Heisenberg Uncertainty

Outline

- Heisenberg Microscope
- Measurement Uncertainty

- Example: Hydrogen Atom
- Example: Single Slit Diffraction
- Example: Quantum Dots

TRUE / FALSE

A photon (quantum of light) is reflected from a mirror.

(A) Because a photon has a zero mass, it does not exert a force on the mirror. ____

(B) Although the photon has energy, it cannot transfer any energy to the surface because it has zero mass. ____

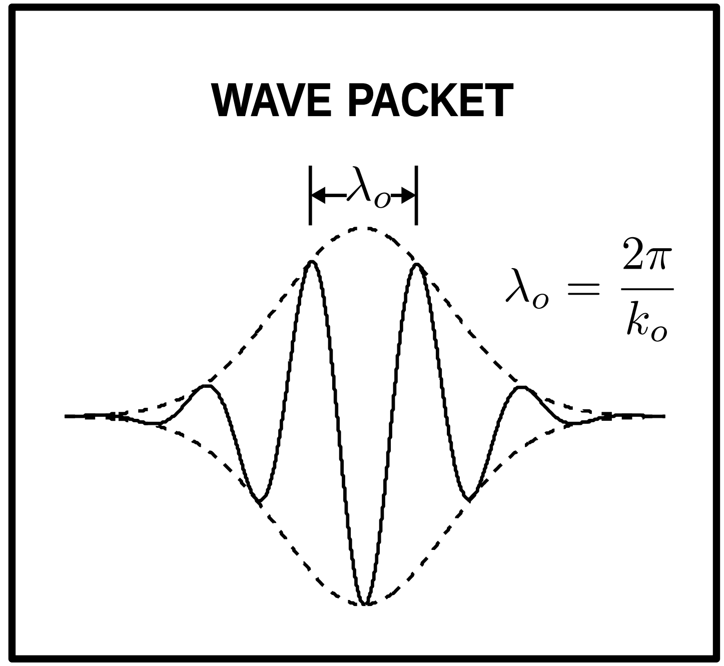
(C) The photon carries momentum, and when it reflects off the mirror, it undergoes a change in momentum and exerts a force on the mirror. ____

(D) Although the photon carries momentum, its change in momentum is zero when it reflects from the mirror, so it cannot exert a force on the mirror. ____

Gaussian Wavepacket in Space

$$E(z,t) = E_o exp\left(-\frac{\sigma_k^2}{2}(ct-z)^2\right)\cos(\omega_o t - k_o z)$$

GAUSSIAN ENVELOPE

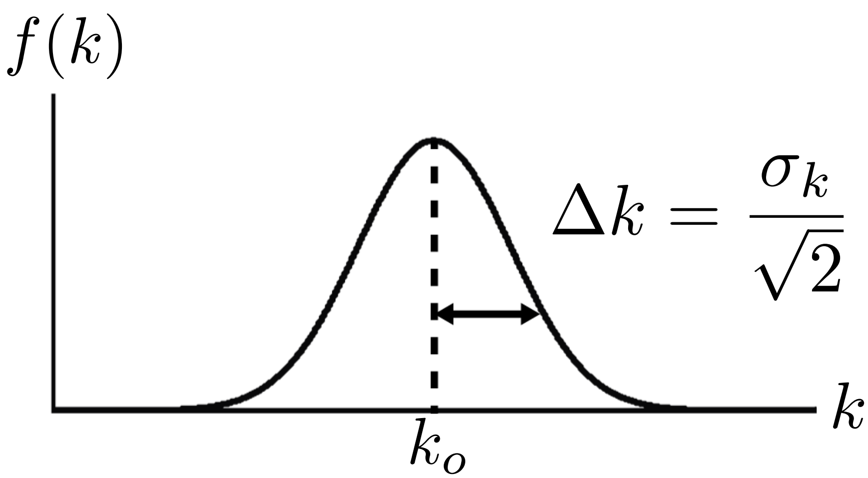


In free space ...

$$k = \frac{\omega}{c} = \frac{2\pi E}{hc}$$

... this plot then shows the PROBABILITY OF WHICH k (or ENERGY) EM WAVES are MOST LIKELY TO BE IN THE WAVEPACKET

$$\Delta z = \frac{1}{\sqrt{2}\sigma_k}$$

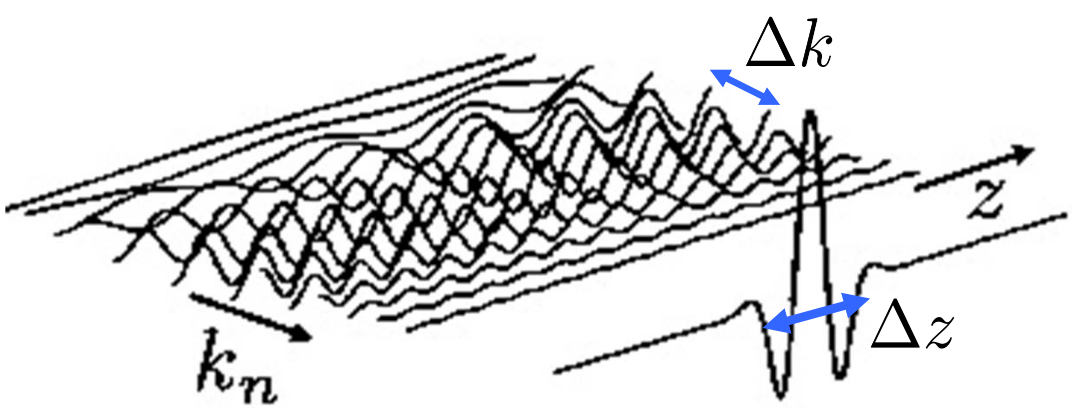


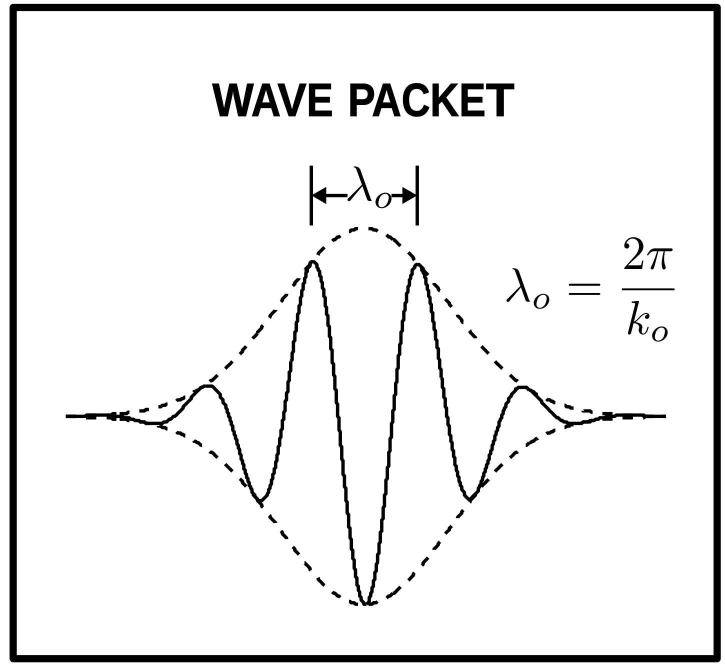
$$\Delta k = \frac{\sigma_k}{\sqrt{2}}$$

$$\Delta k \Delta z = 1/2$$

Gaussian Wavepacket in Time

$$E(z,t) = E_o exp\left(-\frac{\sigma_k^2}{2}(ct-z)^2\right)\cos(\omega_o t - k_o z)$$





UNCERTAINTY RELATIONS

$$\Delta z = \frac{c}{n}\Delta t$$

$$\Delta k = \frac{n}{c}\Delta\omega$$

$$\Delta k \Delta z = 1/2$$

$$\Delta\omega\Delta t = 1/2$$

$$\Delta p \Delta z = \hbar/2$$

$$\Delta E \Delta t = \hbar/2$$

Heisenberg's Uncertainty Principle

uncertainty in momentum

$$\Delta x \Delta p \geq \frac{h}{4\pi} = \frac{\hbar}{2}$$

uncertainty in position

The more accurately you know the position (i.e., the smaller Δx is), the less accurately you know the momentum (i.e., the larger Δp is); and vice versa

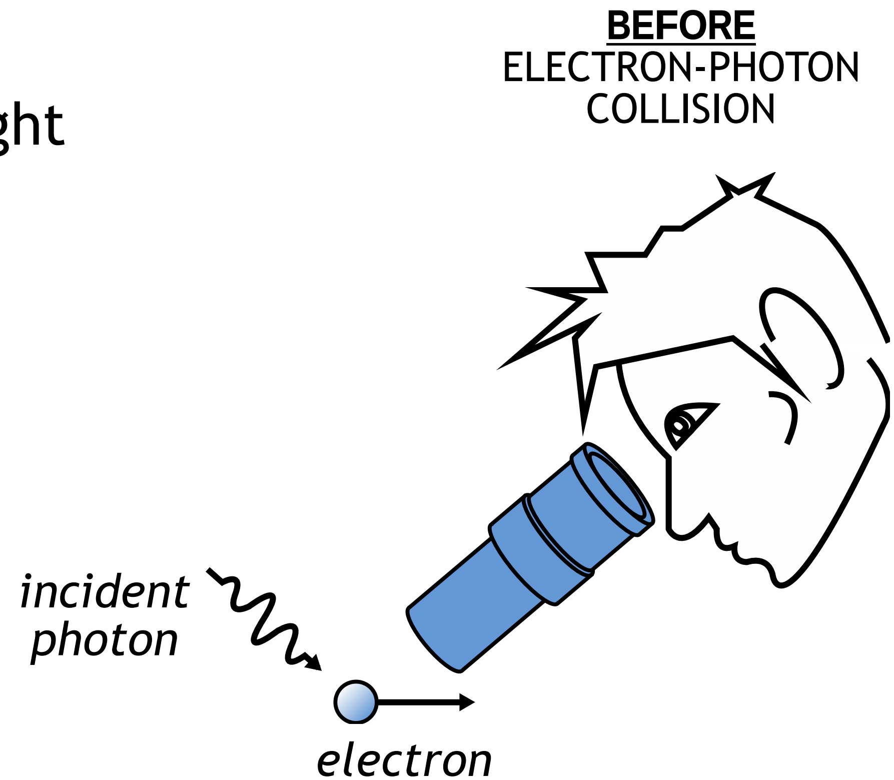
Heisenberg realised that ...

