October 13, 2005 - Quiz #1

Name: SOLUTIONS

Recitation:

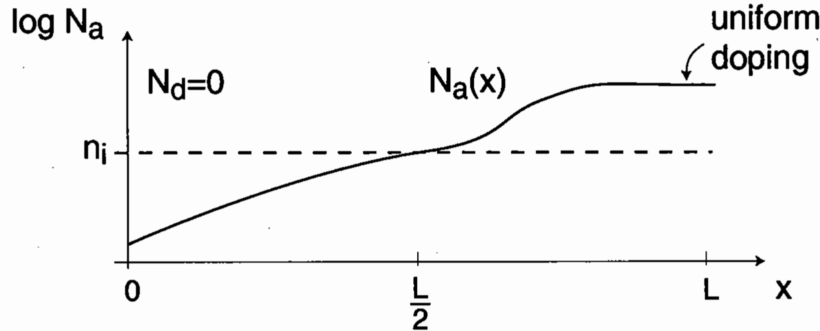
problem	grade
1	
2	
3	
4	
total	

General guidelines (please read carefully before starting):

- Make sure to write your name on the space designated above.
- **Open book**: you can use any material you wish.
- All answers should be given in the space provided. Please do not turn in any extra material. If you need more space, use the back page.
- You have **120 minutes** to complete your quiz.
- Make reasonable approximations and *state them*, i.e. quasi-neutrality, depletion approximation, etc.
- Partial credit will be given for setting up problems without calculations. **NO** credit will be given for answers without reasons.
- Use the symbols utilized in class for the various physical parameters, i.e. μ_n, I_D, E, etc.
- Every numerical answer must have the proper units next to it. Points will be subtracted for answers without units or with wrong units.
- Use $\phi = 0$ at $n_o = p_o = n_i$ as potential reference.
- Use the following fundamental constants and physical parameters for silicon and silicon dioxide at room temperature:

$$n_i = 1 \times 10^{10}\ cm^{-3}$$
$$kT/q = 0.025\ V$$
$$q = 1.60 \times 10^{-19}\ C$$
$$\epsilon_s = 1.05 \times 10^{-12}\ F/cm$$
$$\epsilon_{ox} = 3.45 \times 10^{-13}\ F/cm$$

1. *(25 points)* A bar of silicon is doped with acceptors as shown below. The doping density variers smoothly and motononically in the x direction from $N_A \ll n_i$ at $x = 0$ to $N_A \gg n_i$ at $x = L$. $N_a(L/2) = n_i$. Around $x = L$, the acceptor profile becomes uniform. The donor density is zero everywhere. This is a thermal equilibrium situation.



On the basis of this description, answer the following questions by circling the correct answer. Write a brief justification for your choice below.

(1a) *(2 points)* Where is the hole concentration the greatest?

$x = 0$ $\quad$ $0 < x < L/2$ $\quad$ $x = L/2$ $\quad$ $L/2 < x < L$ $\quad$ $x = L$ (circled) $\quad$ uniform

The hole concentration is greatest where Na is greatest.

(1b) *(2 points)* Where is the electron concentration the greatest?

$x = 0$ (circled) $\quad$ $0 < x < L/2$ $\quad$ $x = L/2$ $\quad$ $L/2 < x < L$ $\quad$ $x = L$ $\quad$ uniform

The electron concentration is greatest where the hole concentration is lowest which is where Na is lowest.

(1c) *(2 points)* In which direction does the hole diffusion current flow?

$-\vec{x}$ (circled) $\quad$ no current $\quad$ $+\vec{x}$

Holes diffuse down the gradient, from right to left. Hence, the hole diffusion current flows from right to left.

(1d) *(2 points)* In which direction does the hole drift current flow?

$-\vec{x}$ no current $+\vec{x}$ (circled)

In thermal equilibrium hole drift current has to be opposite to hole diffusion current.

(1e) *(2 points)* In which direction does the electron diffusion current flow?

$-\vec{x}$ (circled) no current $+\vec{x}$

Electrons diffusion from left to right. Then, the diffusion current flows from right to left.

