Lecture 3 - Semiconductor Physics (II)

Carrier Transport

September 15, 2005

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1. Thermal motion
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Reading assignment:

Howe and Sodini, Ch. 2, §§2.4-2.6

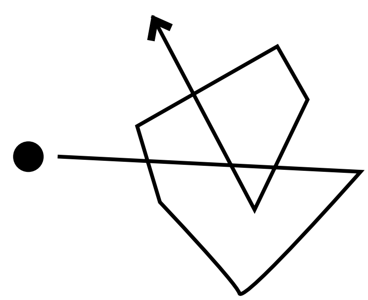
Key questions

- What are the physical mechanisms responsible for current flow in semiconductors?
- How do electrons and holes in a semiconductor behave in an electric field?
- How do electrons and holes in a semiconductor behave if their concentration is non-uniform in space?

1. Thermal Motion

In thermal equilibrium, carriers are not sitting still:

- undergo collisions with vibrating Si atoms (*Brownian motion*)
- electrostatically interact with charged dopants and with each other



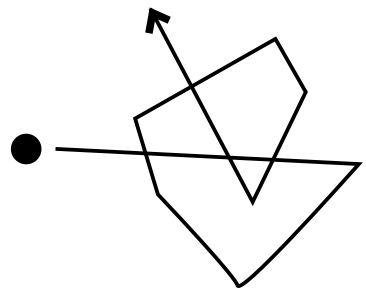
Characteristic time constant of thermal motion - mean free time between collisions:

$$\tau_c \equiv \textit{collision time}\ [s]$$

In between collisions, carriers acquire high velocity:

$$v_{th} \equiv \textit{thermal velocity}\ [cm/s]$$

...but get nowhere!
on average



Characteristic length of thermal motion:

$$\lambda \equiv \ mean\ free\ path\ [cm]$$

$$\lambda = v_{th}\tau_c$$

Put numbers for Si at 300 K:

$$\tau_c \simeq 10^{-14} \sim 10^{-13}\ s$$ (<< 1 ps)

$$v_{th} \simeq 10^7\ cm/s$$

$$\Rightarrow \lambda \ \simeq 1 \sim 10\ nm$$

For reference, state-of-the-art MOSFETs today:

$$L_g \simeq \sout{0.1}\ \mu m$$ ≃ 50 nm

⇒ carriers undergo many collisions in modern devices

(but picture changing over next few years)

2. Carrier Drift

Apply electric field to semiconductor:

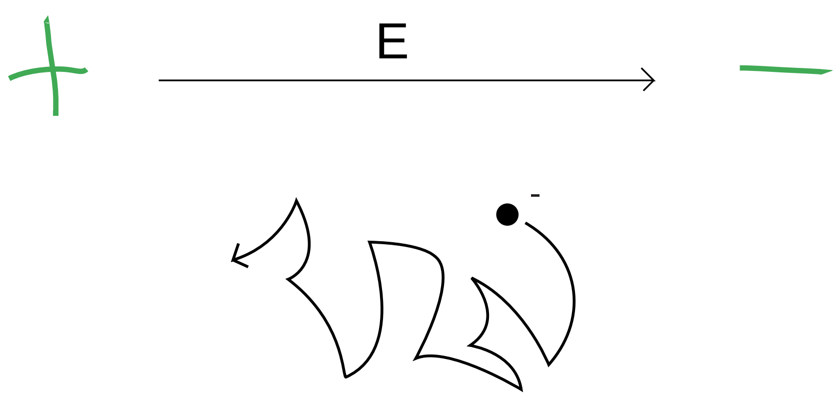
$$E \equiv \textit{electric field}\,[V/cm]$$