- In the world of very small particles, one cannot measure any property of a particle without interacting with it in some way
- This introduces an unavoidable uncertainty into the result
- One can never measure all the properties exactly

Werner Heisenberg (1901-1976)
Image in the Public Domain

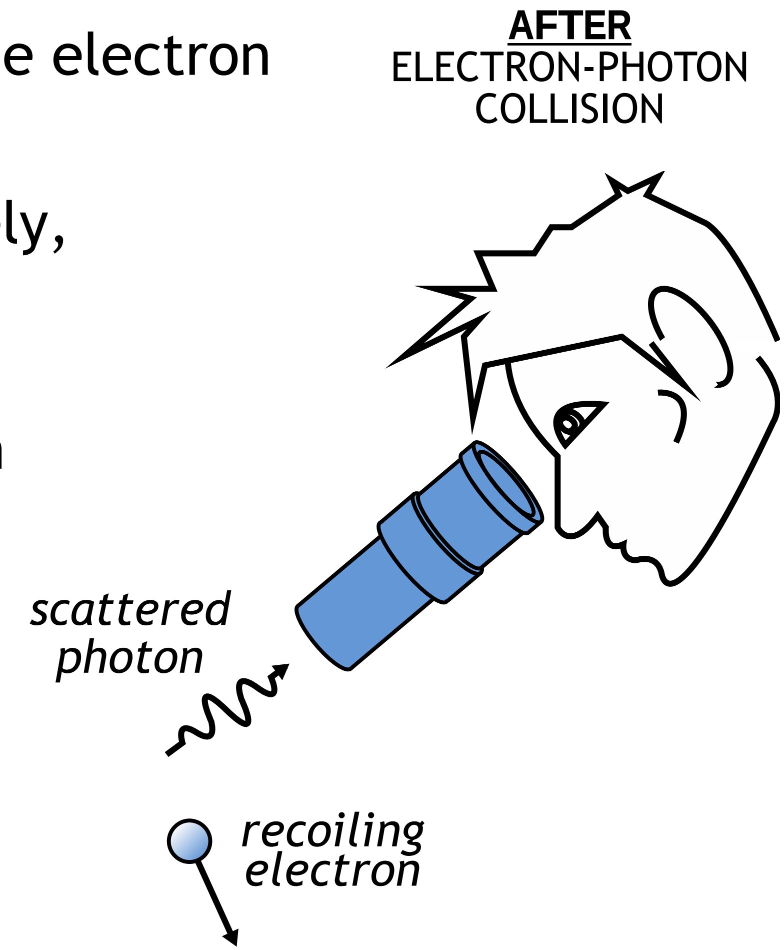
Measuring Position and Momentum of an Electron

- Shine light on electron and detect reflected light using a microscope
- Minimum uncertainty in position is given by the wavelength of the light
- So to determine the position accurately, it is necessary to use light with a short wavelength

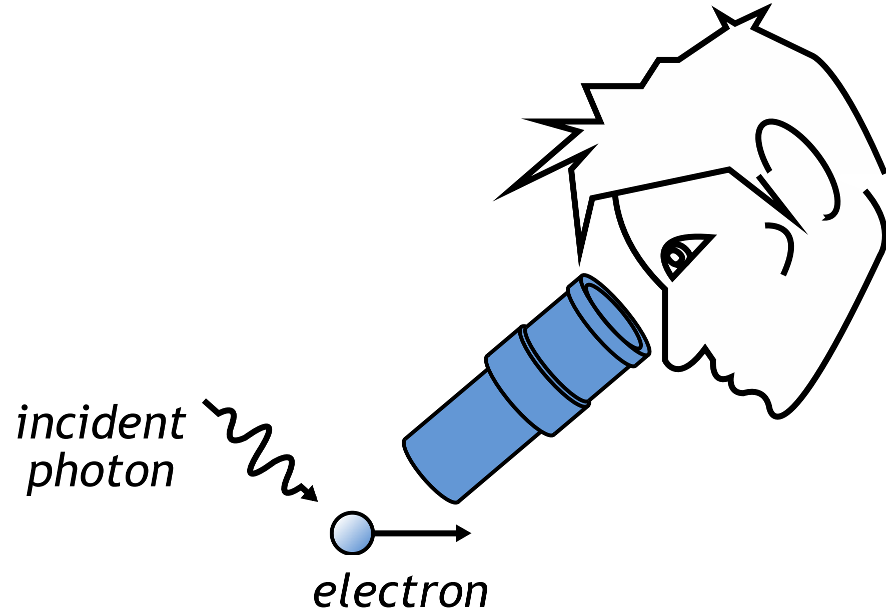


Measuring Position and Momentum of an Electron

- By Planck's law $E = hc/\lambda$, a photon with a short wavelength has a large energy
- Thus, it would impart a large 'kick' to the electron
- But to determine its momentum accurately, electron must only be given a small kick
- This means using light of long wavelength



Light Microscopes

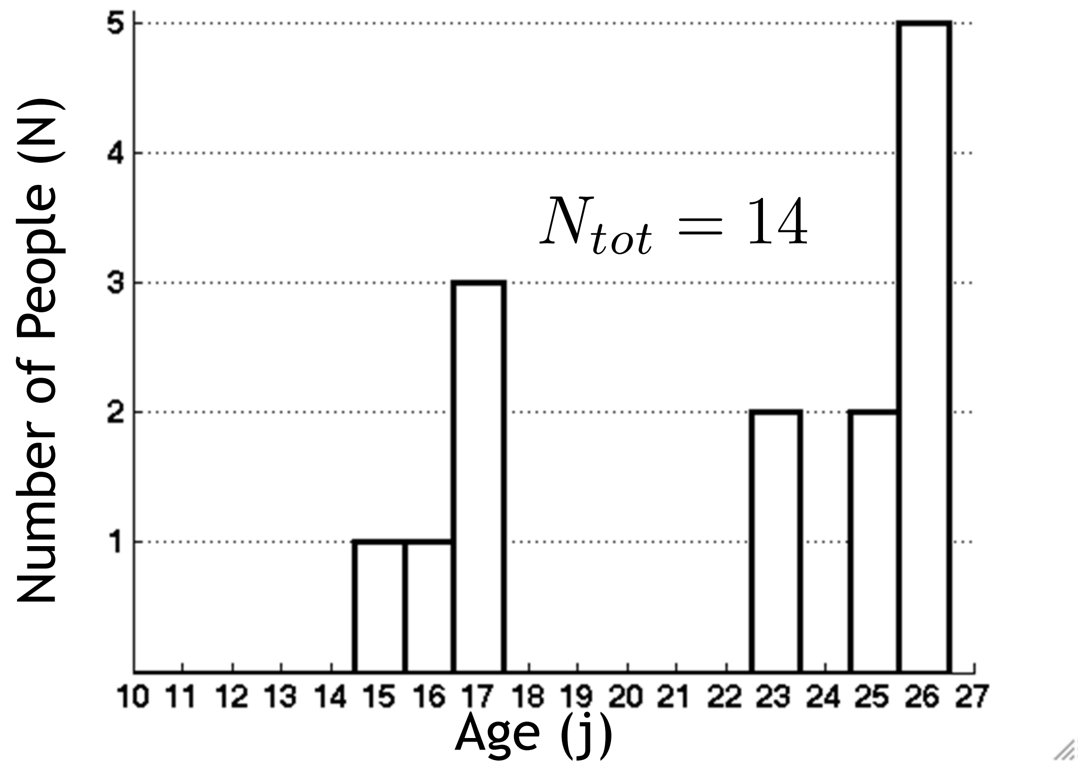


- Suppose the positions and speeds of all particles in the universe are measured to sufficient accuracy at a particular instant in time
- It is possible to predict the motions of every particle at any time in the future (or in the past for that matter)

"An intelligent being knowing, at a given instant of time, all forces acting in nature, as well as the momentary positions of all things of which the universe consists, would be able to comprehend the motions of the largest bodies of the world and those of the smallest atoms in one single formula, provided it were sufficiently powerful to subject all the data to analysis; to it, nothing would be uncertain, both future and past would be present before its eyes."

