Lecture 4 - PN Junction and MOS Electrostatics (I)

Semiconductor Electrostatics in Thermal Equilibrium

September 20, 2005

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1. Non-uniformly doped semiconductor in thermal equilibrium
2. Quasi-neutral situation
3. Relationships between $\phi(x)$ and equilibrium carrier concentrations (*Boltzmann relations*), "60 mV Rule"

Reading assignment:

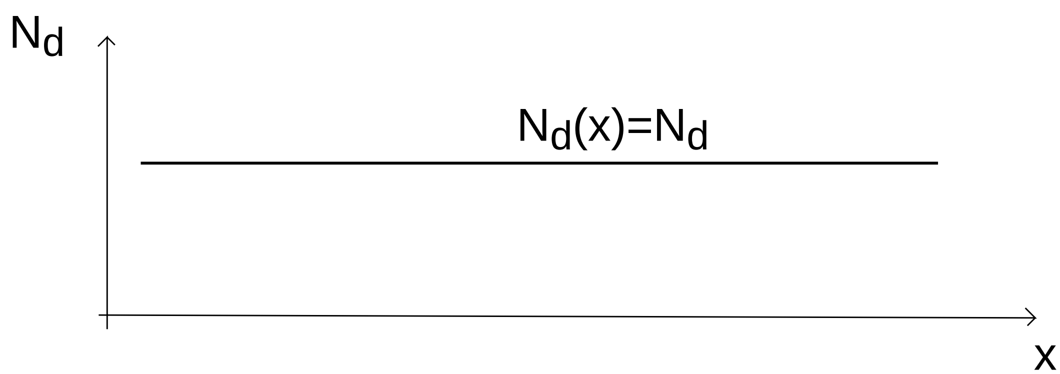
Howe and Sodini, Ch. 3, §§3.1-3.2

Key questions

- Is it possible to have an electric field inside a semiconductor in thermal equilibrium?
- If there is a doping gradient in a semiconductor, what is the resulting majority carrier concentration in thermal equilibrium?

1. Non-uniformly doped semiconductor in thermal equilibrium

Consider first *uniformly doped* n-type Si in thermal equilibrium:

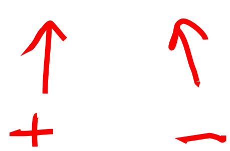


n-type ⇒ lots of electrons, few holes
⇒ focus on electrons

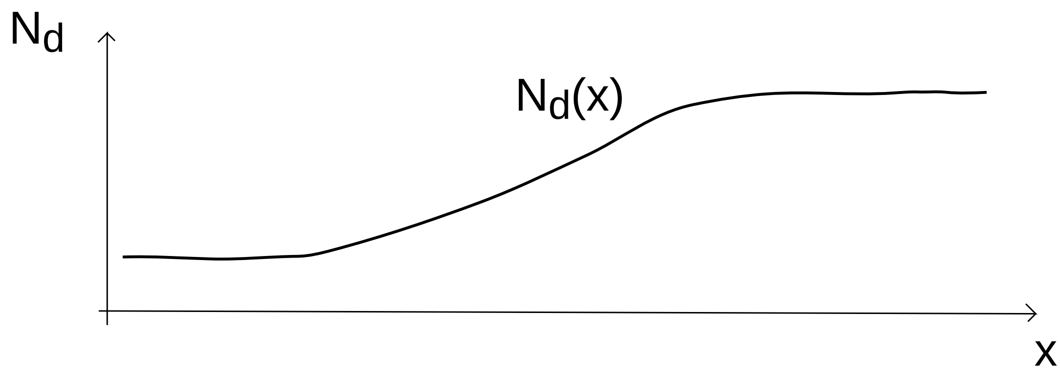
$$n_o = N_d \quad \text{independent of } x$$

Volume charge density $[C/cm^3]$:

$$\rho = q(N_d - n_o) = 0$$



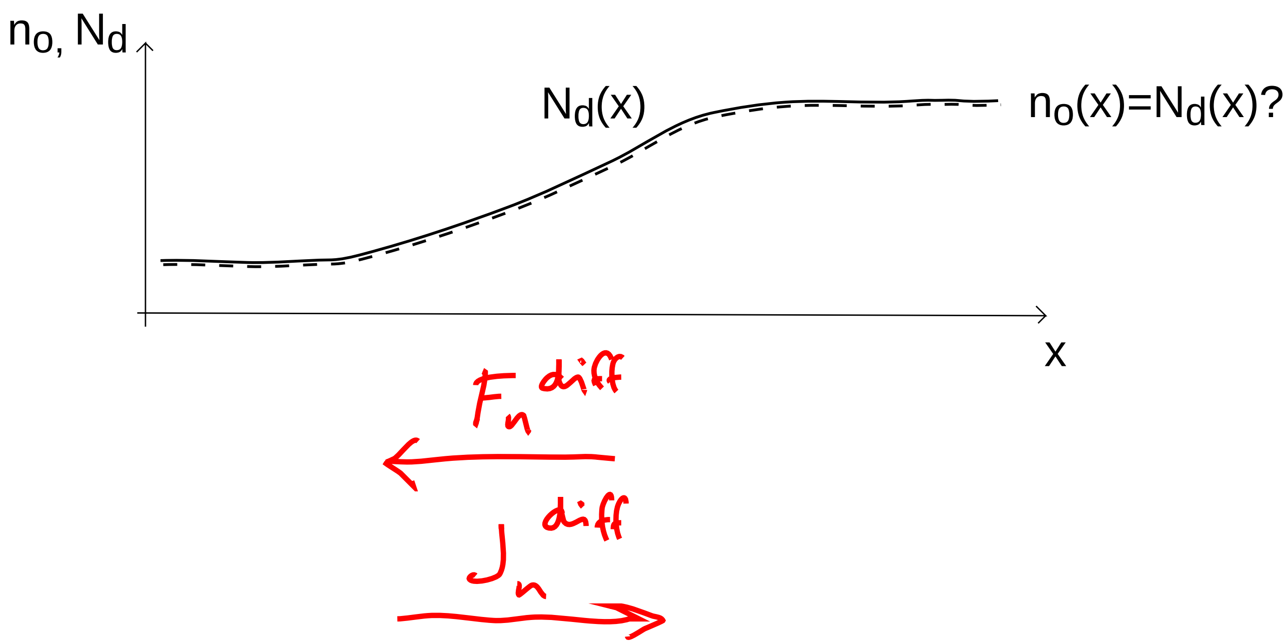
Next, consider piece of n-type Si in thermal equilibrium with *non-uniform dopant distribution*:



What is the resulting electron concentration in thermal equilibrium?

OPTION 1: Every donor gives out one electron ⇒

$$n_o(x) = N_d(x)$$

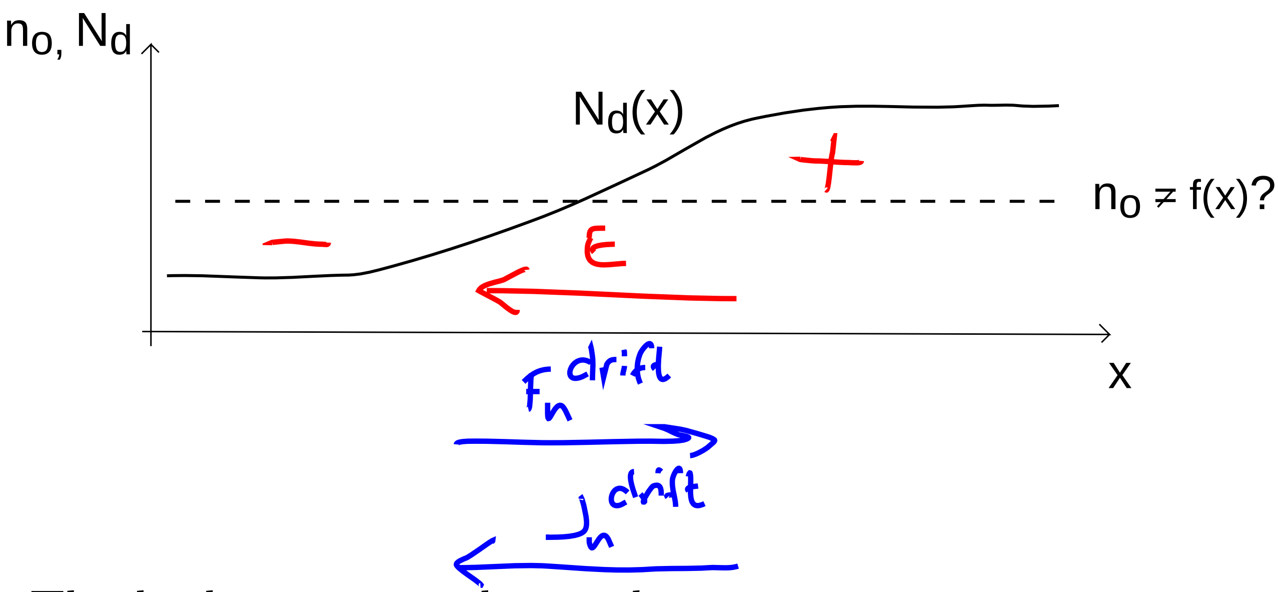


Gradient of electron concentration:

⇒ <u>net electron diffusion</u>
⇒ not thermal equilibrium!

OPTION 2: Electron concentration uniform in space:

$$n_o = n_{ave} \neq f(x)$$



Think about space charge density:

