Wavepackets

Outline

- Review: Reflection & Refraction
- Superposition of Plane Waves
- Wavepackets
- $\Delta k - \Delta x$ Relations

Sample Midterm 2

(one of these would be Student X's Problem)

Q1: Midterm 1 re-mix

(Ex: actuators with dielectrics)

Q2: Lorentz oscillator

(absorption / reflection / dielectric constant / index of refraction / phase velocity)

Q3: EM Waves

(Wavevectors / Poynting / Polarization / Malus' Law / Birefringence /LCDs)

Q4: Reflection & Refraction

(Snell's Law, Brewster angle, Fresnel Equations)

Q5: Interference / Diffraction

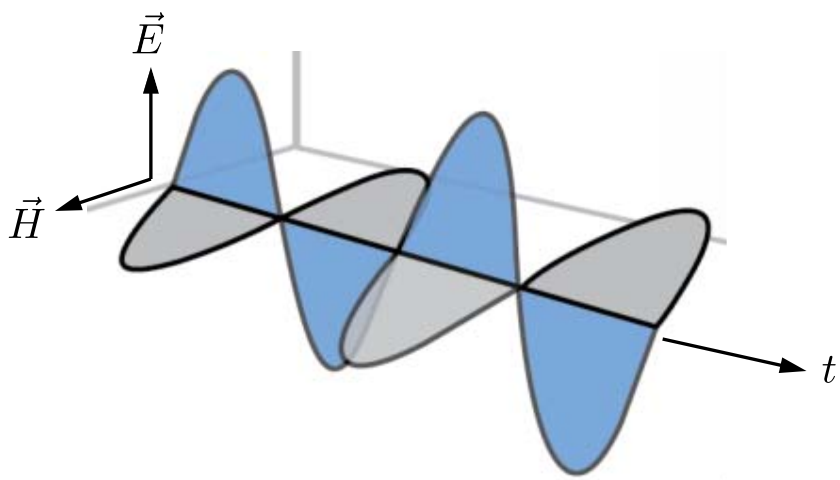
Electromagnetic Plane Waves

The Wave Equation

$$\frac{\partial^2 E_y}{\partial z^2} = \epsilon\mu \frac{\partial^2 E_y}{\partial t^2} \qquad\qquad k = \frac{\omega}{c}$$

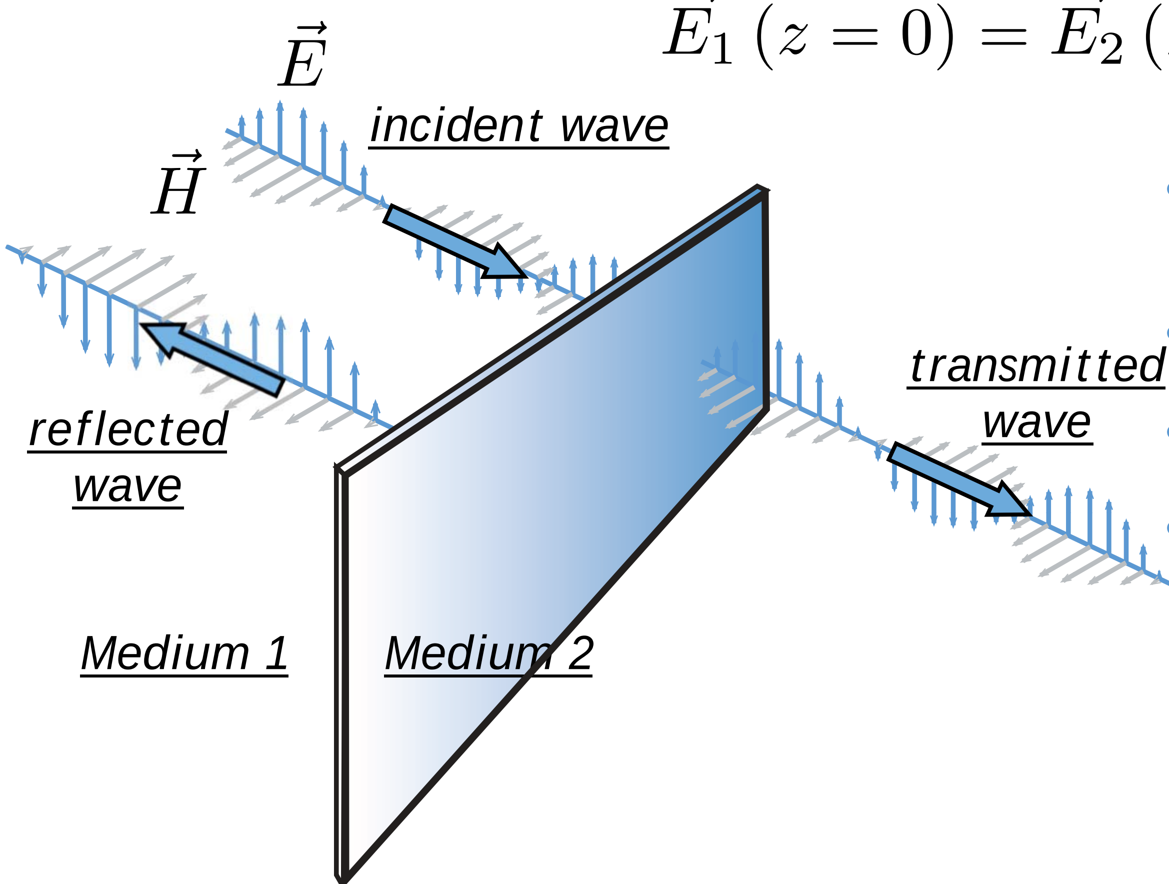
$$E_y = A_1 \cos(\omega t - kz) + A_2 \cos(\omega t + kz)$$