Pierre Simon Laplace

Review: Probability - a closer look at Δx



Probability of someone with age j:

$$P(j) = \frac{N(j)}{N_{tot}}$$

where $\sum_{0}^{\infty} P(j) = 1$

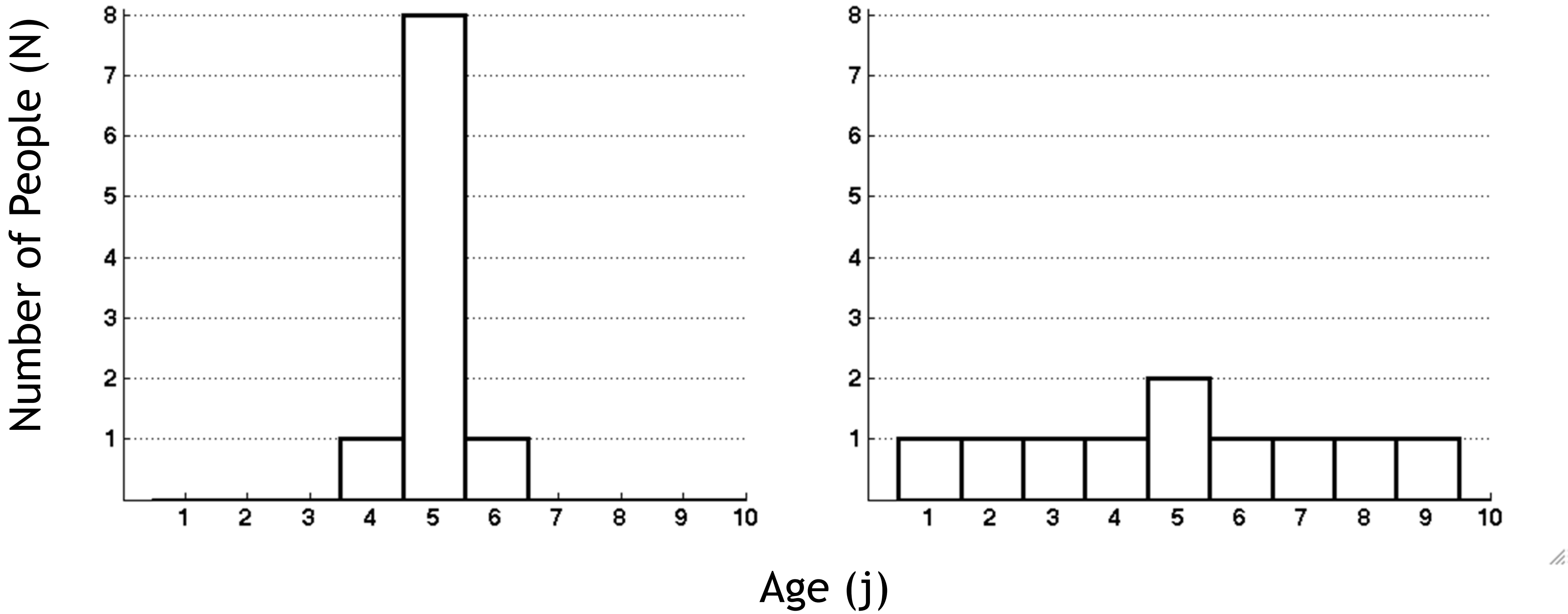
Average age (expected age):

$$< j >= \frac{\sum_j N(j)}{N_{tot}} = \sum_{0}^{\infty} jP(j)$$

General expected value:

$$< f(j) >= \sum_{0}^{\infty} f(j)P(j)$$

Review: Probability



One distribution is more "spread out" than the other.

$$< \Delta j^2 > = < j^2 > - < j >^2$$

Variance

Expected value of j^2

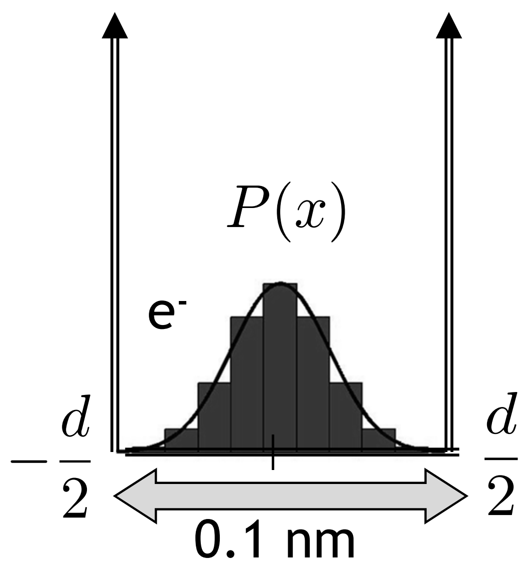
(Expected value of j)^2

What we've been loosely calling Δj is actually

$$\sqrt{< \Delta j^2 >}$$

Standard deviation

Review: Probability
A closer look at Δx



Probability of being at position x:

$$\sum_{x=-\infty}^{\infty} P(x) = 1$$

General expected value:

$$< f(x) >= \sum_{-\infty}^{\infty} f(x) P(x)$$

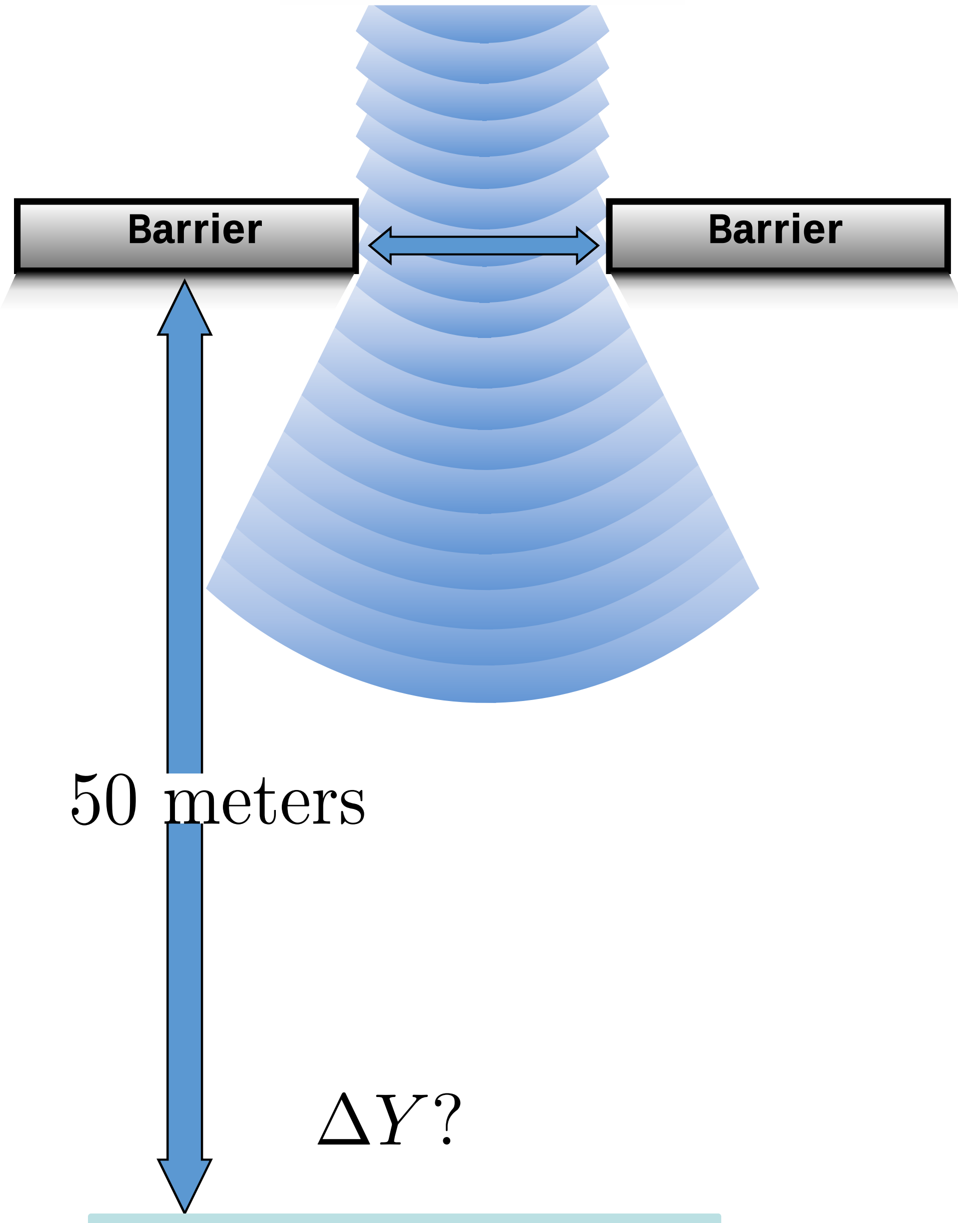
Expected (average) position:

$$< x >= \sum_{-\infty}^{\infty} x P(x)$$

Uncertainty in position:

$$< \Delta x^2 >=< x^2 > - < x >^2$$

Heisenberg Example: Diffraction



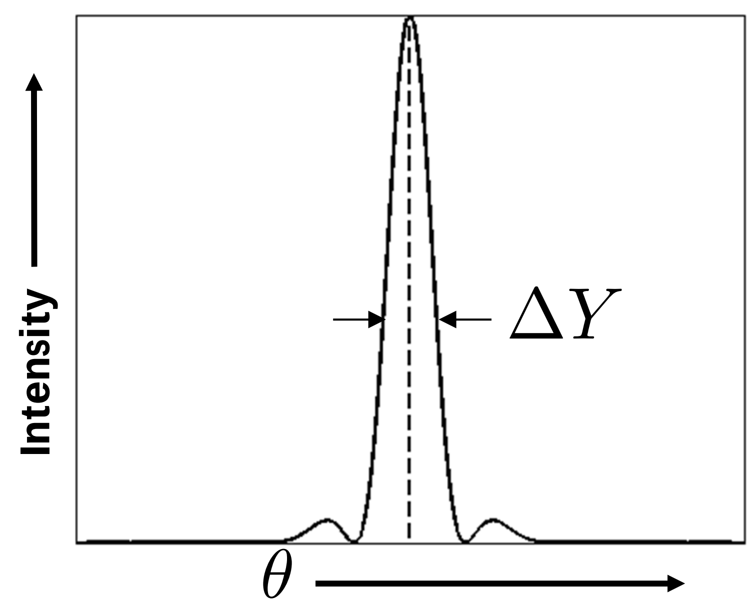
What is the spread of electrons on the screen ?