$$\rho(x) = q[N_d(x) - n_o(x)]$$

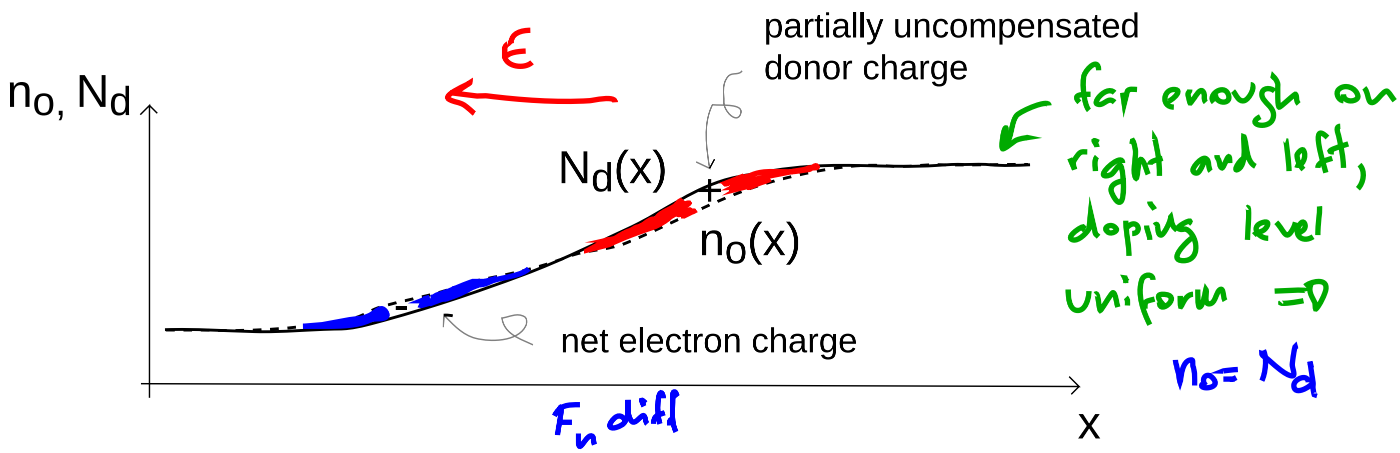
If $N_d(x) \neq n_o(x) \Rightarrow \rho(x) \neq 0$
$\Rightarrow$ electric field
$\Rightarrow$ net electron drift
$\Rightarrow$ not thermal equilibrium!

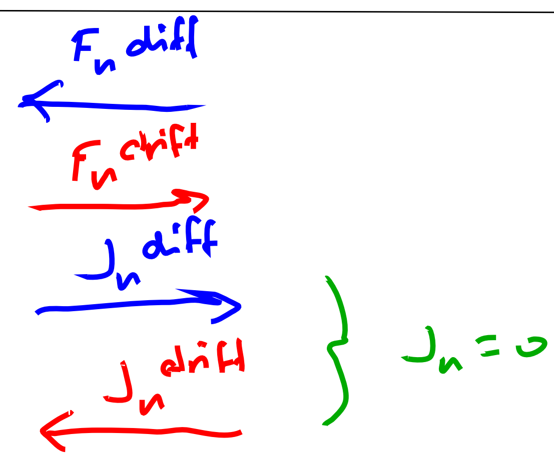
OPTION 3: Demand $J_e = 0$ in thermal equilibrium (and $J_h = 0$ too) at every $x \Rightarrow$ ← required by definition of thermal equilibrium

Diffusion precisely balances drift:

$$J_e(x) = J_e^{drift}(x) + J_e^{diff}(x) = 0 \quad \text{at all } x$$

What is $n_o(x)$ that satisfies this condition?





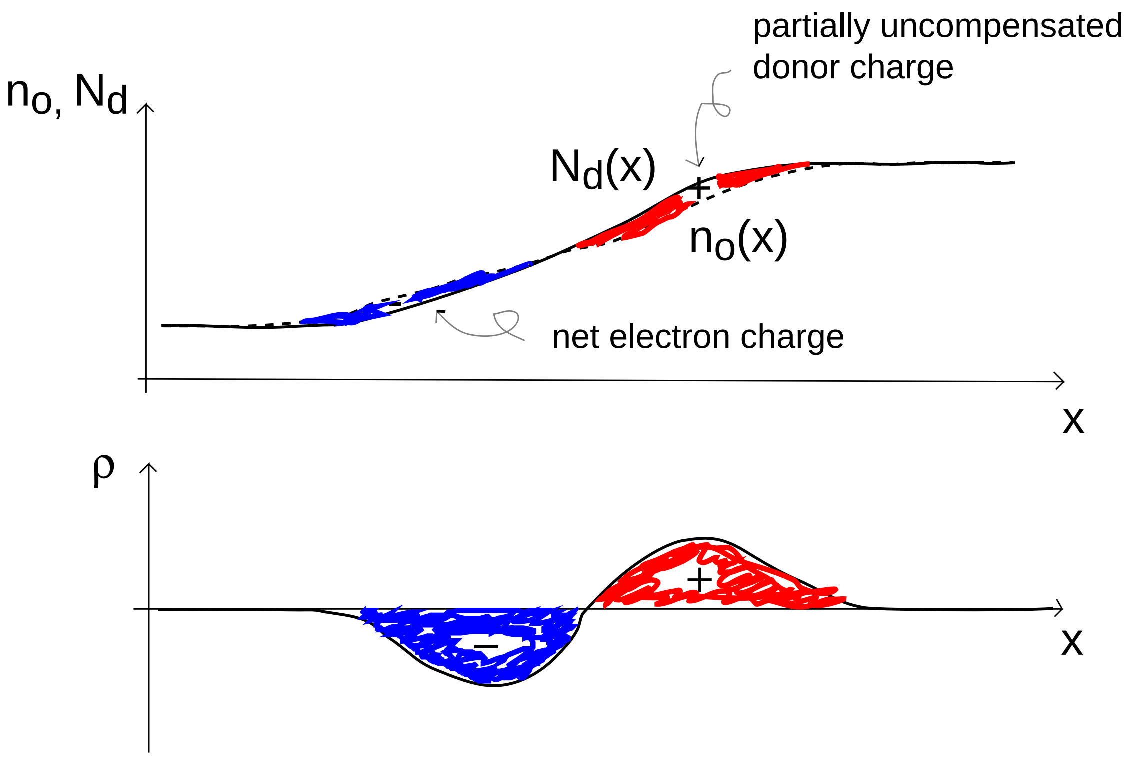
In general, then:

$$n_o(x) \neq N_d(x)$$

What are the implications of this?

