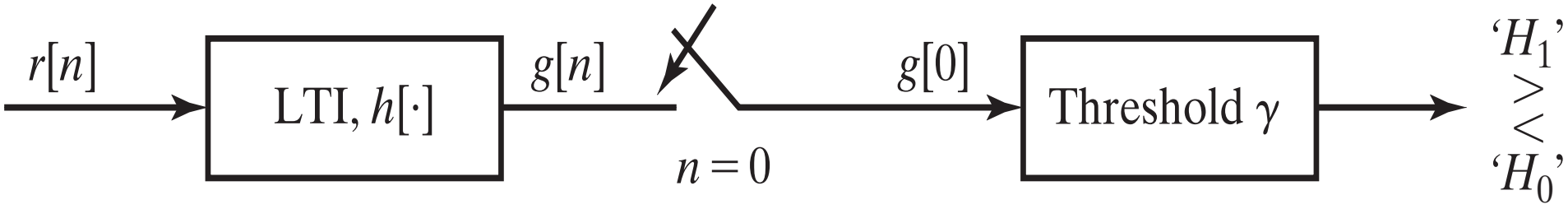
Matched filtering

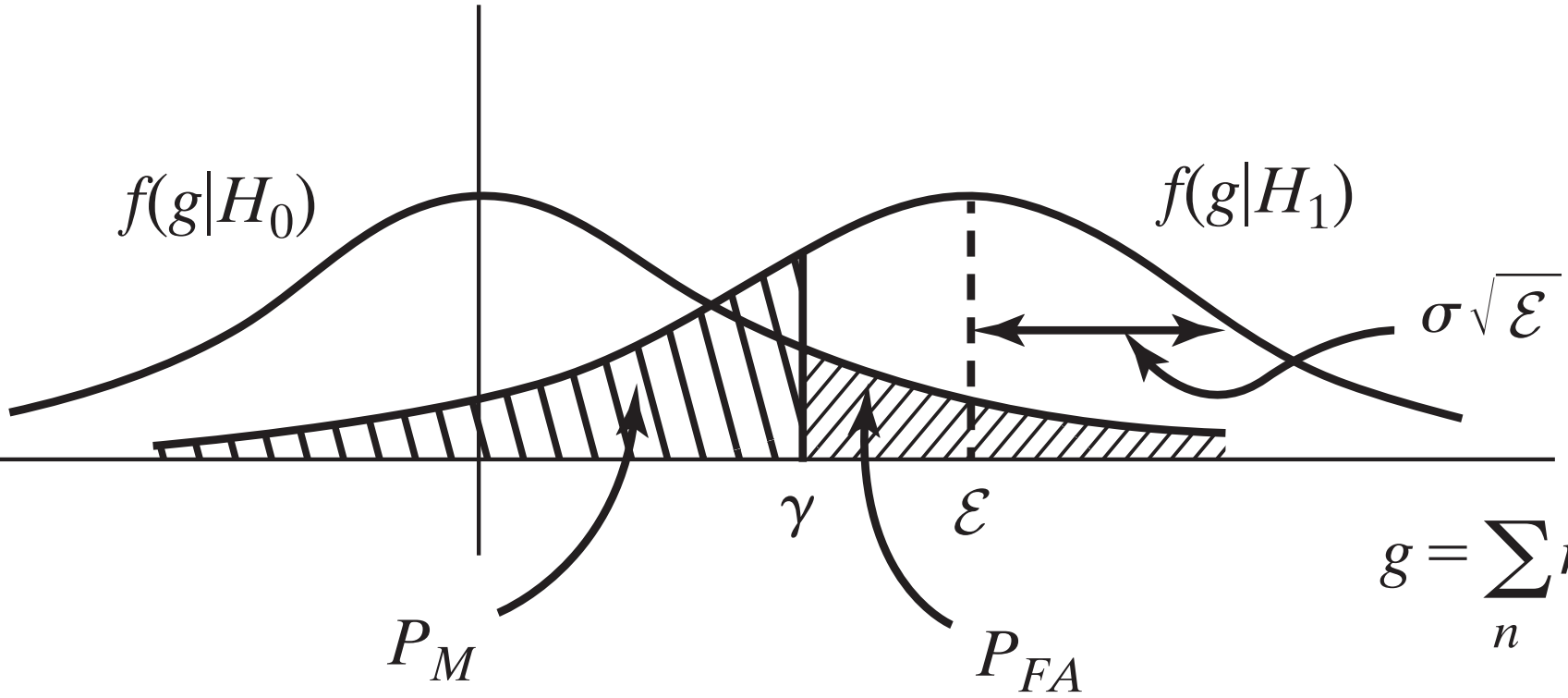
6.011, Spring 2018

Lec 24

Matched filtering for detecting known signal in white Gaussian noise



Matched filter performance



$$g = \sum_n r[n]s[n]$$

Q(.) function for area in Gaussian tail

The tail area to the right of x under a Gaussian PDF of mean 0 and standard deviation 1 is tabulated as the **tail-probability function**:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-v^2/2}\, dv$$