(1f) *(2 points)* In which direction does the electron drift current flow?

$-\vec{x}$ no current $+\vec{x}$ (circled)

Electron drift current has to be opposite to electron diffusion current.

(1g) *(2 points)* In which direction does the electric field point?

$-\vec{x}$ no field $+\vec{x}$ (circled)

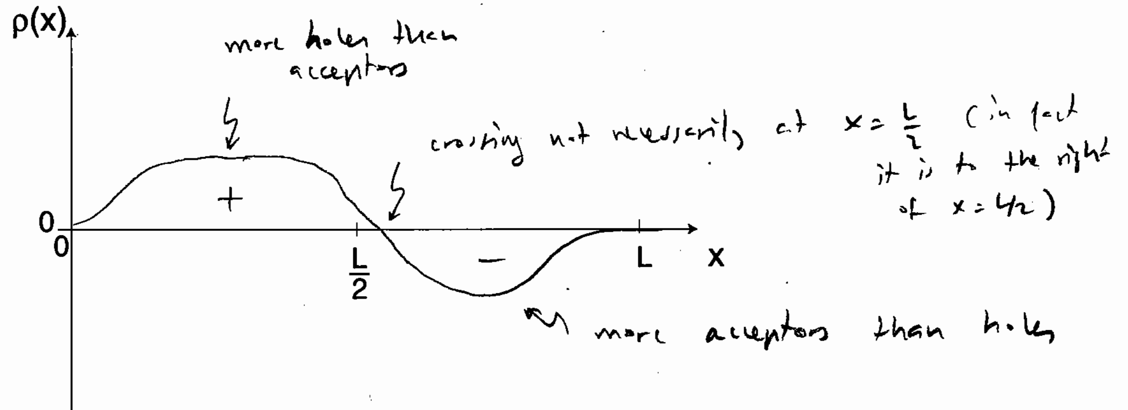
This is the sign that gets holes to drift to right and electrons to drift to the left.

(1h) *(2 points)* Where is the electrostatic potential the greatest?

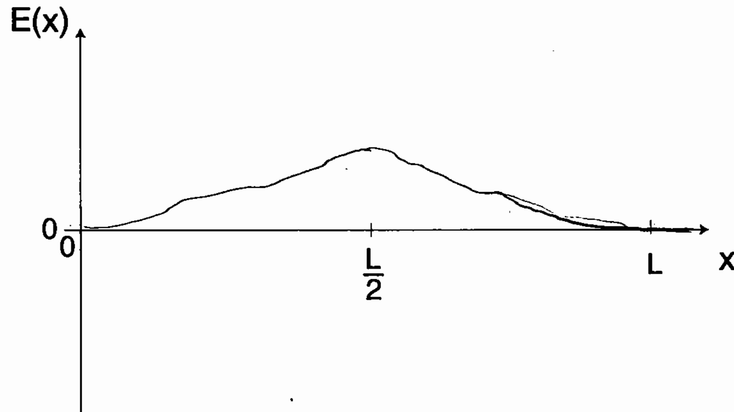
$x = 0$ (circled) $0 < x < L/2$ $x = L/2$ $L/2 < x < L$ $x = L$ uniform

The potential is highest on the left end of the sample because the electric field is positive everywhere.

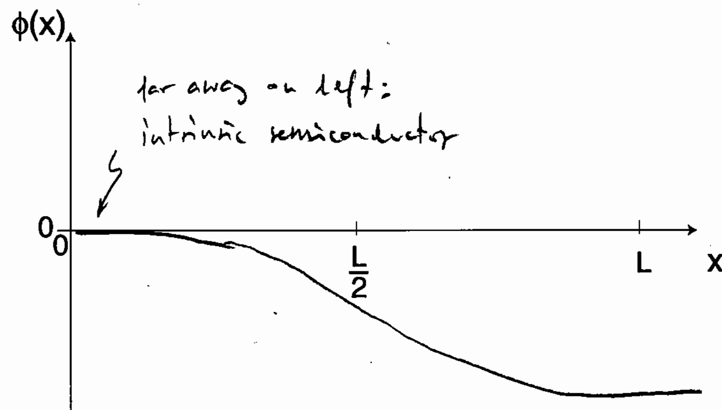
(1i) *(3 points)* In the axis provided below, sketch the volume charge density along x.