$$H_x = -\frac{A_1}{\eta} \cos(\omega t - kz) + \frac{A_2}{\eta} \cos(\omega t + kz)$$



Reflection of EM Waves at Boundaries

$$\vec{E}_1(z=0) = \vec{E}_2(z=0)$$

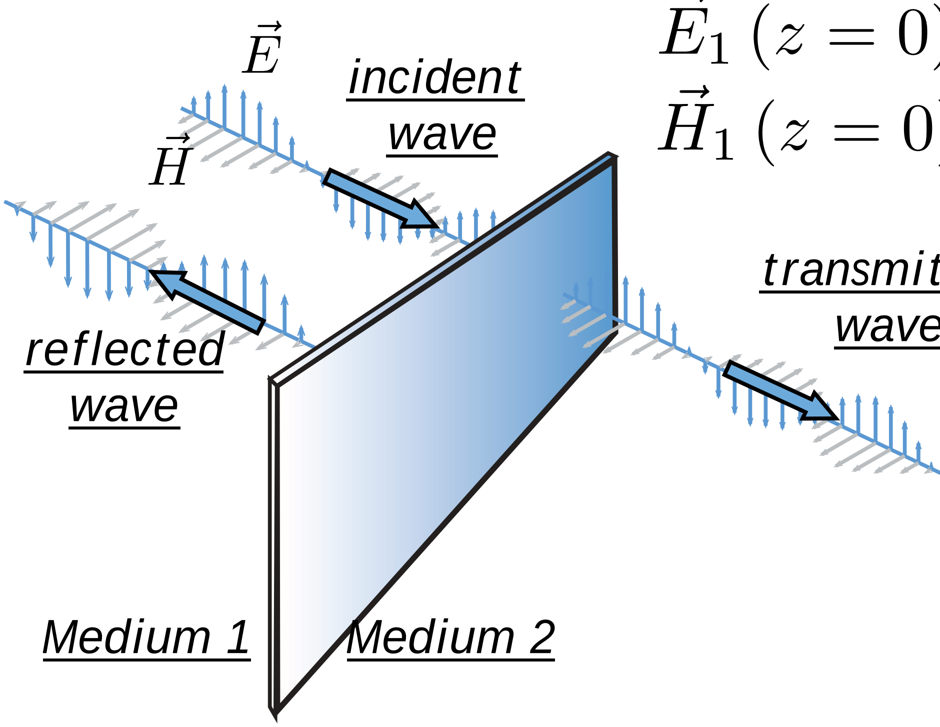


- Write traveling wave terms in each region
- Determine boundary condition
- Infer relationship of $\omega_{1,2}$ & $k_{1,2}$
- Solve for E_{ro} (r) and E_{to} (t)

$$\vec{E}_1 = \vec{E}_i + \vec{E}_r$$
$$= \hat{a}_x\left(E_{i0}e^{-jk_1z} + E_{r0}e^{+jk_1z}\right)$$
$$= \hat{a}_x\left(E_{i0}e^{-jk_1z} + rE_{i0}e^{+jk_1z}\right)$$

$$\vec{E}_2 = \vec{E}_t$$
$$= \hat{a}_xE_{t0}e^{-jk_2z}$$
$$= \hat{a}_xtE_{i0}e^{-jk_2z}$$