The slit gives information about y…

bound on Δy

spread in Δp_y

… and if you look closely you will see …

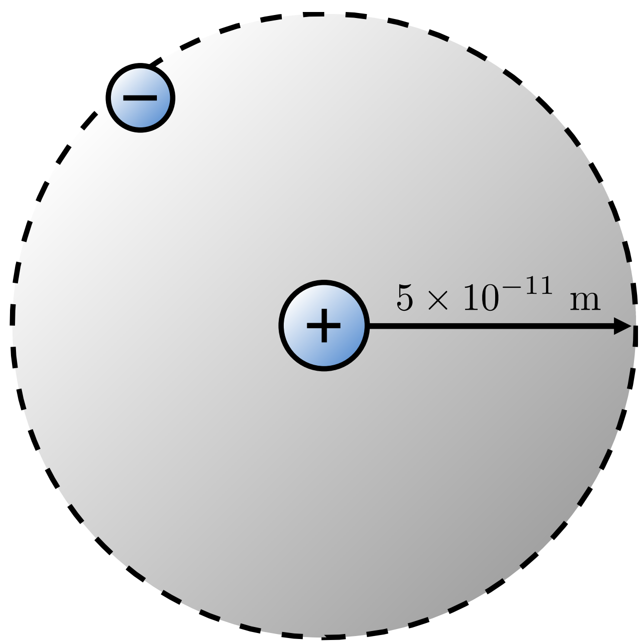


Classical Hydrogen Atom

Classically we know that negatively charged electron is attracted to the proton, and it was suggested that the electron circles the proton.

But if an electron is circling, every-time it changes direction it is accelerated, and an accelerating charge emits EM radiation (light). Classically, it can be calculated that the radiation of the electron would cause it to gradually loose its rotational kinetic energy and collapse on top of the proton within 10^{-9} seconds !

$$V(r) = -\frac{e^2}{4\pi\epsilon_o r}$$



Hydrogen Atom

Consider a single hydrogen atom:

an electron of *charge* = -*e* free to move around
in the electric field of a fixed proton of *charge* = +*e*
(proton is ~2000 times heavier than electron, so we consider it fixed).

The electron has a potential energy due to the attraction to proton of:

$$V(r) = -\frac{e^2}{4\pi\epsilon_o r}$$

where *r* is the electron-proton separation

The electron has a kinetic energy of $K.E. = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

The total energy is then $E(r) = \frac{p^2}{2m} - \frac{e^2}{4\pi\epsilon_o r}$

Classically, the minimum energy of the hydrogen atom is – ∞

the state in which the electron is on top of the proton → p = 0, r = 0.

Quantum mechanically, the uncertainty principle forces the electron to have non-zero momentum and non-zero expectation value of position.

If ***a*** is an average distance electron-proton distance, the uncertainty principle informs us that the minimum electron momentum is on the order of ***ħ/a***.

The energy as a function of ***a*** is then:

$$E(a) = \frac{\hbar^2}{2ma^2} - \frac{e^2}{4\pi\epsilon_o a}$$

If we insist on placing the electron right on top of the proton (a=0), the potential energy is still - ∞ , just as it is classically, but the total energy is:

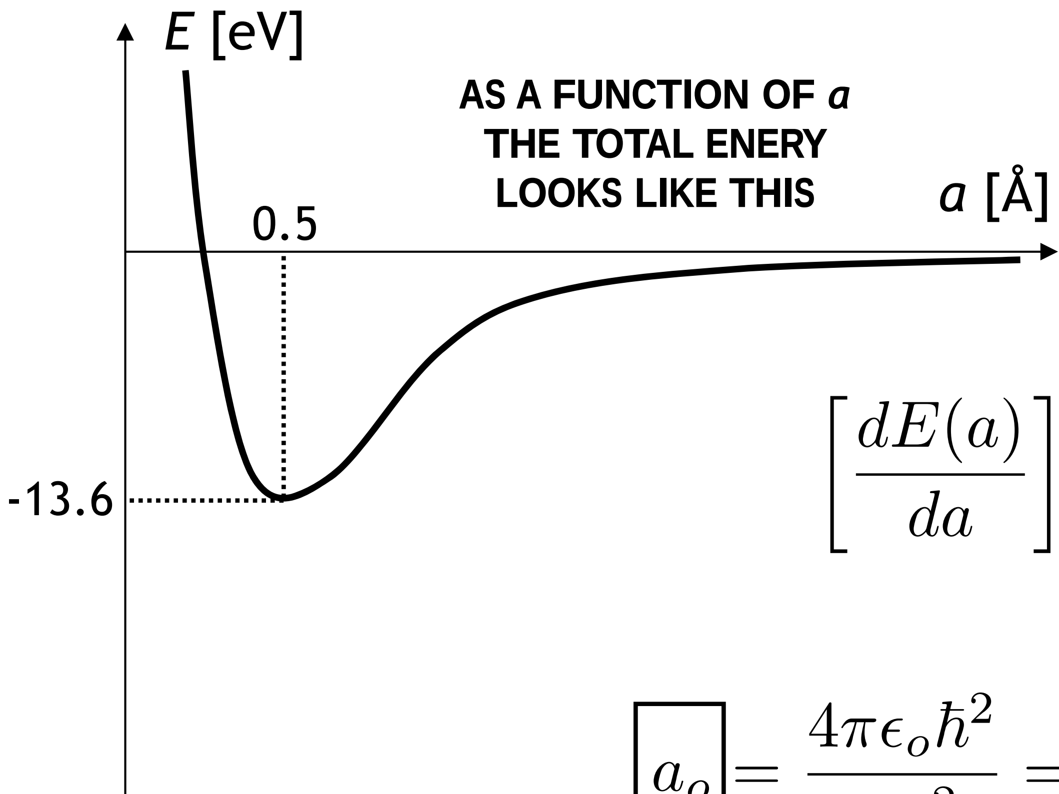
$$E(0) \approx \lim_{a\to 0}\left[\frac{\hbar^2}{2ma^2} - \frac{e^2}{4\pi\epsilon_o a}\right]$$

$$\approx \lim_{a\to 0}\left[\frac{2\pi\epsilon_o\hbar^2 - me^2 a}{4\pi\epsilon_o m a^2}\right]$$

$$\boxed{\to +\infty}$$

→ Quantum mechanics tells us that an **ATOM COULD NEVER COLLAPSE** as it would take an infinite energy to locate the electron on top of the proton

The minimum energy state, quantum mechanically, can be estimated by calculating the value of $a=a_o$ for which $E(a)$ is minimized:



$$E(a) = \frac{\hbar^2}{2ma^2} - \frac{e^2}{4\pi\epsilon_o a}$$

$$\left[\frac{dE(a)}{da}\right]_{a_o} = -\frac{\hbar^2}{ma_o^3} + \frac{e^2}{4\pi\epsilon_o a_o^2} = 0$$

$$\boxed{a_o} = \frac{4\pi\epsilon_o\hbar^2}{me^2} = \frac{10^{-10}\cdot 10^{-68}}{10^{-30}\cdot 2\cdot 10^{-38}} m \approx \boxed{0.5\ \mathring{A}}$$

By preventing localization of the electron near the proton, the Uncertainty Principle

RETARDS THE CLASSICAL COLLAPSE OF THE ATOM,

PROVIDES THE CORRECT DENSITY OF MATTER,

and YIELDS THE PROPER BINDING ENERGY OF ATOMS

One might ask:

“If light can behave like a particle, might particles act like waves”?

YES !

Particles, like photons, also have a wavelength given by:

$$\lambda = h/p = h/mv$$

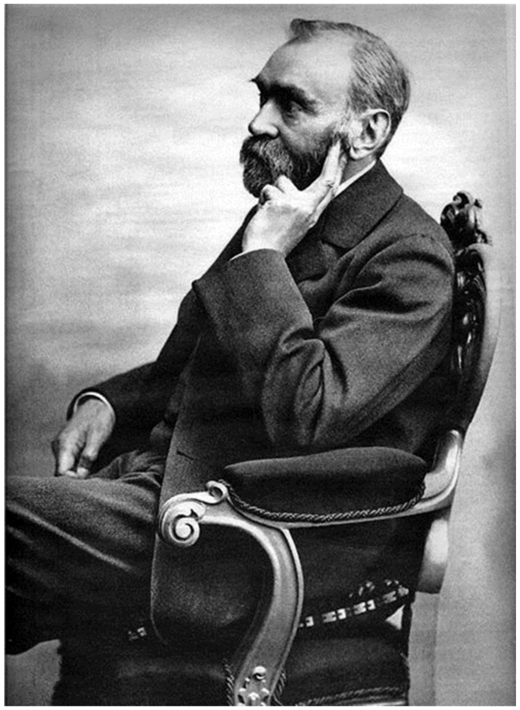
de Broglie wavelength

The wavelength of a particle depends on its momentum, just like a photon!

The main difference is that matter particles have mass, and photons don’t !