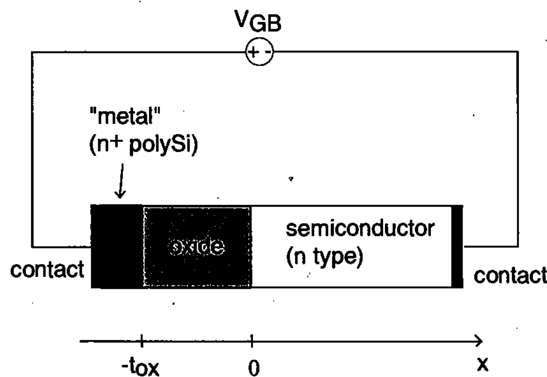
(1j) *(3 points)* In the axis provided below, sketch the electric field distribution along x.



(1k) *(3 points)* In the axis provided below, sketch the electrostatic potential distribution along x. Use as reference $\phi = 0$ at $n_o = p_o = n_i$.

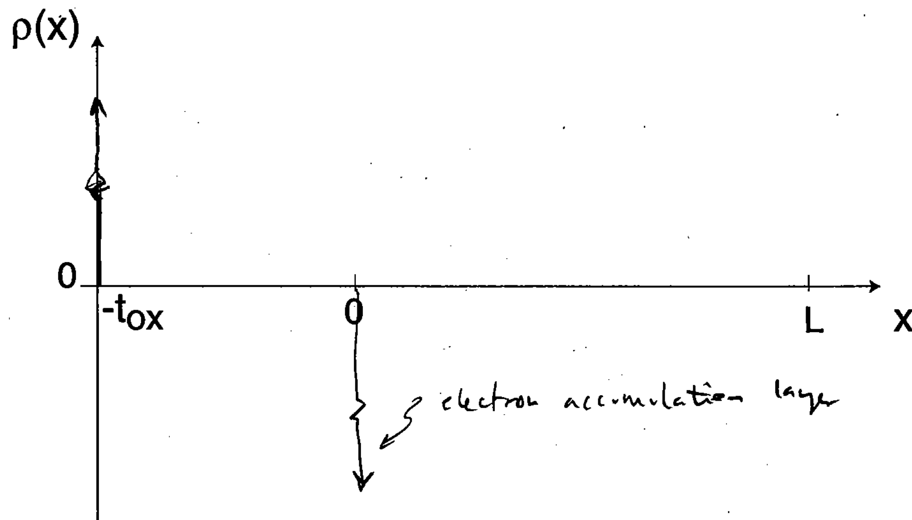


2. *(10 points)* Consider an MOS structure on an n-type substrate:



The doping level in the substrate (or body) is $N_D = 10^{17}\ cm^{-3}$. The doping level in the gate is $N_D^+ = 10^{20}\ cm^{-3}$.

(2a) *(5 points)* In the axis below, *qualitatively* sketch the volume charge density across this structure at zero bias. Explain your result.



The built-in potential of this structure is:

$$\phi_B = \phi_g - \phi_n = \phi_{n+} - \frac{kT}{q} \ln \frac{N_D}{n_i} = 0.55 - 0.025 \ln \frac{10^{17}}{10^{10}} =$$

$$= 0.15\ V$$

This is positive. This demands a positive sheet of charge at the metal-oxide interface and negative charge in the semiconductor. The only way to do this is to have an accumulation layer of electrons at the oxide-semiconductor interface.