Reflection of EM Waves at Boundaries



$$\vec{E}_1(z=0) = \vec{E}_2(z=0)$$
$$\vec{H}_1(z=0) = \vec{H}_2(z=0)$$

- Write traveling wave terms in each region
- Determine boundary condition
- Infer relationship of $\omega_{1,2}$ & $k_{1,2}$
- Solve for E_{ro} (r) and E_{to} (t)

$$\vec{E}_1 = \hat{a}_x\left(E_{i0}e^{-jk_1z} + E_{r0}e^{+jk_1z}\right)$$

$$\vec{H}_1 = \hat{a}_y\left(\frac{E_{i0}}{\eta_1}e^{-jk_1z} - \frac{E_{r0}}{\eta_1}e^{+jk_1z}\right)$$

$$\vec{E}_2 = \hat{a}_x E_{t0}e^{-jk_2z}$$

$$\vec{H}_2 = \hat{a}_y\frac{E_{t0}}{\eta_2}e^{-jk_2z}$$

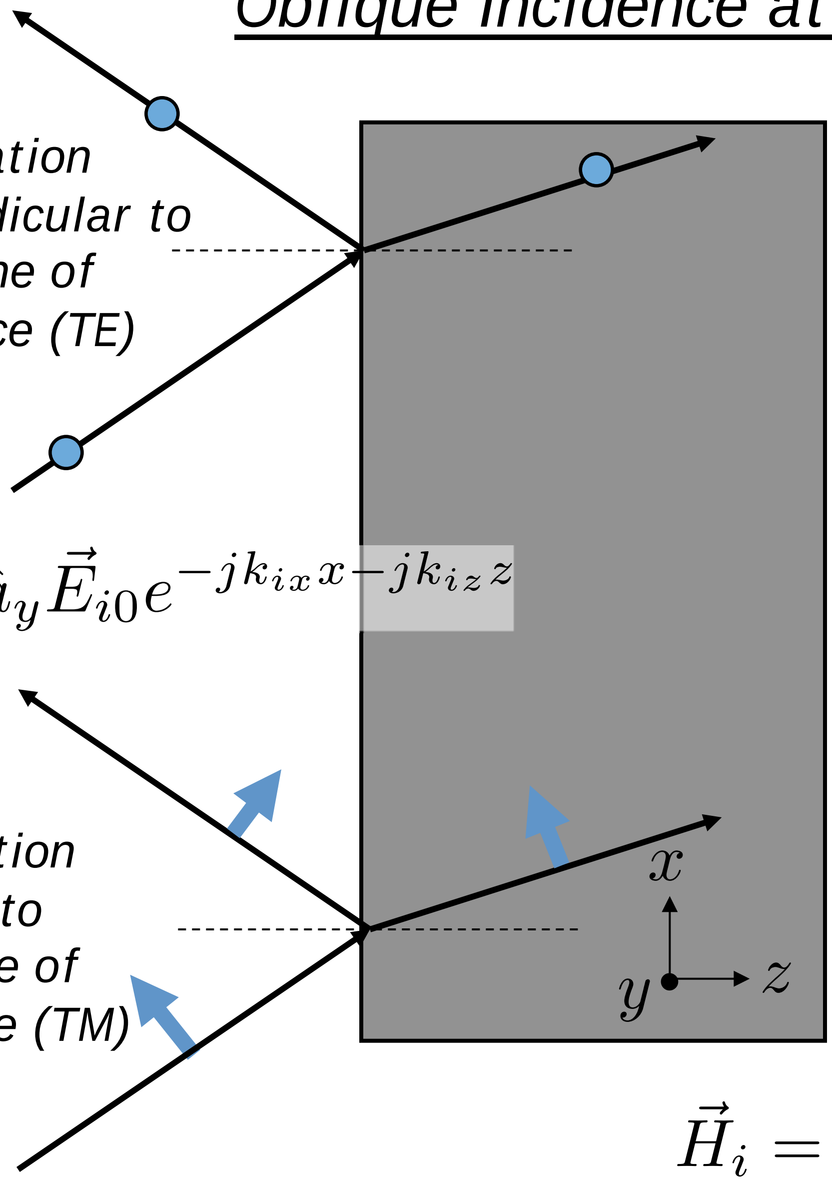
$$r = \frac{E_0^r}{E_0^i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

At normal incidence..