• *Space charge density*:

$$\rho(x) = q[N_d(x) - n_o(x)]$$



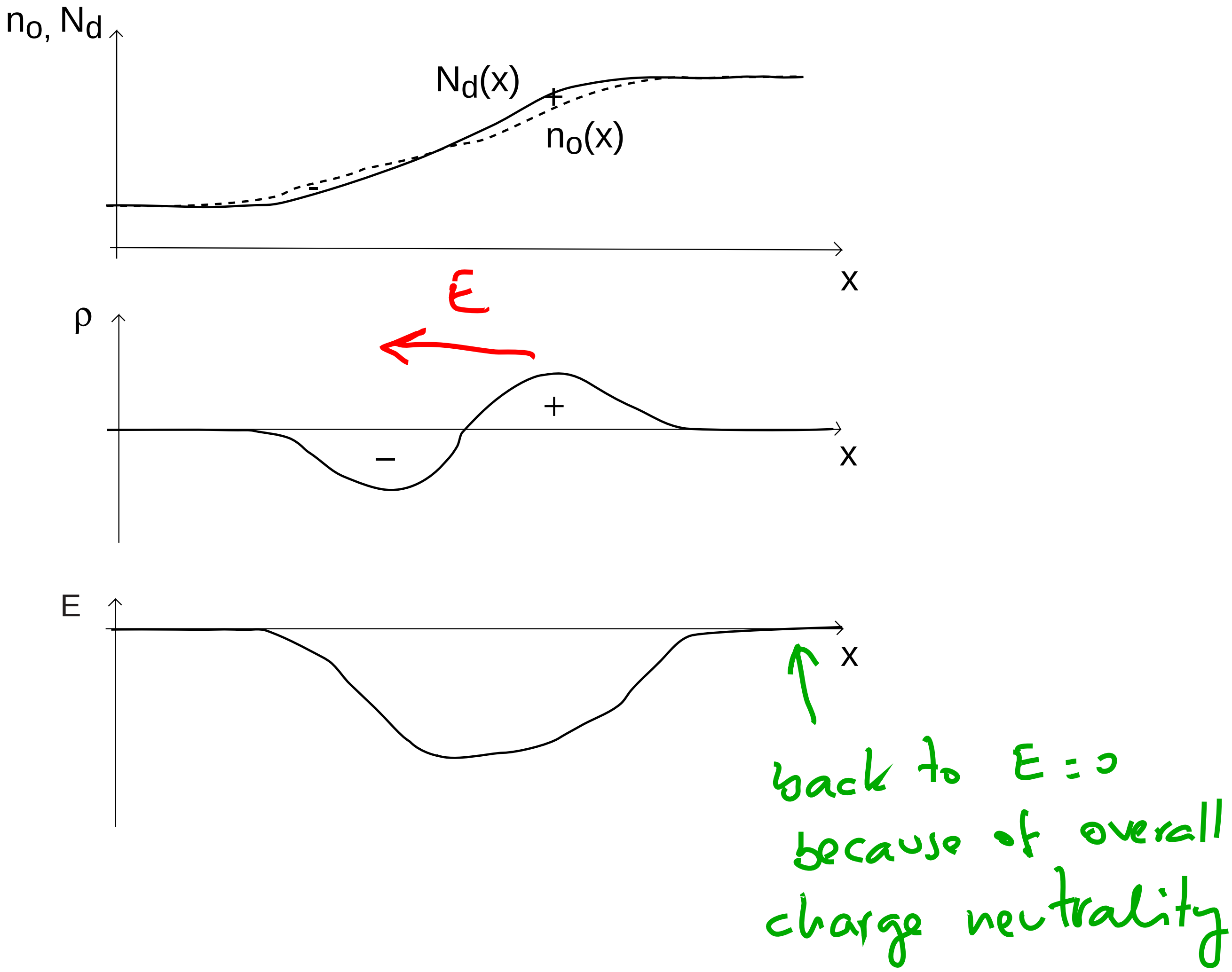
• *Electric field*:

Gauss' equation:

$$\frac{dE}{dx} = \frac{\rho}{\epsilon_s}$$

Integrate from $x = 0$ to x:

$$E(x) - E(0) = \frac{1}{\epsilon_s} \int_0^x \rho(x) dx$$



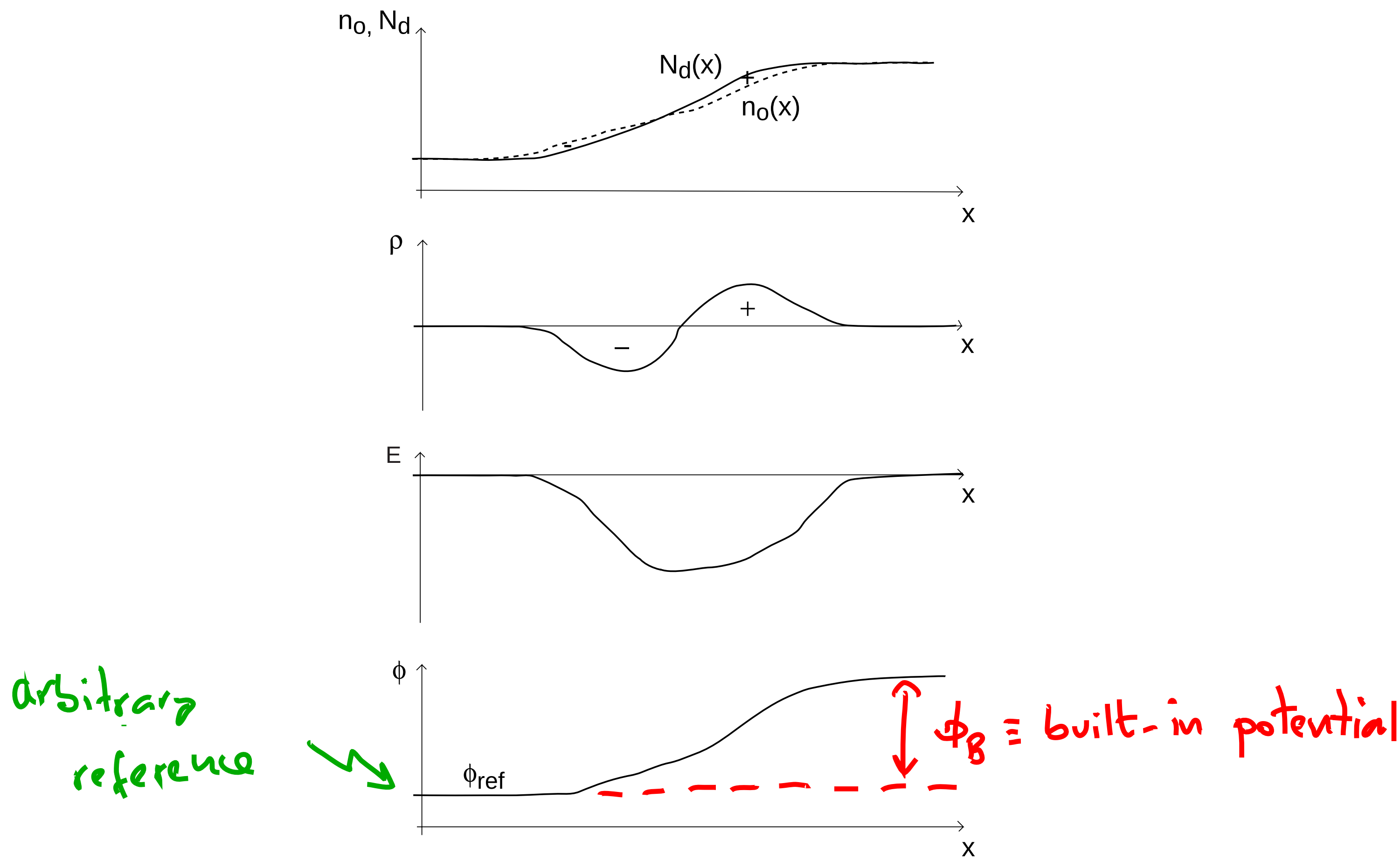
- *Electrostatic potential*:

$$\frac{d\phi}{dx} = -E$$

Integrate from $x = 0$ to x:

$$\phi(x) - \phi(0) = -\int_0^x E(x)dx$$

Need to select reference (physics is in potential difference, not in absolute value!); select $\phi(x = 0) = \phi_{ref}$:



Given $N_d(x)$, want to know $n_o(x)$, $\rho(x)$, $E(x)$, and $\phi(x)$.

Equations that describe problem:

$$J_e = q\mu_n n_o E + qD_n \frac{dn_o}{dx} = 0$$

$$\frac{dE}{dx} = \frac{q}{\epsilon_s}(N_d - n_o)$$

Express them in terms of ϕ:

$$-q\mu_n n_o \frac{d\phi}{dx} + qD_n \frac{dn_o}{dx} = 0 \qquad (1)$$

$$\frac{d^2\phi}{dx^2} = \frac{q}{\epsilon_s}(n_o - N_d) \qquad (2)$$

Plug [1] into [2]:

$$\frac{d^2(\ln n_o)}{dx^2} = \frac{q^2}{\epsilon_s kT}(n_o - N_d) \qquad (3)$$

One equation with one unknown. Given $N_d(x)$, can solve for $n_o(x)$ and all the rest, but...
... no analytical solution for most situations!