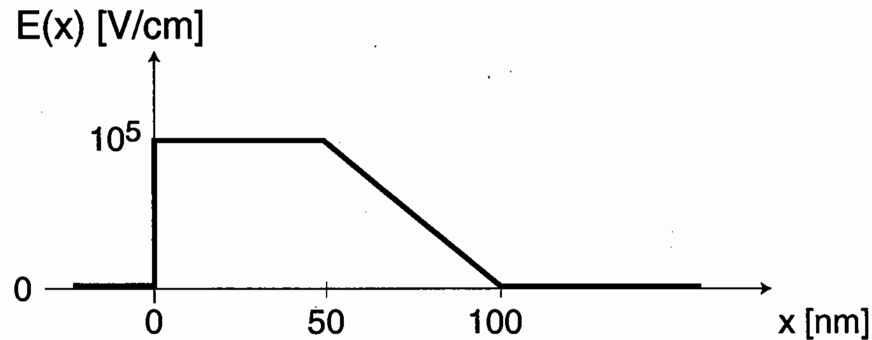
(2b) *(5 points)* Calculate the flatband voltage of this structure *(numerical answer with appropriate sign and units expected).*

The flatband voltage is simply:

$$V_{FB} = -\phi_B = -0.15 \text{ V}$$

One has to apply a negative value of V_{FB} to wipe out the electron accumulation layer.

3. *(30 points)* Consider a pn junction at zero bias with an electric field distribution as sketched below. The metallurgical junction is placed at $x = 0$.



(3a) *(10 points)* Calculate the depletion capacitance at zero bias *(numerical answer with appropriate sign and units expected).*

At zero bias, the depletion region thickness is 100 nm. The depletion capacitance is then:

$$C_j = \frac{\epsilon_s}{x_d} = \frac{1.05 \times 10^{-12}\ F/cm}{100 \times 10^{-7}\ cm} = 1.1 \times 10^{-7}\ F/cm^2$$

(3b) *(5 points)* Calculate the built-in potential *(numerical answer with appropriate sign and units expected).*

This is simply the integral of the electric field at zero bias:

$$\phi_B = 10^5 \frac{V}{cm} \left(50 \times 10^{-7} + \frac{1}{2}\, 50 \times 10^{-7} \right) \cdot cm = 0.75\ V$$

(3c) *(5 points)* Estimate the doping type and doping level of the region between $50 < x < 100$ *nm* *(numerical answer with appropriate sign and units expected).*

Since the field is positive for $x > 0$, then this is the <u>p region</u> of the p-n junction. This is because the negative charge required there is supported by uncompensated acceptors.

In the $50 < x < 100$ nm region there is a ramp. This suggests a uniform charge concentration given by a uniform p-type doping level. The volume charge density is:

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\epsilon_s} \Rightarrow \rho = \epsilon_s \frac{d\mathcal{E}}{dx} = -1.05\times10^{-12}\,\frac{F}{cm} \times \frac{10^5\ V/cm}{50\times10^{-7}\ cm} =$$

$$= -2.1\times10^{-2}\ C/cm^3$$

The doping level is then:

$$\rho = -qN_a \Rightarrow N_a = -\frac{\rho}{q} = +\frac{2.1\times10^{-2}\ C/cm^3}{1.6\times10^{-19}\ C} = 1.3\times10^{17}\ cm^{-3}$$

(3d) *(5 points)* What can you say about the doping type and doping level of the region between $0 < x < 50\ nm$?

In this region the doping type is also p-type (we are told the metallurgical junction is placed at $x=0$). In this region, the electric field doesn't change. Hence the volume charge density is negligible in the scale of the value we just estimated in the previous section. Hence, the best we can say is:

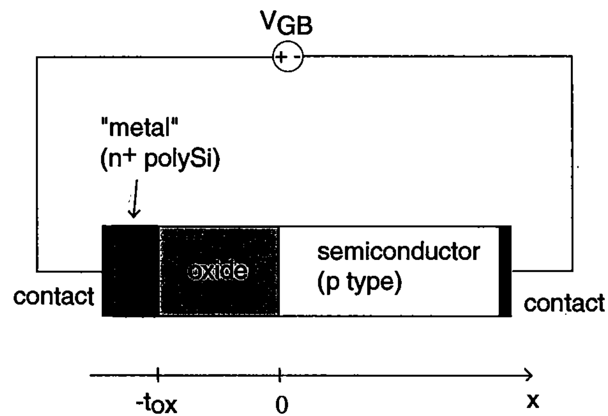
$$N_a \ll 1.3 \times 10^{17}\ cm^{-3}$$

(3e) *(5 points)* What can you say about the doping type and doping level of the region defined as $x < 0$?