Oblique Incidence at Dielectric Interface

- Write traveling wave terms in each region
- Determine boundary condition
- Infer relationship of $\omega_{1,2}$ & $k_{1,2}$
- Solve for E_{ro} (r) and E_{to} (t)

E-field polarization perpendicular to the plane of incidence (TE)



$$\vec{E}_i = \hat{a}_y \vec{E}_{i0} e^{-jk_{ix}x - jk_{iz}z}$$

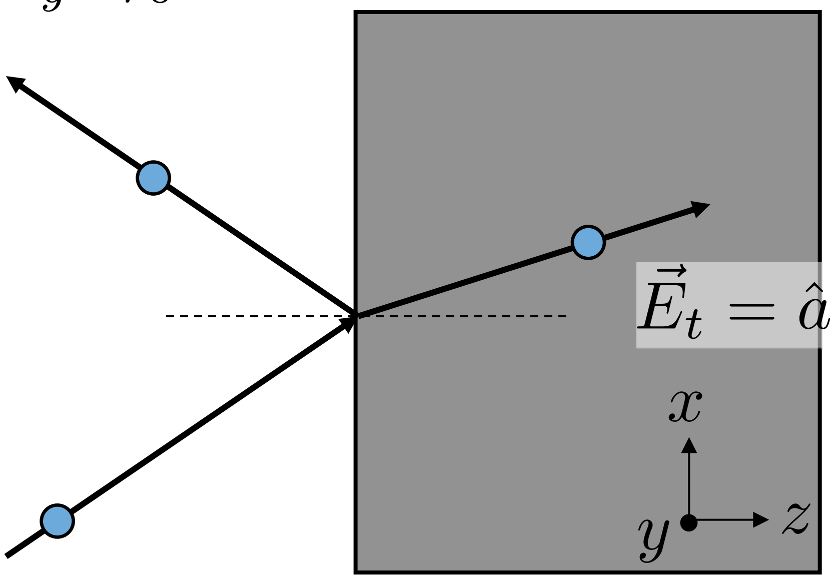
E-field polarization parallel to the plane of incidence (TM)

$$\vec{H}_i = \hat{a}_y H_{i0} e^{-jk_{ix}x - jk_{iz}z}$$

Oblique Incidence at Dielectric Interface

- *Write traveling wave terms in each region*
- *Determine boundary condition*
- *Infer relationship of $\omega_{1,2}$ & $k_{1,2}$*
- *Solve for E_{ro} (r) and E_{to} (t)*

$$\vec{E}_r = \hat{a}_y E_{r0} e^{-jk_{rx}x + jk_{rz}z}$$



$$\vec{E}_t = \hat{a}_y E_{t0} e^{-jk_{tx}x - jk_{tz}z}$$

$$\vec{E}_i = \hat{a}_y E_{i0} e^{-jk_{ix}x - jk_{iz}z}$$

Oblique Incidence at Dielectric Interface

- Write traveling wave terms in each region
- Determine boundary condition
- Infer relationship of $\omega_{1,2}$ & $k_{1,2}$
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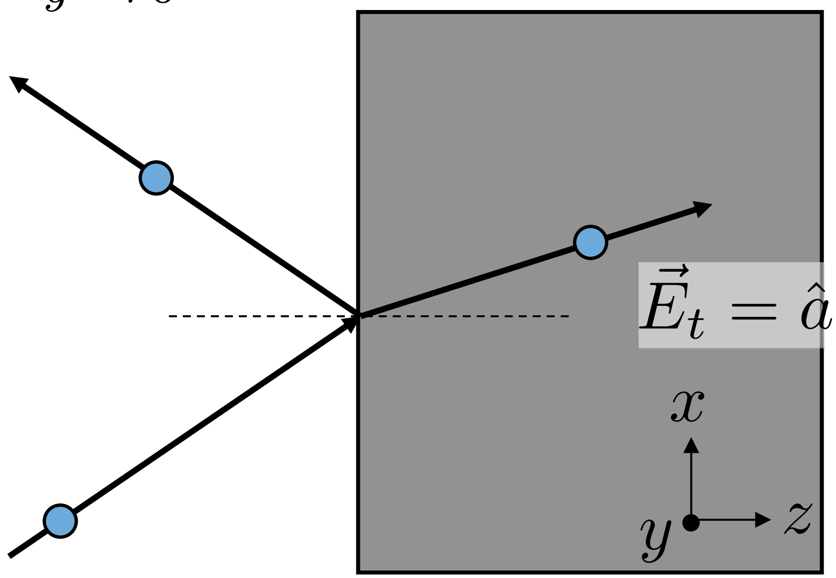