2. Quasi-neutral situation

$$\frac{d^2(\ln n_o)}{dx^2} = \frac{q^2}{\epsilon_s kT}(n_o - N_d)$$

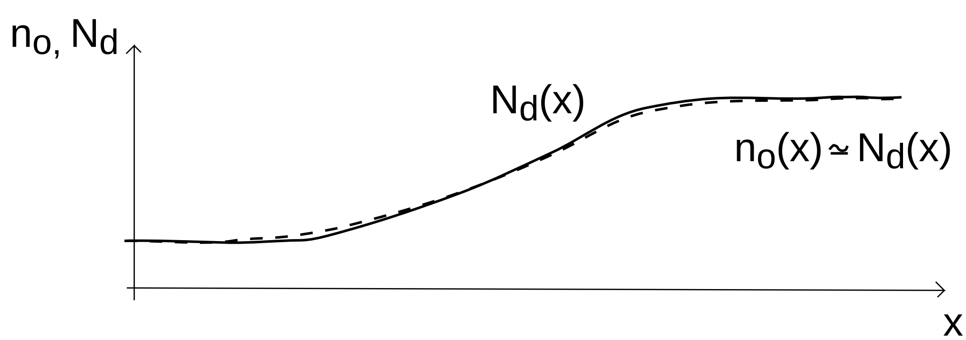
If $N_d(x)$ changes slowly with x:

$\Rightarrow n_o(x)$ also changes slowly with x

$\Rightarrow \frac{d^2(\ln n_o)}{dx^2}$ small

$$\Longrightarrow \boxed{n_o(x) \simeq N_d(x)}$$

$n_o(x)$ tracks $N_d(x)$ well $\Rightarrow$ minimum space charge $\Rightarrow$ semiconductor is *quasi-neutral*



Quasi-neutrality good if:

$$\left|\frac{n_o - N_d}{n_o}\right| \ll 1 \quad \text{or} \quad \left|\frac{n_o - N_d}{N_d}\right| \ll 1$$

3. Relationships between $\phi(x)$ and equilibrium carrier concentrations (Boltzmann relations)

From [1]:

$$\frac{\mu_n}{D_n}\frac{d\phi}{dx} = \frac{1}{n_o}\frac{dn_o}{dx}$$

Using Einstein relation:

$$\frac{q}{kT}\frac{d\phi}{dx} = \frac{d(\ln n_o)}{dx}$$

Integrate:

$$\frac{q}{kT}(\phi - \phi_{ref}) = \ln n_o - \ln n_o(ref) = \ln \frac{n_o}{n_o(ref)}$$

Then:

$$n_o = n_o(ref)e^{q(\phi-\phi_{ref})/kT}$$

Any reference is good.

In 6.012, $\phi_{ref} = 0$ at $n_o(ref) = n_i$.

Then:

$$n_o = n_i e^{q\phi/kT}$$

Boltzmann relations

If do same with holes (starting with $J_h = 0$ in thermal equilibrium, or simply using $n_o p_o = n_i^2$):

$$p_o = n_i e^{-q\phi/kT}$$

Can rewrite as:

$$\phi = \frac{kT}{q} \ln \frac{n_o}{n_i}$$

and

$$\phi = -\frac{kT}{q} \ln \frac{p_o}{n_i}$$

□ **"60 mV" Rule**:

At room temperature for Si:

$$\phi = (25\ mV) \ln \frac{n_o}{n_i} = (25\ mV) \ln(10) \log \frac{n_o}{n_i}$$

Or

$$\boxed{\phi \simeq (60\ mV) \log \frac{n_o}{10^{10}}}$$

For every decade of increase in n_o*,* ϕ *increases by 60 mV at 300K.*

- Example 1:

$$n_o = 10^{18}\ cm^{-3} \Rightarrow \phi = (60\ mV) \times 8 = 480\ mV$$

With holes:

$$\phi = -(25\ mV)\ln\frac{p_o}{n_i} = -(25\ mV)\ln(10)\log\frac{p_o}{n_i}$$

Or

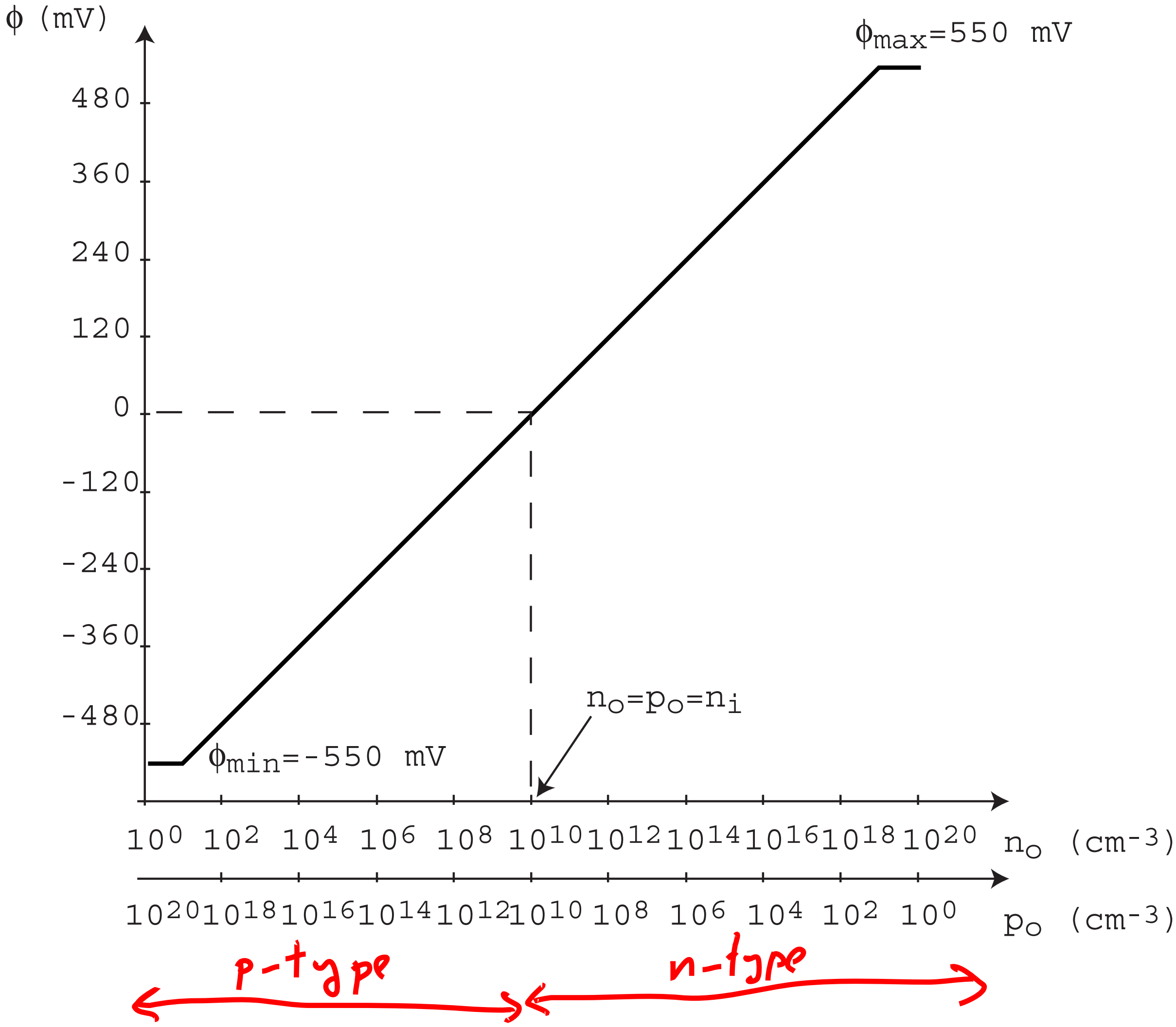
$$\boxed{\phi \simeq -(60\ mV)\log\frac{p_o}{10^{10}}}$$