Nobel Prize

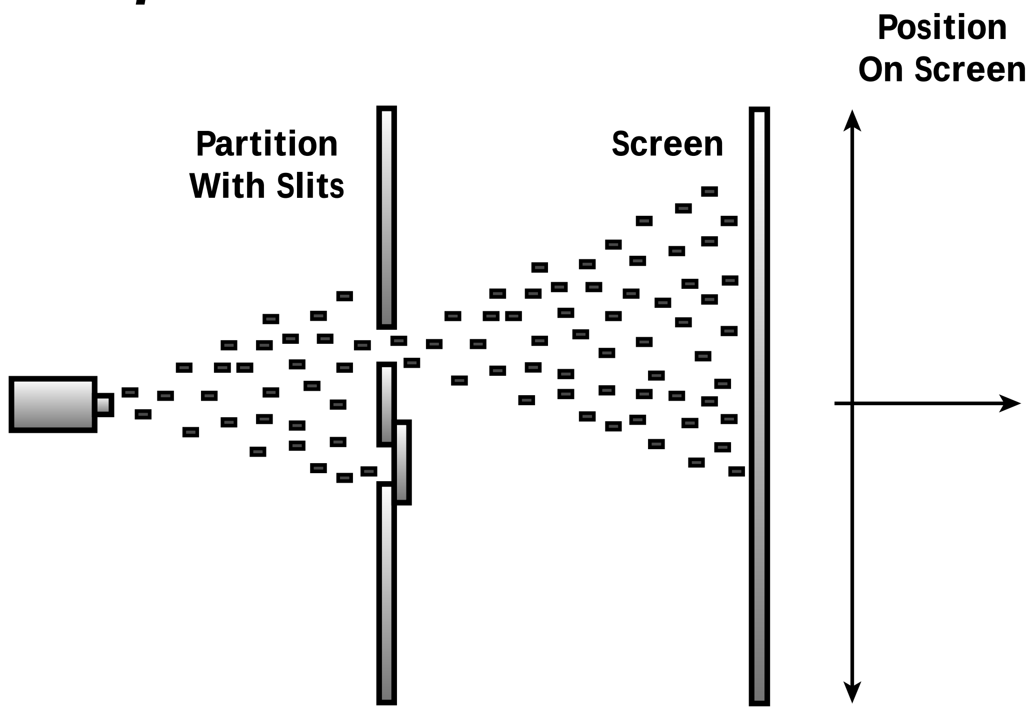
- Alfred Nobel was born in1833 in Stockholm, Sweden. He was a chemist, engineer, and inventor. In 1894 Nobel purchased the Bofors iron and steel mill, which he converted into a major armaments manufacturer. Nobel amassed a fortune during his lifetime, most of it from his 355 inventions, of which dynamite is the most famous.
- In 1888, Alfred was astonished to read his own obituary in a French newspaper. It was actually Alfred's brother Ludvig who had died. The article disconcerted Nobel and made him apprehensive about how he would be remembered. This inspired him to change his to bequeathed 94% of his total assets (US$186 million in 2008) to establish the five Nobel Prizes. Today the Nobel Foundation has US$560 million.
- Winners receive a diploma, medal, and monetary award. In 2009, the monetary award was US$1.4 million.



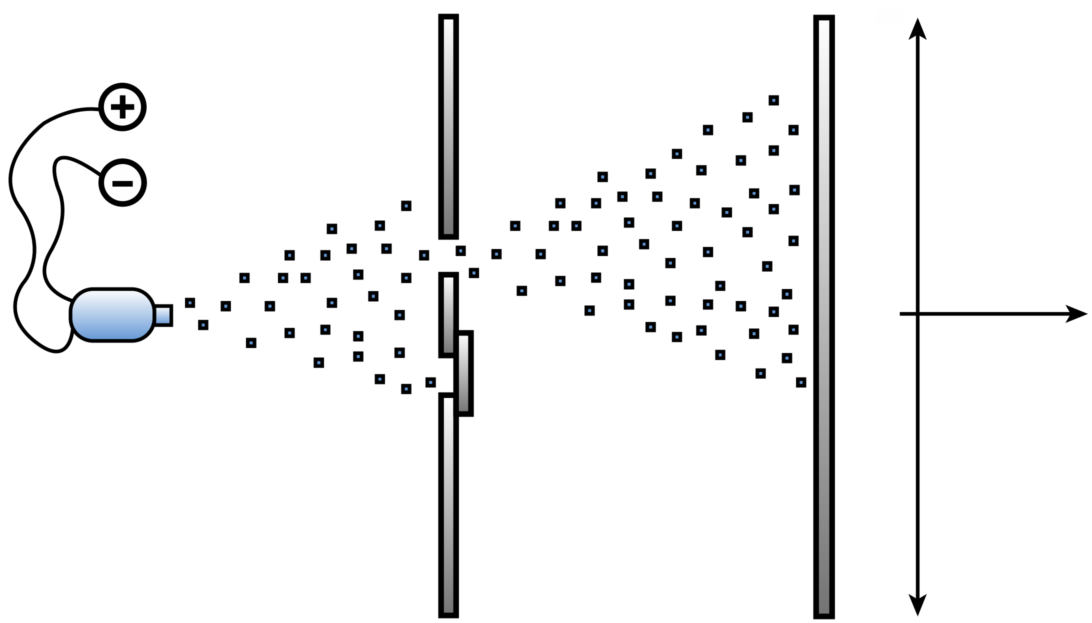
Alfred Nobel

Image in the Public Domain

Do particles exhibit interference ?

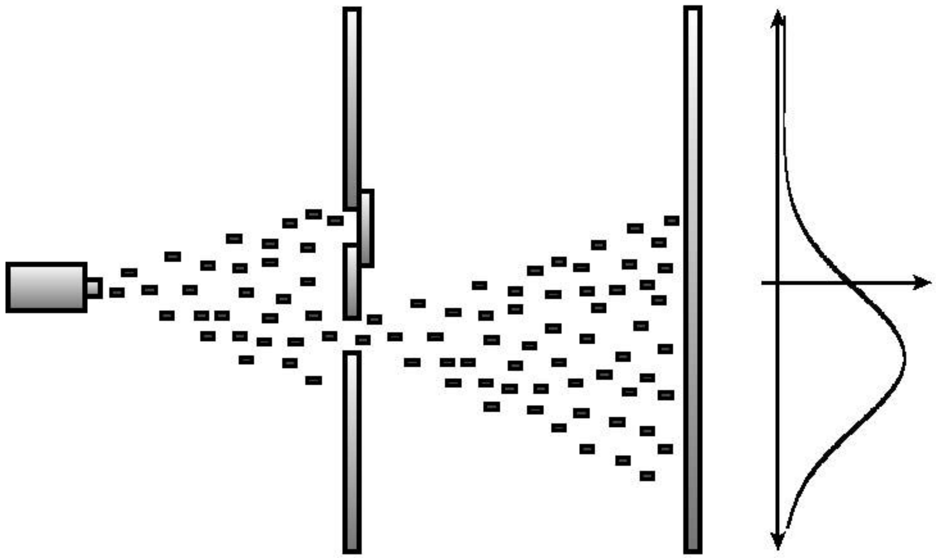


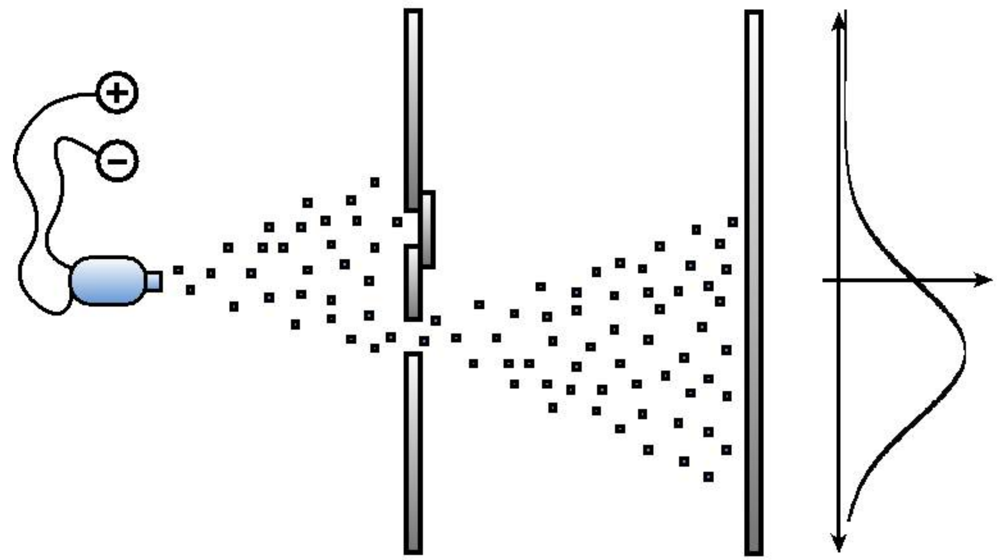
Firing bullets at a "double -slit" with one slit closed



Firing electrons at a "double -slit" with one slit closed

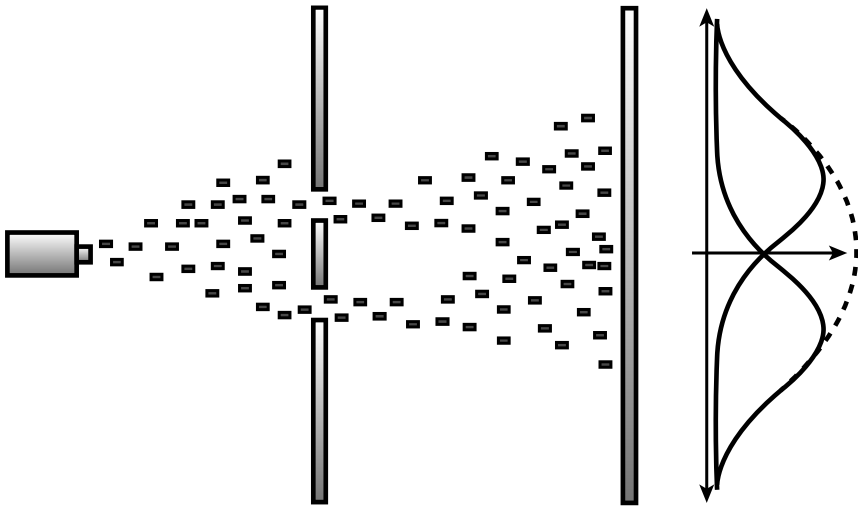
What about the other slit ?



