and large reverse current flows.

4. Is there anything interesting we haven't talked about?

Yes, the charge stored in the depletion regions. It is very interesting and important. For starters, the net charge stored on the positive side of the junction is negative:

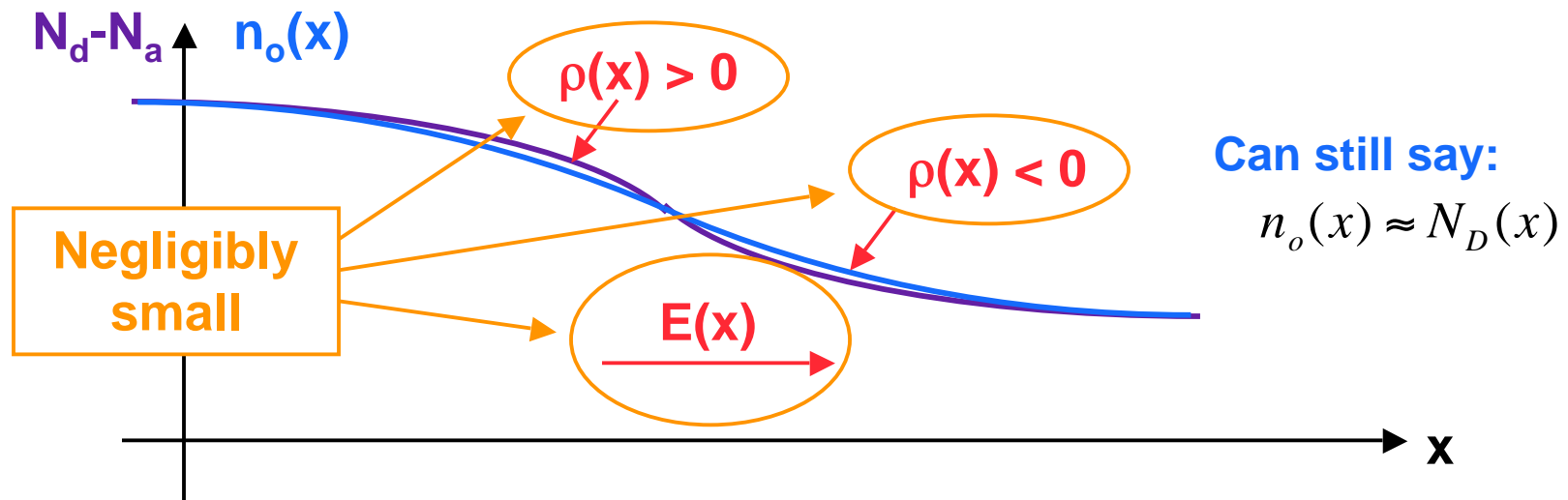
$$Q_p = -qAN_{Ap}x_p = -\sqrt{2q\epsilon_{Si}(\phi_b - v_{AB})N_{Ap}N_{Dn}/(N_{Ap} + N_{Dn})}$$

Non-uniform doping in thermal equilibrium, cont.:

Case II: Slowly varying doping profiles

Detailed solutions of Poisson's equation in semiconductors teach us that if the doping variation, $|N_d(x)-N_a(x)|$ is slow enough*, then quasineutrality holds, and the majority carrier concentration closely tracks the net doping profile.

The net charge densities and electric field are all negligible.



* Note: What is meant by "slowly" can be quantified using the extrinsic Debye length - see the course text.

Non-uniform doping in thermal equilibrium, cont.:

Case II: Slowly varying doping profiles, cont.

If $|N_d(x) - N_a(x)|$ is slowly varying*, then quasineutrality holds, and the majority carrier concentration closely tracks the net doping profile.

- In n-type samples:

$$n_o(x) \approx N_d(x) - N_a(x) \quad \text{and} \quad p_o(x) = n_i^2 / n_o(x)$$

$$\text{Also, } \phi_n(x) = \frac{kT}{q} \ln \left[\frac{N_d(x) - N_a(x)}{n_i} \right]$$

- In p-type samples:

$$p_o(x) \approx N_a(x) - N_d(x) \quad \text{and} \quad n_o(x) = n_i^2 / p_o(x)$$

$$\text{Also, } \phi_p(x) = -\frac{kT}{q} \ln \left[\frac{N_a(x) - N_d(x)}{n_i} \right]$$

* Note: What is meant by “slowly” can be quantified using the extrinsic Debye length - see the course text.

Lecture 4 - p-n Junctions: Electrostatics - Summary

- **Abrupt p-n junction (electrostatics)**

Depletion region forms for $-x_p < x < x_n$:

$$x_p/x_n = N_{Dn}/N_{Ap}$$

$$w = x_p + x_n = [(2\varepsilon/q) \phi_b \{(N_{Ap} + N_{Dn})/N_{Ap}N_{Dn}\}]^{1/2}$$

$$|E_{pk}| = |E(0)| = [(2q/\varepsilon) \phi_b \{N_{Ap}N_{Dn}/(N_{Ap} + N_{Dn})\}]^{1/2}$$

$$\phi_b \equiv (kT/q) \ln (N_{Ap}N_{Dn}/n_i^2)$$

Observations:

1. Greatest depletion is into most lightly doped side
2. Depletion width, w , goes down as doping is increased
3. Peak electric field, $|E_{pk}|$, goes up as doping is increased
4. Asymmetric junction: assume $N_{Ap} \gg N_{Dn}$, then

$$x_n \gg x_p, w \approx x_n \approx [(2\varepsilon/q)\phi_b N_{Dn}]^{1/2}, |E_{pk}| \approx [(2q/\varepsilon)\phi_b N_{Dn}]^{1/2}$$

- **Applying bias to a p-n junction (what happens?)**

Want all applied bias, v_{AB} , to fall across depletion region

Two changes:

1. $\Delta\phi$ crossing junction changes, and thus so do w , and E_{pk} :
 $(\phi_b - v_{AB})$ replaces ϕ_b , so that both w and E_{pk}
 increase with increasing reverse bias

2. Currents flow

(The topic of Lectures 5 and 6)

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