This is the n-type region. The extent of the depletion region there appears negligible. Hence the doping level is very high in the scale of the doping level in the $50 < x < 100\ nm$ region. Hence, the best we can say is:

$$N_d \gg 1.3 \times 10^{17}\ cm^{-3}$$

4. *(40 points)* Consider a MOS structure as sketched below:



The oxide thickness os $t_{ox} = 50\ nm$ and the doping level in the substrate is $N_a = 10^{16}\ cm^{-3}$.

This problem is about calculating the hole concentration at $x = 0$ (the oxide-semiconductor interface) under the following conditions:

(4a) *(10 points)* At flatband *(numerical answer with appropriate sign and units expected).*

At flatband, by definition, the hole concentration at $x=0$ is equal to the doping in the body:

$$p(x=0) = N_a = 10^{16}\ cm^{-3}$$

(4b) *(10 points)* At threshold *(numerical answer with appropriate sign and units expected).*

At threshold, by definition, the electron concentration at $x=0$ is equal to the doping level of the body:

$$n(x=0) = N_a = 10^{16}\ cm^{-3}$$

Since in the MOS structure under bias $np = n_i^2$, then

$$p(x=0) = \frac{n_i^2}{n(x=0)} = \frac{10^{20}}{10^{16}} = 10^4\ cm^{-3}$$

(4d) *(10 points)* At a condition when the capacitance per unit area of the MOS structure is 50 nF/cm^2 *(numerical answer with appropriate sign and units expected).*

First, let us compute the capacitance of the oxide:

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.45 \times 10^{-13}\ F/cm}{50 \times 10^{-7}\ cm} = 6.9 \times 10^{-8}\ F/cm^2 = 69\ nF/cm^2$$

This is a higher number than the capacitance given above, hence this structure is in depletion. We can compute the extent of the depletion region by realizing that we have two capacitors in series:

$$\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_s}$$

Then

$$C_s = \frac{1}{\frac{1}{C} - \frac{1}{C_{ox}}} = \frac{1}{\frac{1}{50} - \frac{1}{69}} = 182\ nF/cm^2 = \frac{\epsilon_s}{x_d}$$

Then

$$x_d = \frac{\epsilon_s}{C_s} = \frac{1.05 \times 10^{-12}\ F/cm}{182 \times 10^{-9}\ F/cm^2} = 5.8 \times 10^{-6}\ cm = 58\ nm$$

The built-in potential across this depletion region is given by

$$V_B = \frac{qN_a x_d^2}{2\epsilon_s} = \frac{1.6 \times 10^{-19}\ C \times 10^{16}\ cm^{-3} \times (5.8 \times 10^{-6}\ cm)^2}{2 \times 1.05 \times 10^{-12}\ F/cm} = 2.6 \times 10^{-2}\ V$$

We now do a calculation similar to that of part 4C:

$$p(x=0) = p(x_d)\ \exp - \frac{qV_B}{kT} = 10^{16} \exp - \frac{2.6 \times 10^{-2}}{0.025} = 3.5 \times 10^{15}\ cm^{-3}$$

(4c) *(10 points)* At a condition in which the potential build up from the quasi-neutral body of the semiconductor to $x = 0$ is 0.5 V *(numerical answer with appropriate sign and units expected).*

We can use the Boltzmann relation to relate carrier concentration across the depletion region of a MOS structure under bias:

$$V_B = \phi(x=0) - \phi(x = x_d) = \frac{kT}{q} \ln \frac{p(x_d)}{p(0)}$$

Solving for $p(x=0)$:

$$p(x=0) = p(x_d) \exp -\frac{q}{kT} [\phi(x=0) - \phi(x=x_d)] =$$

$$= 10^{16} \exp - \frac{0.5}{0.025} = 2.1 \times 10^{7} \text{ cm}^{-3}$$

This is because $p(x_d) = N_a = 10^{16} \text{ cm}^{-3}$.