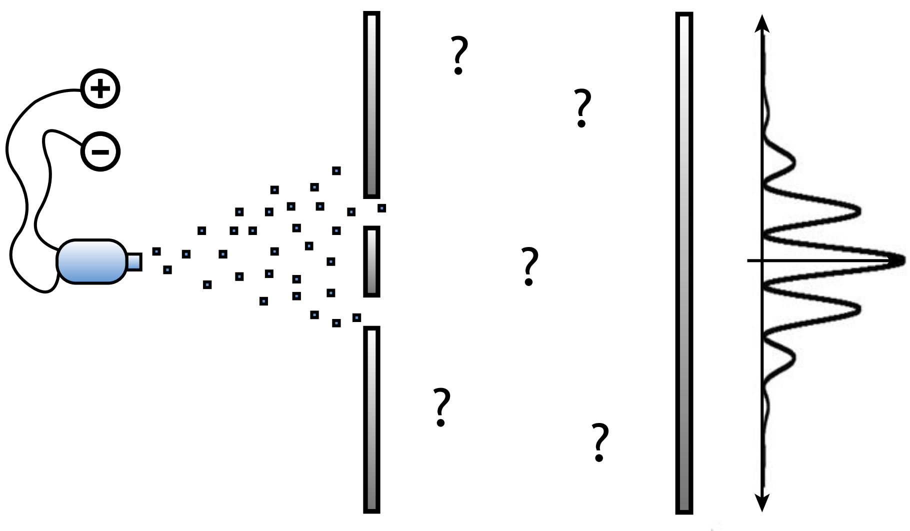


Again, you just get a rather expected result …

What if both slits are open ?



With bullets, you get what appears to be a simple "sum" of the two intensity distributions.

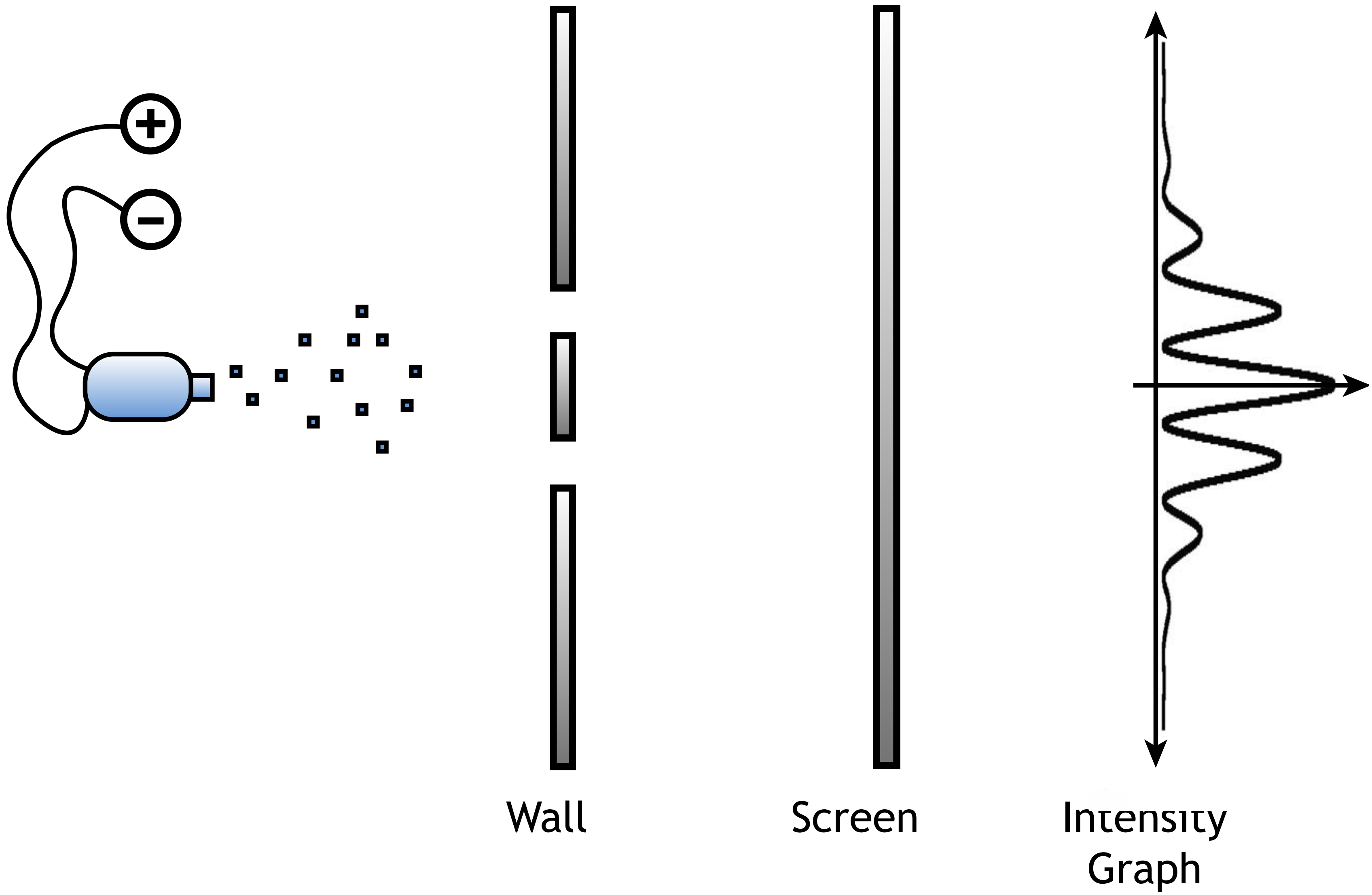


With electrons, you find an "interference" pattern, just like with light waves ?

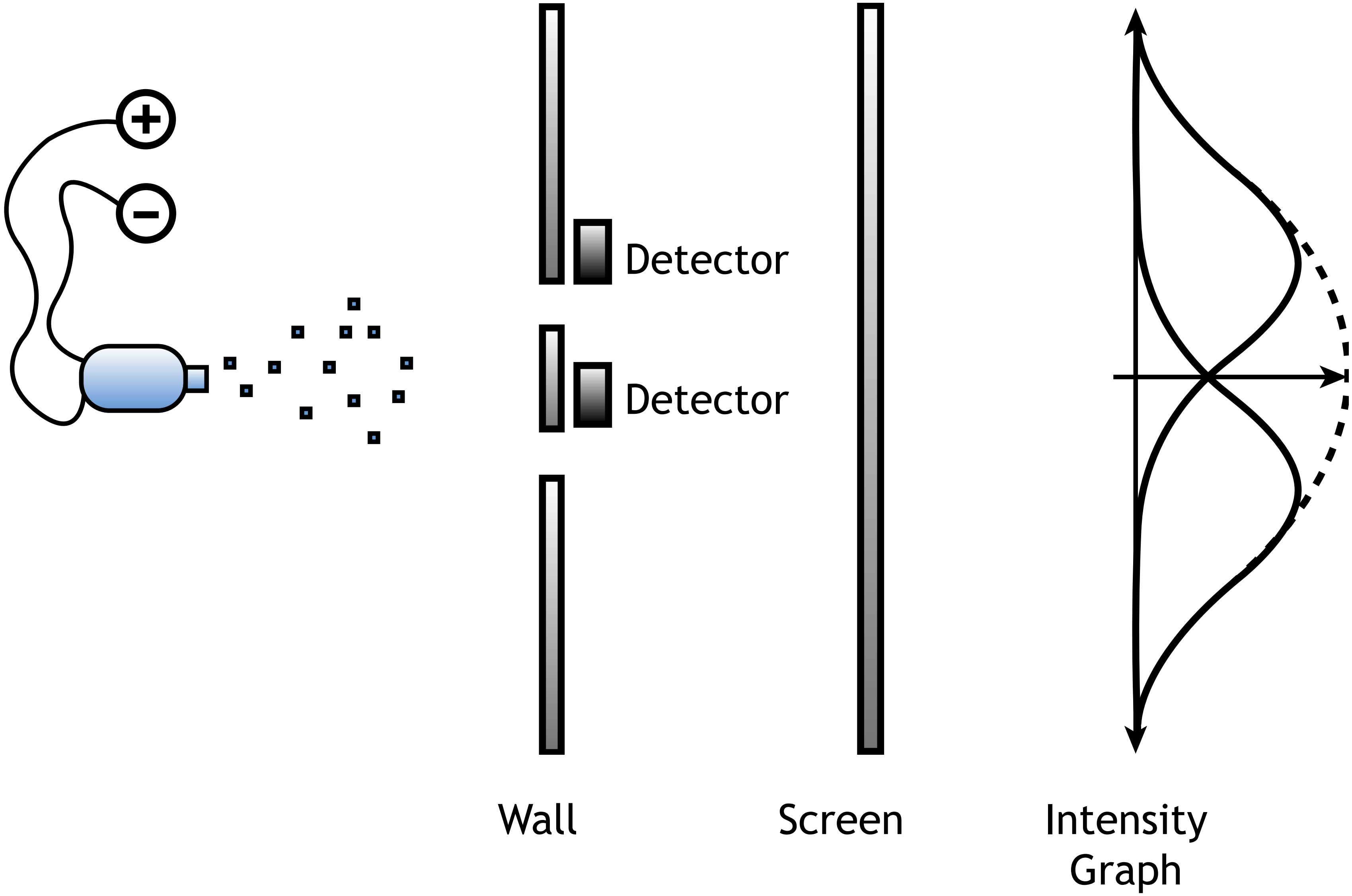
Huh ? Come again ?