- EXAMPLE 2:

$$n_o = 10^{18}\ cm^{-3}\ \Rightarrow\ p_o = 10^2\ cm^{-3}$$

$$\Rightarrow \phi = -(60\ mV)\times(-8) = 480\ mV$$

Relationship between ϕ and n_o and p_o:



Note: ϕ cannot exceed $550\,mV$ or be smaller than $-550\,mV$ (beyond these points, different physics come into play).

• EXAMPLE 3: Compute potential difference in thermal equilibrium between region where $n_o = 10^{17}\ cm^{-3}$ and region where $p_o = 10^{15}\ cm^{-3}$:

$$\phi(n_o = 10^{17}\ cm^{-3}) = 60 \times 7 = 420\ mV$$

$$\phi(p_o = 10^{15}\ cm^{-3}) = -60 \times 5 = -300\ mV$$

$$\phi(n_o = 10^{17}\ cm^{-3}) - \phi(p_o = 10^{15}\ cm^{-3}) = 720\ mV$$

• EXAMPLE 4: Compute potential difference in thermal equilibrium between region where $n_o = 10^{20}\ cm^{-3}$ and region where $p_o = 10^{16}\ cm^{-3}$:

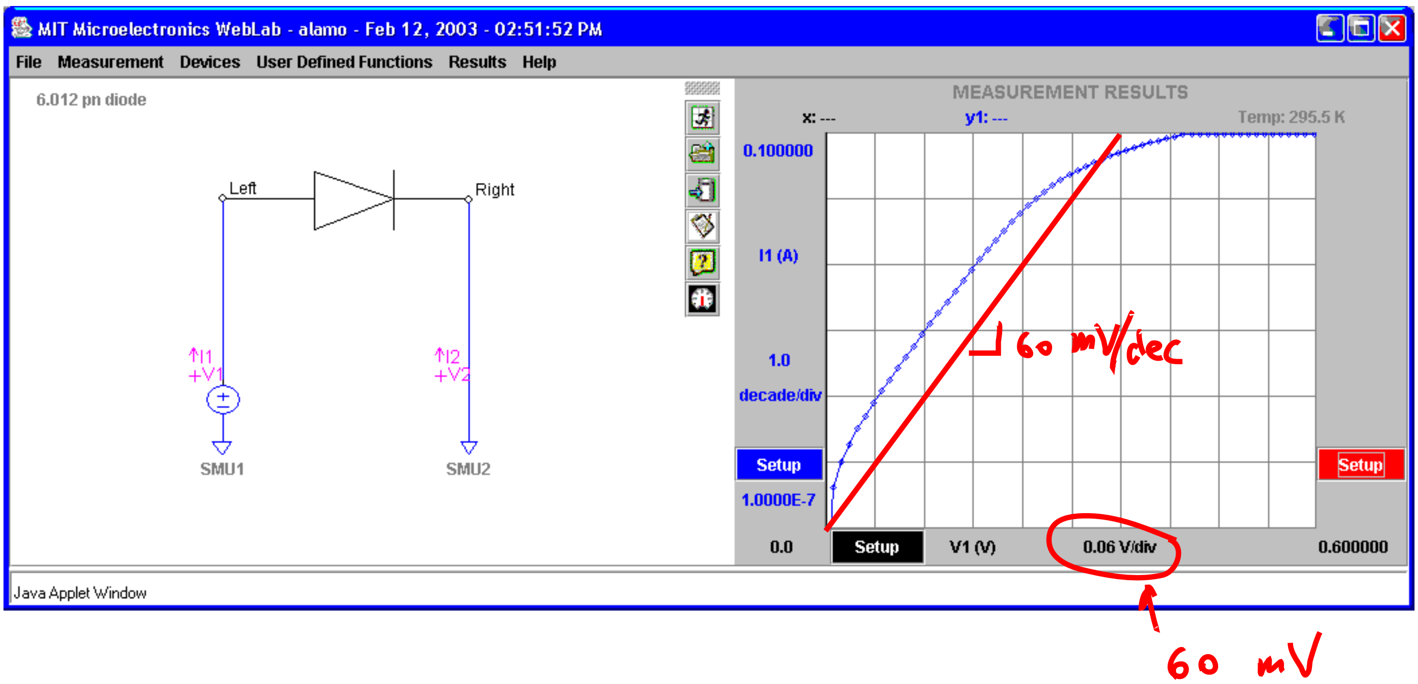
$$\phi(n_o = 10^{20}\ cm^{-3}) = \phi_{max} = 550\ mV$$