$\Rightarrow$ net force on carrier

holes

$$F = \pm qE$$

electrons

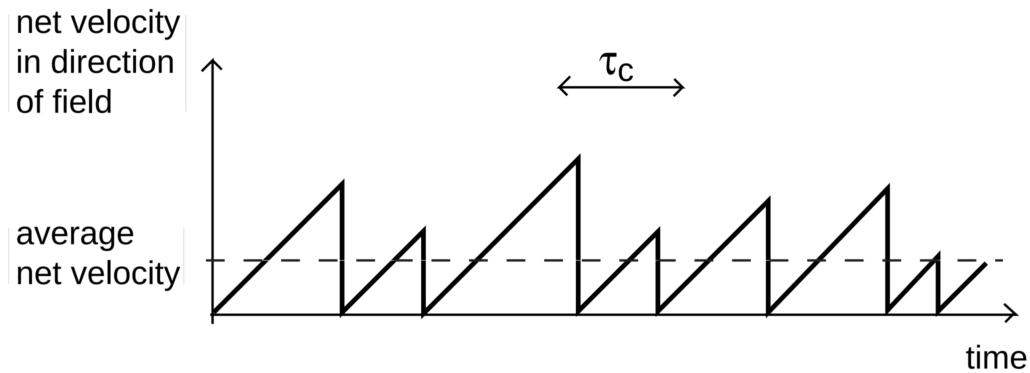


Between collisions, carriers accelerate in direction of field:

$$v(t) = at = -\frac{qE}{m_n}t \quad \text{for electrons}$$

$$v(t) = \frac{qE}{m_p}t \quad \text{for holes}$$

But velocity randomized every τ_c (on average):



Then, average net velocity in direction of field:

holes → $+$; electrons → $-$

$$\overline{v} = v_d = \pm\frac{qE}{2m_{n,p}}\tau_c = \pm\frac{q\tau_c}{2m_{n,p}}E$$

$m_n \equiv$ electrons

$m_p \equiv$ holes ← account for carrier interactions with "perfect" crystal

This is called ***drift velocity*** $[cm/s]$.

Define:

$$\mu_{n,p} = \frac{q\tau_c}{2m_{n,p}} \equiv \textit{mobility}\ [cm^2/V \cdot s]$$

Any imperfections: $\tau_c \downarrow \Rightarrow \mu_n \downarrow$

Then, for electrons:

$$v_{dn} = -\mu_n E$$

for holes:

$$v_{dp} = \mu_p E$$

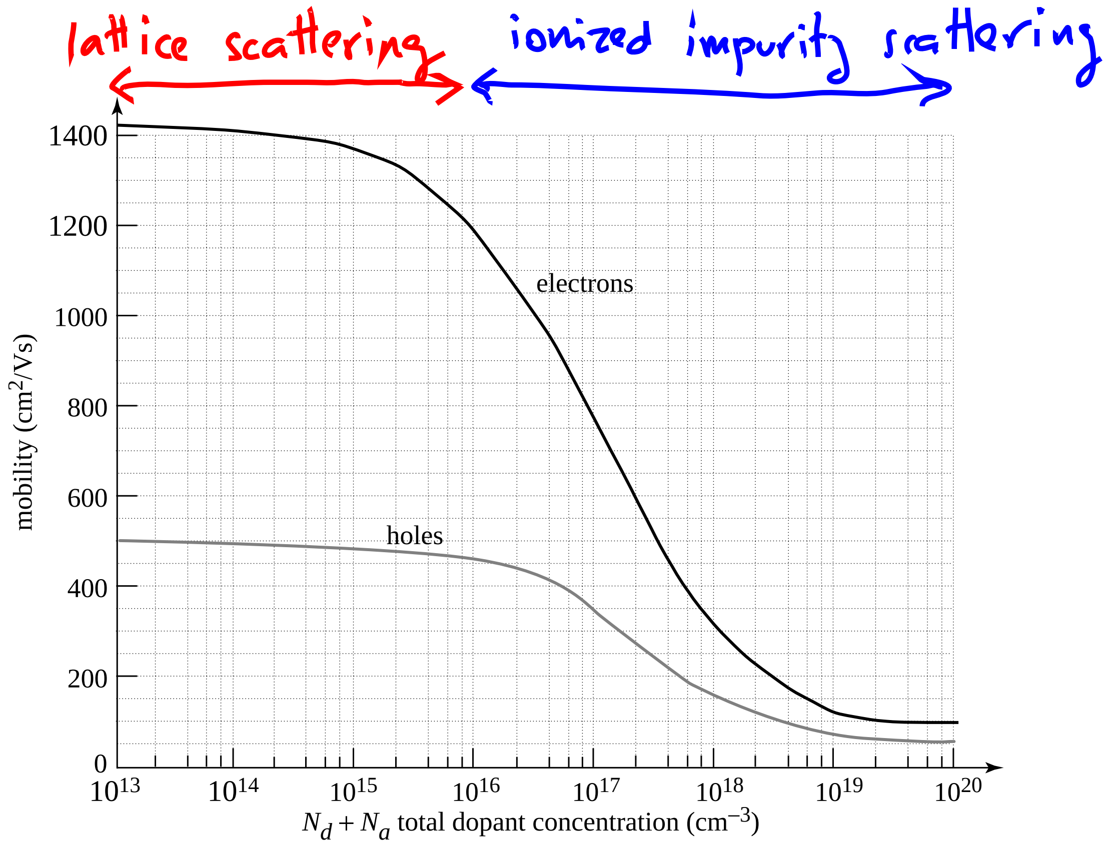
most important result:

drift velocity $\propto$ electric field

Mobility is measure of *ease* of carrier drift:

- if $\tau_c \uparrow$, longer time between collisions $\rightarrow \mu \uparrow$
- if $m \downarrow$, "lighter" particle $\rightarrow \mu \uparrow$

Mobility depends on doping. For Si at 300K:



- for low doping level, μ limited by collisions with lattice
- for medium and high doping level, μ limited by collisions with ionized impurities
- holes "heavier" than electrons:
  $\rightarrow$ for same doping level, $\mu_n > \mu_p$

Drift current

Net velocity of charged particles ⇒ electric current:

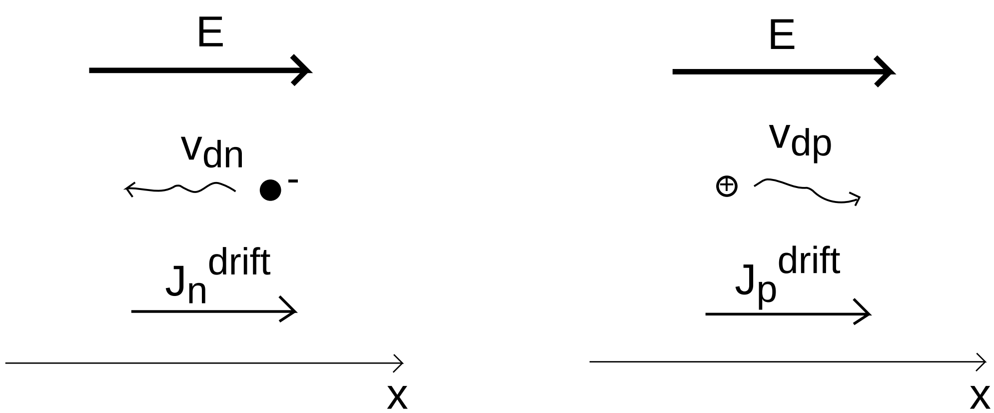
$$\begin{aligned} \textit{Drift current density} &\propto \textit{carrier drift velocity} \\ &\propto \textit{carrier concentration} \\ &\propto \textit{carrier charge} \end{aligned}$$

Drift currents:

$$J_n^{drift} = -qnv_{dn} = qn\mu_n E$$

$$J_p^{drift} = qpv_{dp} = qp\mu_p E$$

Check signs:



Total drift current:

$$J^{drift} = J_n^{drift} + J_p^{drift} = q(n\mu_n + p\mu_p)E$$

Has the shape of *Ohm's Law*:

$$J = \sigma E = \frac{E}{\rho}$$

Where:

$$\sigma \equiv conductivity\ [\Omega^{-1} \cdot cm^{-1}]$$

$$\rho \equiv resistiviy\ [\Omega \cdot cm]$$

Then:

$$\rho = \frac{1}{\sigma} = \frac{1}{q(n\mu_n + p\mu_p)}$$

Resistivity commonly used to specify doping level.

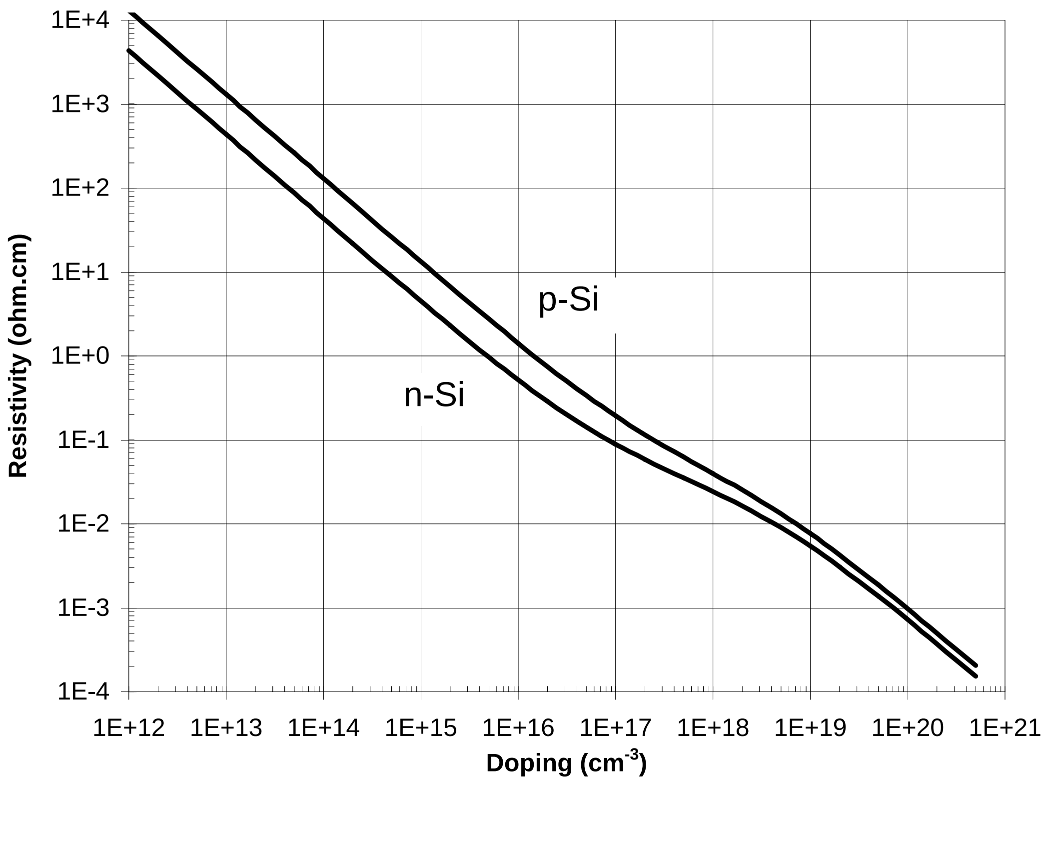
- In n-type semiconductor:

$$\rho_n \simeq \frac{1}{qN_d\mu_n}$$

- In p-type semiconductor:

$$\rho_p \simeq \frac{1}{qN_a\mu_p}$$

For Si at 300K:



Numerical example:

- Si with $N_d = 3 \times 10^{16}\ cm^{-3}$ at 300 K

$$\mu_n \simeq 1000\ cm^2/V \cdot s$$

$$\rho_n \simeq 0.21\ \Omega \cdot cm$$

- apply $|E| = 1\ kV/cm$ (1 V across 10 μm)

$$|v_{dn}| \simeq 10^6\ cm/s \ll v_{th}$$

$$|J_n^{drift}| \simeq 4.8 \times 10^3\ A/cm^2$$

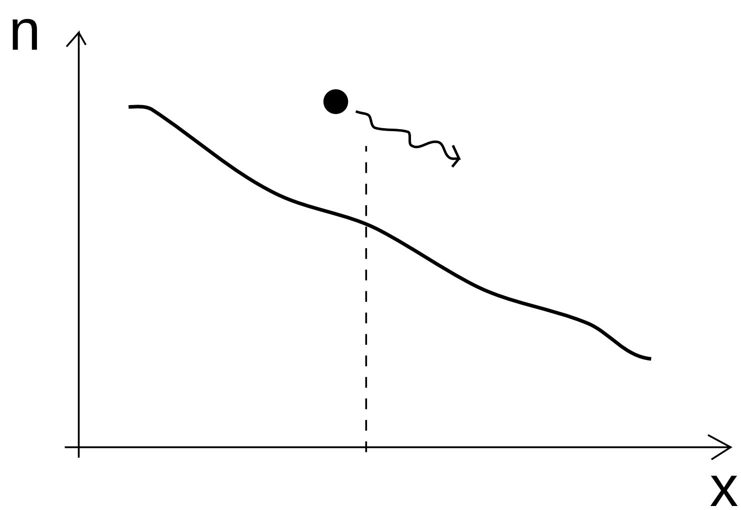
- time to drift through $L = 0.1\ \mu m$:

$$t_d = \frac{L}{v_{dn}} = 10\ ps$$ (~ 16 GHz)

fast!

3. Carrier diffusion

Diffusion: particle movement in response to concentration gradient.



Elements of diffusion:

- a medium (*Si crystal*)
- a gradient of particles (*electrons and holes*) inside the medium
- collisions between particles and medium send particles off in random directions:
  → overall, particle movement down the gradient

Key diffusion relationship (*Fick's first law*):

particles diffuse from high to low concentrations

$$\textit{Diffusion flux} \propto - \textit{concentration gradient}$$

Flux ≡ number of particles crossing unit area per unit time $[cm^{-2} \cdot s^{-1}]$

For electrons:

$$F_n = -D_n \frac{dn}{dx}$$

For holes:

$$F_p = -D_p \frac{dp}{dx}$$

$D_n \equiv$ electron diffusion coefficient $[cm^2/s]$
$D_p \equiv$ hole diffusion coefficient $[cm^2/s]$

D measures the ease of carrier diffusion in response to a concentration gradient: $D \uparrow \Rightarrow F^{diff} \uparrow$.

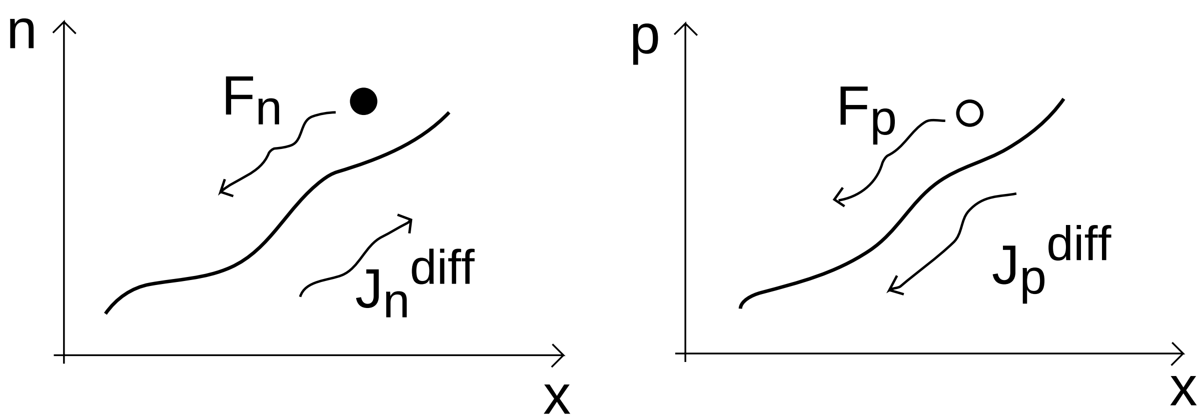
D limited by vibrating lattice atoms and ionized dopants