So, forms of matter do exhibit wave behavior (electrons) and others (bullets) don't ? What's going on here ?

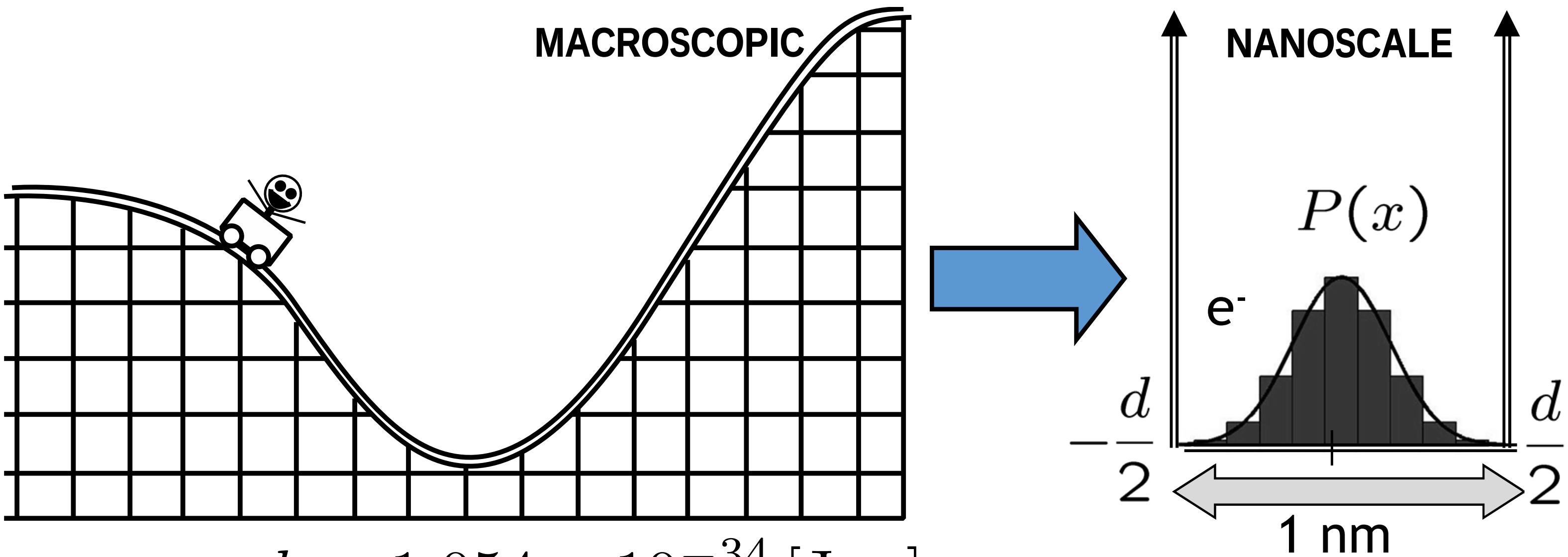
Electron Diffraction



Double-Slit Experiment:
act of observation affects behavior of electron



Another Heisenberg Uncertainty Example: Particle in a Box



$$\hbar = 1.054 \times 10^{-34}\,[\mathrm{J \cdot s}]$$

What is the minimum kinetic energy of the electron in the box?

- A quantum particle can never be in a state of rest, as this would mean we know both its position and momentum precisely
- Thus, the carriage will be jiggling around the bottom of the valley forever

$$< E >= \frac{< p^2 >}{2m} \approx \frac{\hbar^2}{2m < \Delta x^2 >}$$

Example: Engineering Color

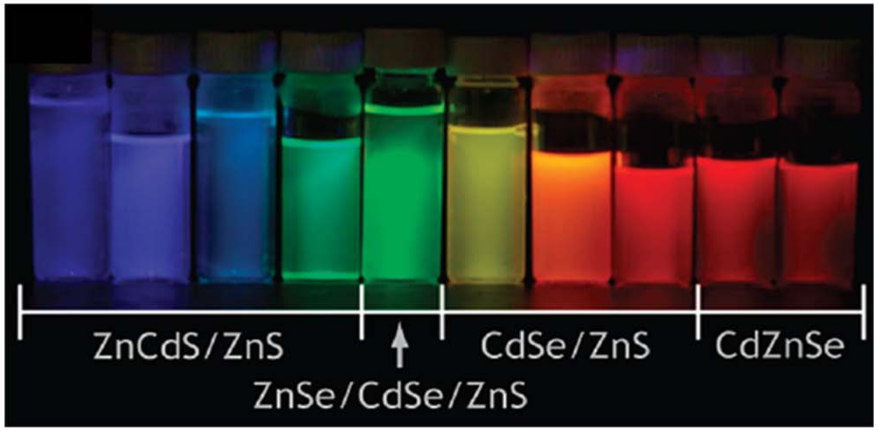


Photo by J. Halpert, Courtesy of M. Bawendi Group, EECS, MIT.

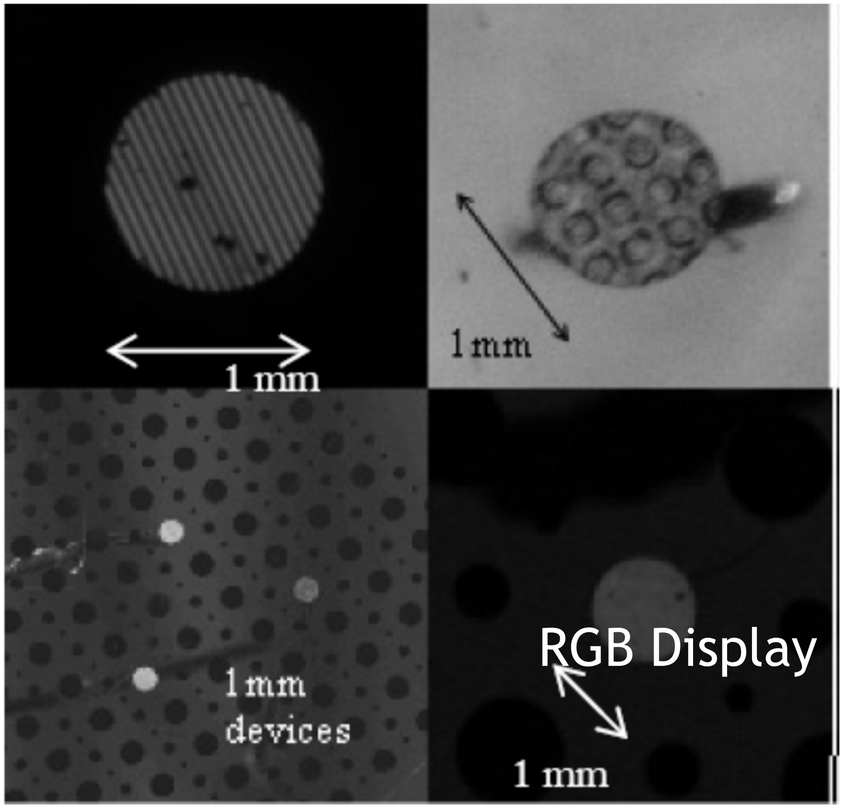
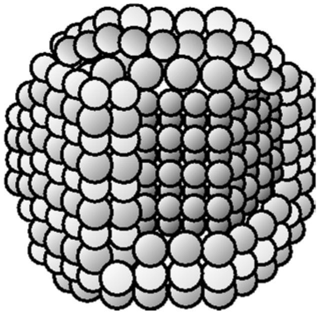


Image courtesy ONE-lab and M. Bawendi Group, EECS, MIT.