Useful bounds:

$$\frac{x}{(1+x^2)} \frac{e^{-x^2/2}}{\sqrt{2\pi}} < Q(x) < \frac{1}{x} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\ , \quad x > 0$$

For a Gaussian random variable of mean value α and standard deviation , the area under the PDF to the right of some value is

$$\frac{1}{\beta\sqrt{2\pi}} \int_\gamma^\infty e^{-(w-\alpha)^2/(2\beta^2)}\, dw = Q\left(\frac{\gamma - \alpha}{\beta}\right)$$

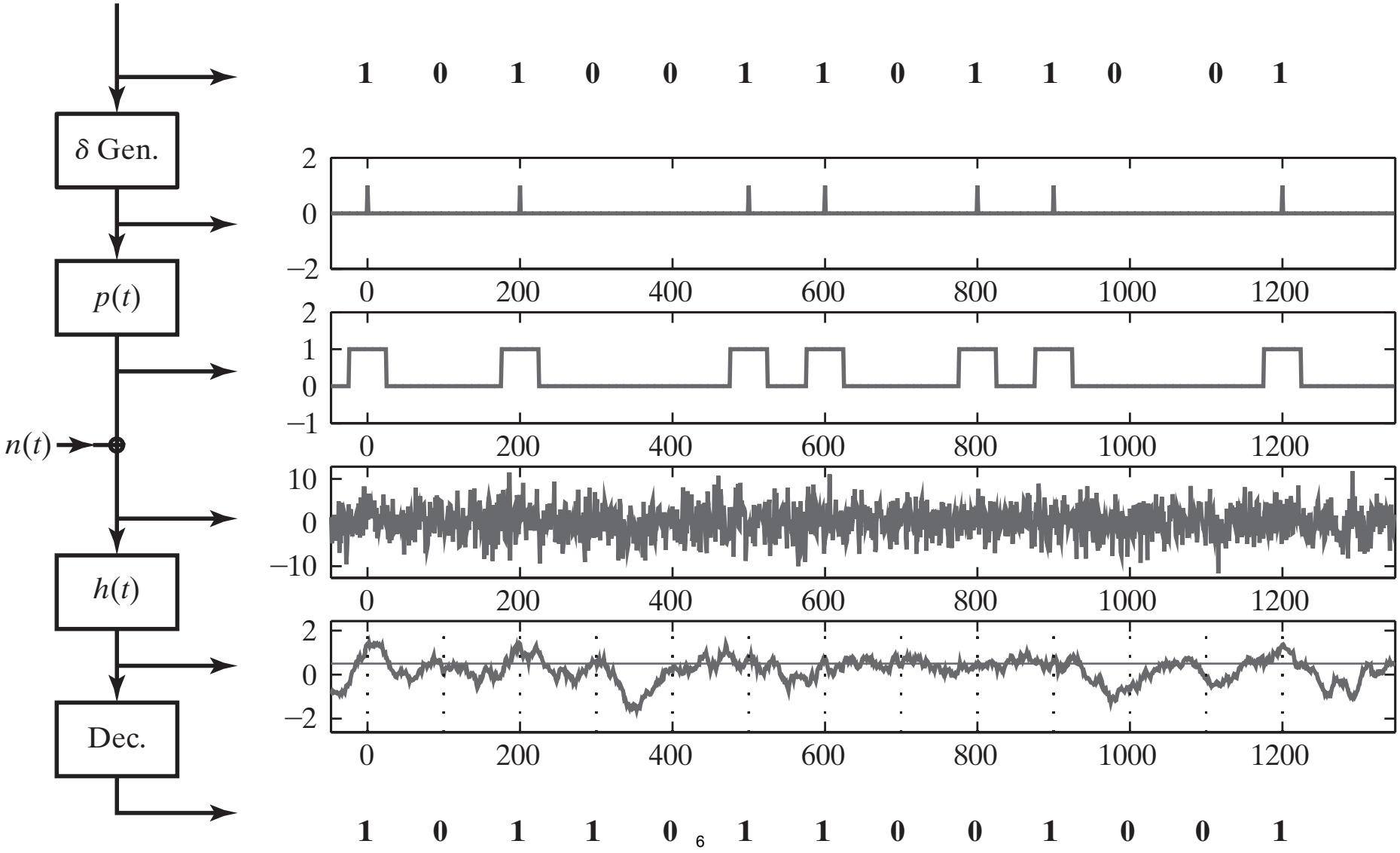
Matched filter properties

- Matched filter output in noise-free case (and before sampling) is the deterministic autocorrelation of the signal:

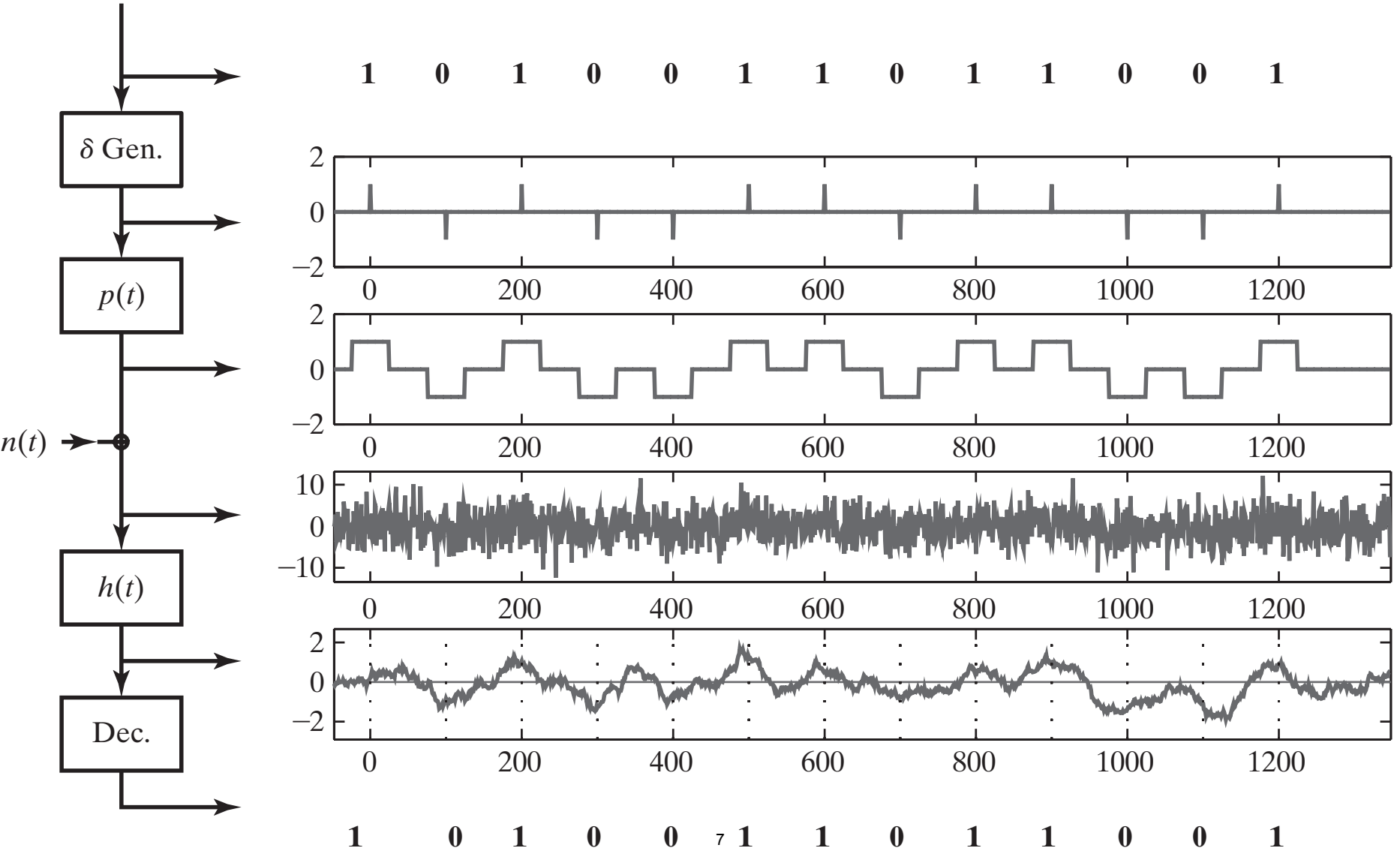
$$g[n] = \overline{R}_{ss}[n]$$

- Matched filter frequency response magnitude accentuates frequencies where signal has strength relative to (spectrally flat) noise
- Matched filter frequency response phase cancels signal phase characteristic to allow all components to contribute at sampling time
- Matched filter maximizes "SNR" of sample fed to threshold test

On-off signaling in noise



Antipodal signaling



Pulse compression for radar

Read the simulation example from
https://en.wikipedia.org/wiki/Pulse_compression

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