$$\phi(p_o = 10^{16}\ cm^{-3}) = -60 \times 6 = -360\ mV$$

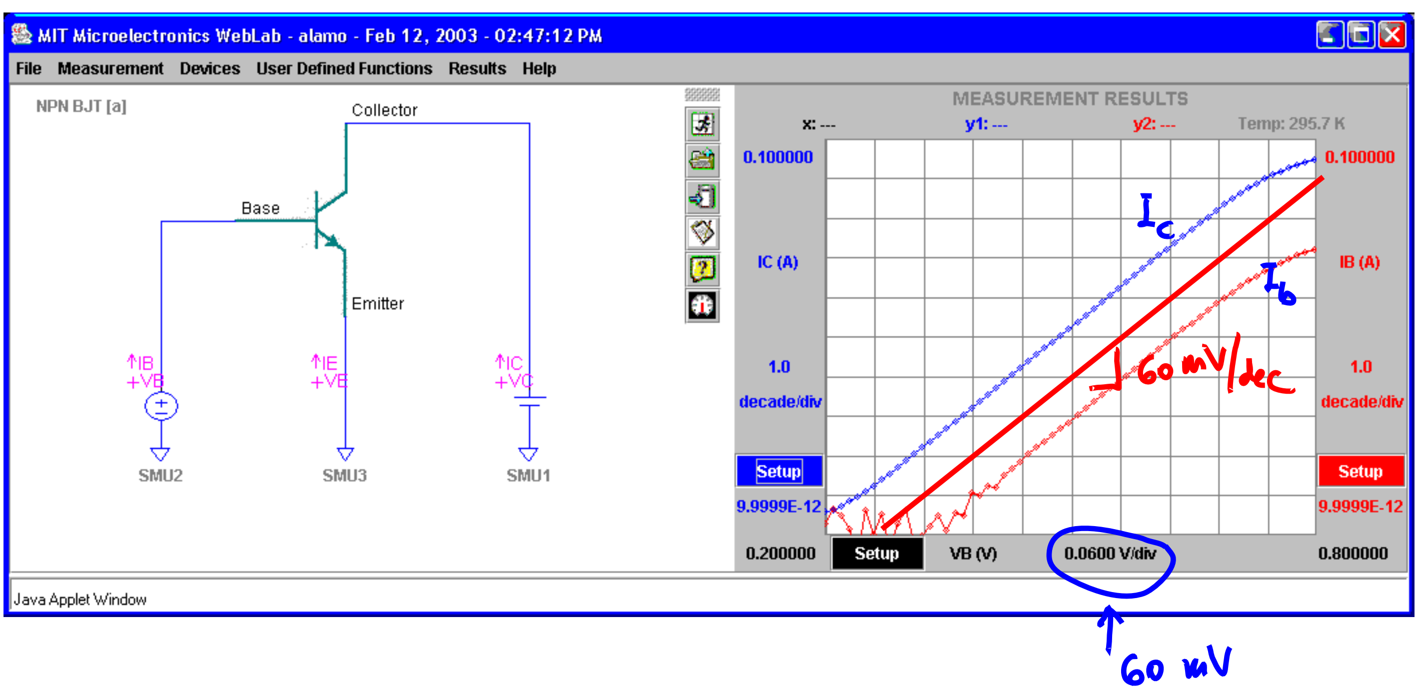
$$\phi(n_o = 10^{20}\ cm^{-3}) - \phi(p_o = 10^{16}\ cm^{-3}) = 910\ mV$$

Boltzmann relations readily seen in device behavior!

☐ pn diode current-voltage characteristics:



☐ Bipolar transistor transfer characteristics:



Key conclusions

- It is possible to have an electric field inside a semiconductor in thermal equilibrium

  ⇒ *non-uniform doping distribution.*

- In a slowly varying doping profile, majority carrier concentration tracks well doping concentration.

- In thermal equilibrium, there are fundamental relationships between $\phi(x)$ and the equilibrium carrier concentrations

  ⇒ *Boltzmann relations* (or "*60 mV Rule*").

↓

readily seen in devices!