Diffusion current density = charge × carrier flux

$$J_n^{diff} = qD_n \frac{dn}{dx}$$

$$J_p^{diff} = -qD_p \frac{dp}{dx}$$

Check signs:



Einstein relation

At the core of diffusion and drift is same physics: collisions among particles and medium atoms

⇒ there should be a relationship between D and μ

Einstein relation [don't derive in 6.012]:

$$\frac{D}{\mu} = \frac{kT}{q}$$

Boltzmann constant
$k = 8.62 \times 10^{-5}$ eV/K

In semiconductors:

$$\boxed{\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{q}}$$

$$\frac{kT}{q} \equiv \textit{thermal voltage}\ [V]$$

At 300 K:

$$\frac{kT}{q} \simeq 25\ mV$$

For example: for $N_d = 3 \times 10^{16}\ cm^{-3}$:

$$\mu_n \simeq 1000\ cm^2/V \cdot s \rightarrow D_n \simeq 25\ cm^2/s$$
$$\mu_p \simeq 400\ cm^2/V \cdot s \rightarrow D_p \simeq 10\ cm^2/s$$

Total current

In general, current can flow by drift and diffusion separately. Total current:

$$J_n = J_n^{drift} + J_n^{diff} = qn\mu_n E + qD_n \frac{dn}{dx}$$

$$J_p = J_p^{drift} + J_p^{diff} = qp\mu_p E - qD_p \frac{dp}{dx}$$

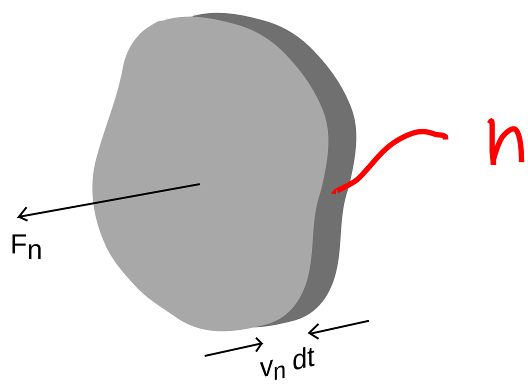
And

$$J_{total} = J_n + J_p$$

Summary: relationship between v, F, and J

In semiconductors: charged particles move
$\Rightarrow$ particle flux $\Rightarrow$ electrical current density

Particle flux: number of particles that cross surface of unit area placed normal to particle flow every unit time



Relationship between particle flux and velocity:

$$F_n = nv_n \qquad F_p = pv_p$$

Current density: amount of charge that crosses surface of unit area placed normal to particle flow every unit time

$$J_n = -qF_n = -qnv_n \qquad J_p = qF_p = qpv_p$$

whether carriers move by drift or diffusion.

Key conclusions

- Electrons and holes in semiconductors are mobile and charged ⇒ *carriers* of electrical current!
- *Drift current*: produced by electric field

$$J^{drift} \propto E$$

- *Diffusion current*: produced by concentration gradient

$$J^{diff} \propto \frac{dn}{dx}, \frac{dp}{dx}$$

- Carriers move fast in response to fields and gradients
- Diffusion and drift currents are sizable in modern devices