Taking color away from chemists and giving it to electrical engineers…

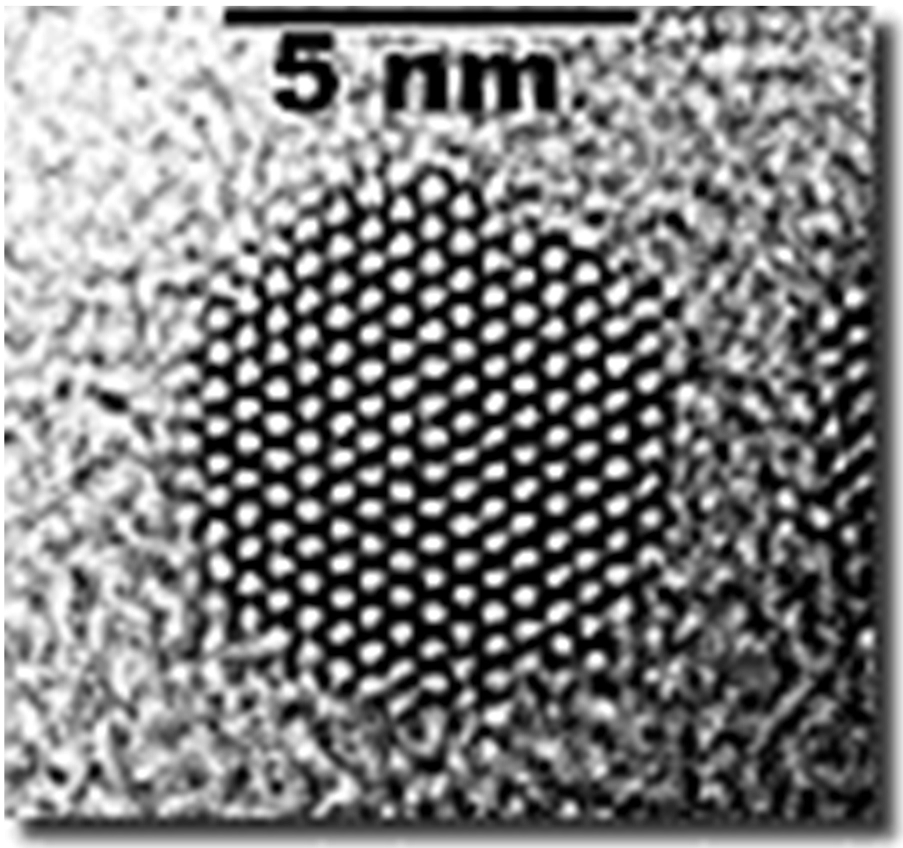


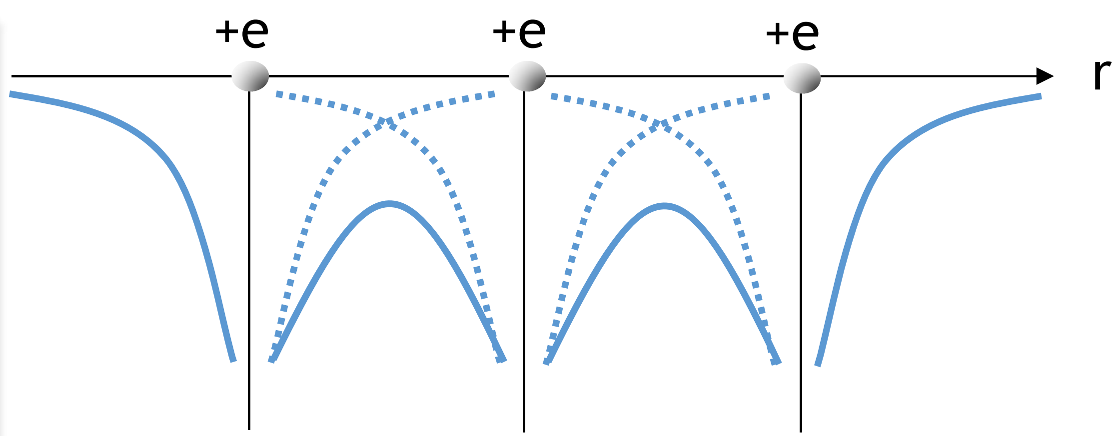
Everything here is a spherical nanoparticle of CdSe !!

Quantum Confinement

another way to know Δx

Transmission Electron Microscopy shows the crystalline arrangement of atoms in a 5nm diameter CdSe nanocrystal quantum dot

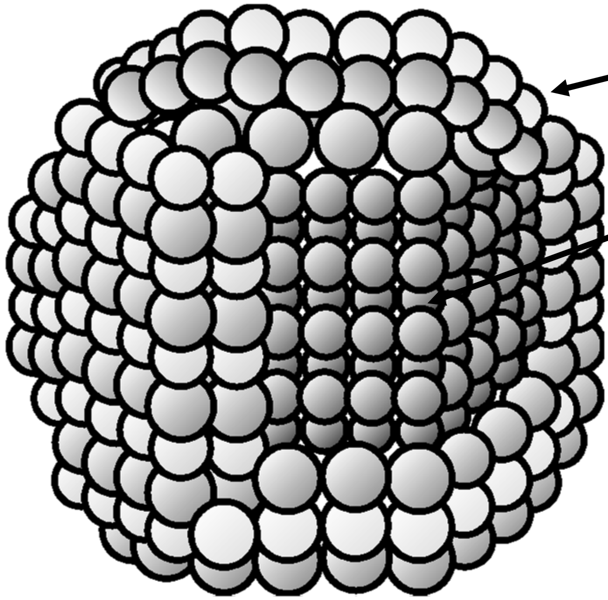




electron can be anywhere in dot

$$< \Delta x^2 > \approx R^2$$

Colloidal Semiconductor Nanoparticles



Red: bigger dots!

Blue: smaller dots!

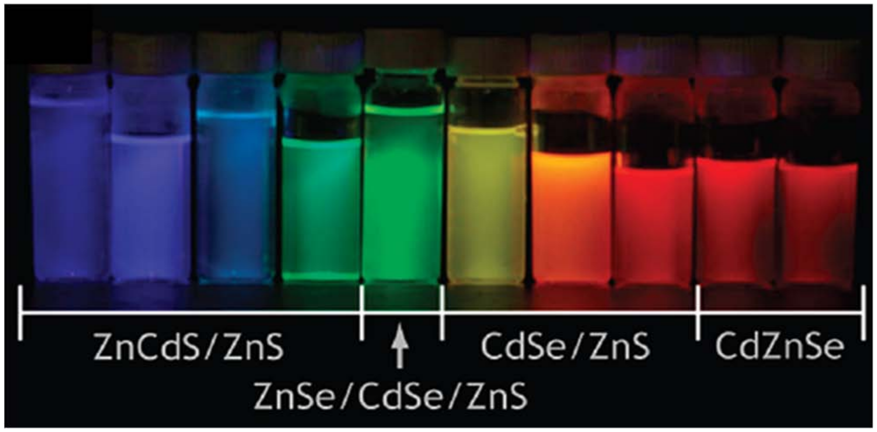


Photo by J. Halpert, Courtesy of M. Bawendi Group, EECS, MIT.

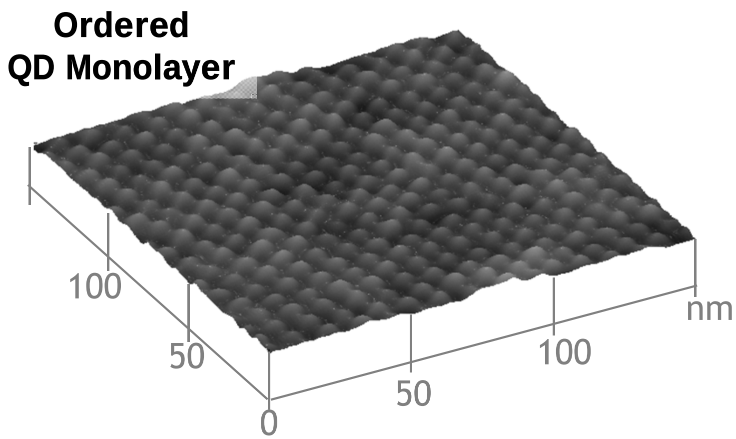
$$< \Delta x^2 > \approx R^2$$

$$< \Delta p^2 > \approx \left(\frac{\hbar}{2R}\right)^2$$

$$< E > = \frac{< p^2 >}{2m} = \frac{< \Delta p^2 >}{2m} \approx \frac{1}{R^2}$$

Quantum Dot Devices

active device region contains a single QD monolayer **~5nm thick**



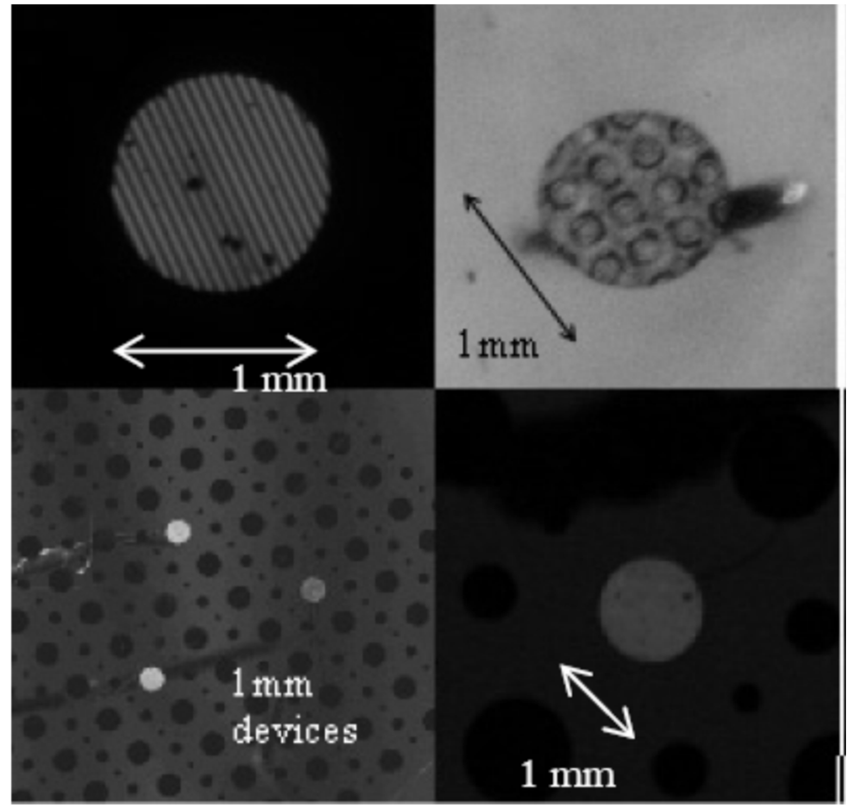


Image courtesy ONE-lab and Moungi Bawendi Group, MIT

Devices:	Advantages:
QD-LEDs	**Color, Pattern, Stability, …**
QD-Photodetectors	**Detectivity**
QD-Solar Cells	**Tunable Stacks, Efficiency**
QD-Floating Gate Memories	**Enable Device Scaling**

Summary

- Photons carry both energy & momentum.

$$E = hc/\lambda \qquad\qquad p = E/c = h/\lambda$$

- Matter also exhibits wave properties. For an object of mass m, and velocity, v, the object has a wavelength, $\lambda = h / mv$

- Heisenberg's uncertainty principle:

uncertainty in momentum

$$\Delta x \Delta p \geq \frac{h}{4\pi} = \frac{\hbar}{2}$$

uncertainty in position

6.007 Electromagnetic Energy: From Motors to Lasers
Spring 2011