$$\vec{E}_t = \hat{a}_y E_{t0} e^{-jk_{tx}x - jk_{tz}z}$$

$$\vec{E}_i = \hat{a}_y E_{i0} e^{-jk_{ix}x - jk_{iz}z}$$

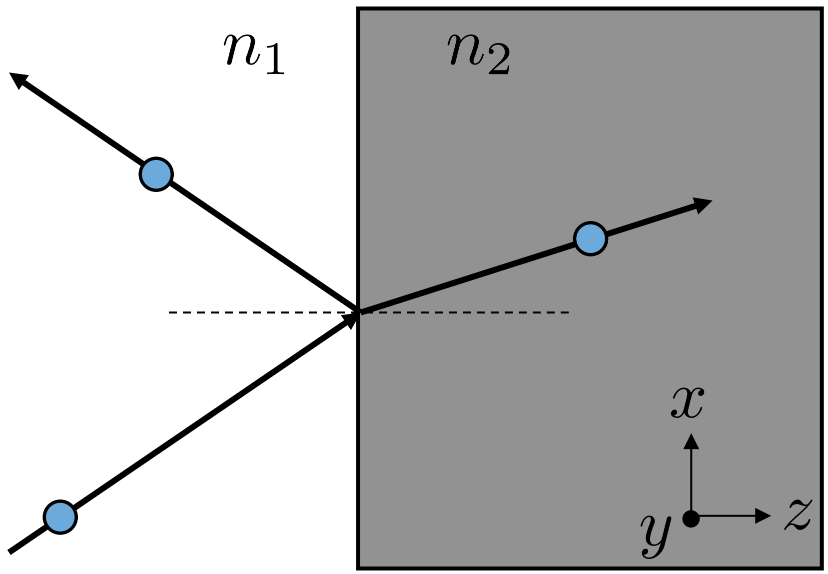
Tangential field is continuous (z=0)..

$$E_{i0} e^{-jk_{ix}x} + E_{r0} e^{-jk_{rx}x} = E_{t0} e^{-jk_{tx}x}$$

Snell's Law

Tangential E-field is continuous ...

$$E_{i0}e^{-jk_{ix}x} + E_{r0}e^{-jk_{rx}x} = E_{t0}e^{-jk_{tx}x}$$



REMINDER:

$$k = \frac{\omega}{v_p} = \frac{\omega n}{c}$$

$$k_{ix} = k_{rx}$$

$$n_1 \sin\theta_i = n_1 \sin\theta_r$$

$$\theta_i = \theta_r$$

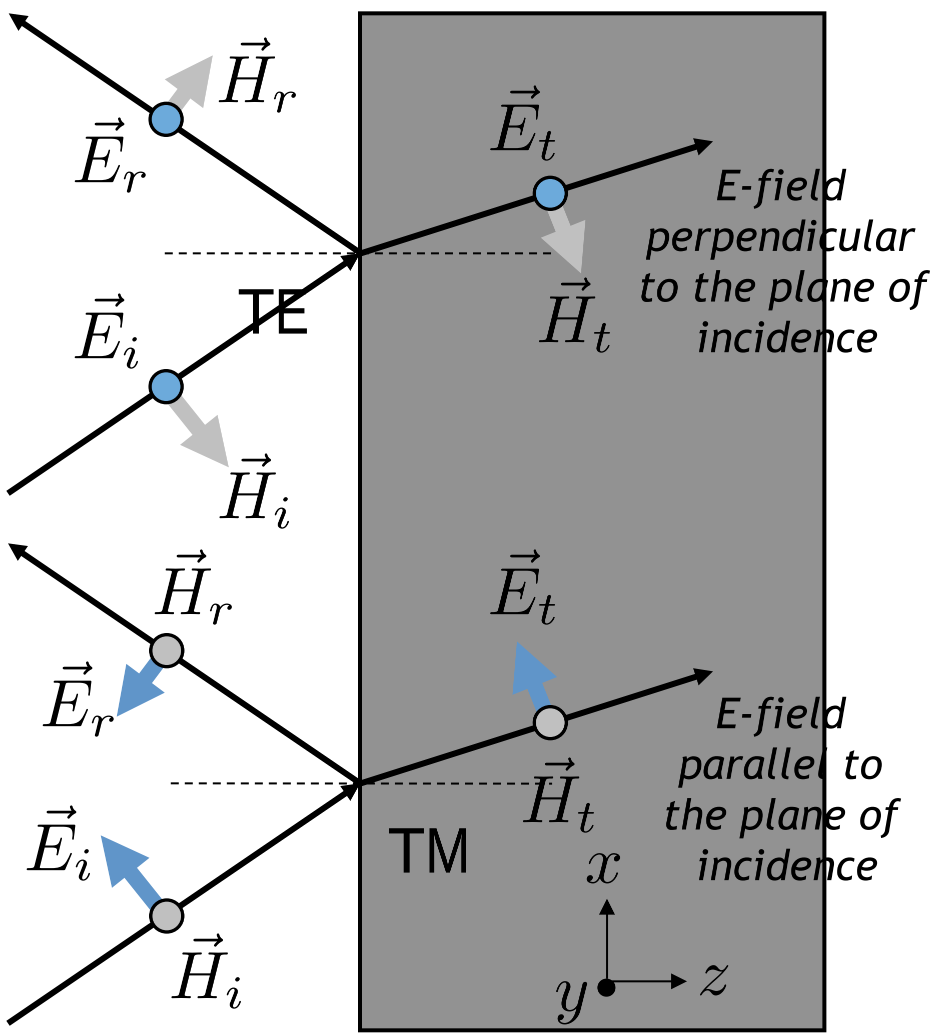
$$k_{ix} = k_{tx}$$

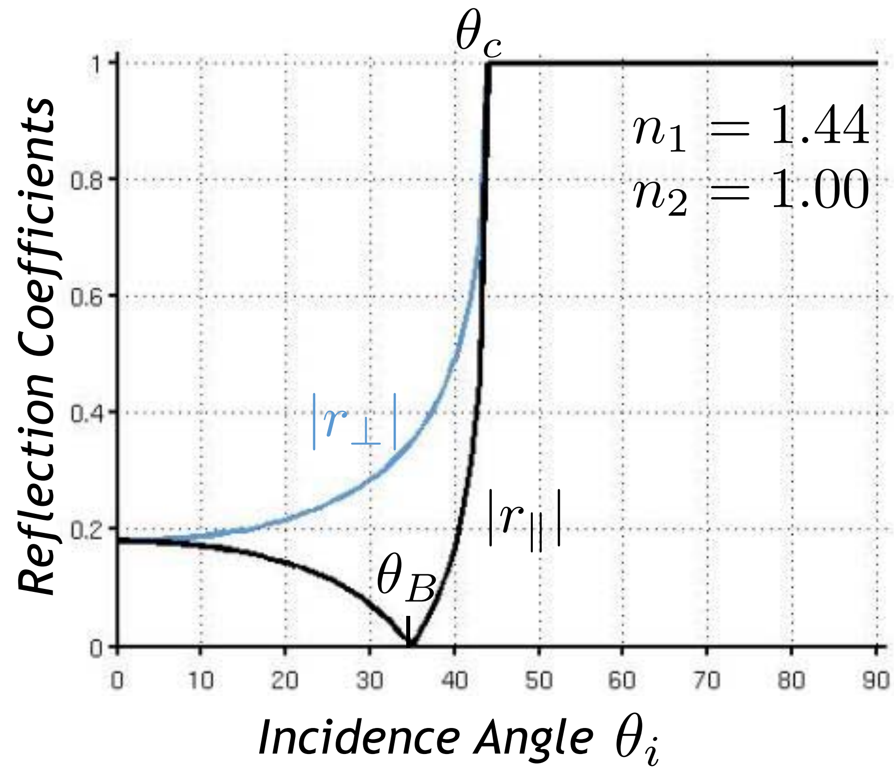
$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

Reflection of Light
(Optics Viewpoint … $\mu_1 = \mu_2$)

TE: $$r_\perp = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}$$

TM: $$r_\parallel = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t}$$





Electromagnetic Plane Waves

The Wave Equation

$$\frac{\partial^2 E_y}{\partial z^2} = \epsilon\mu \frac{\partial^2 E_y}{\partial t^2} \qquad\qquad k = \frac{\omega}{c}$$

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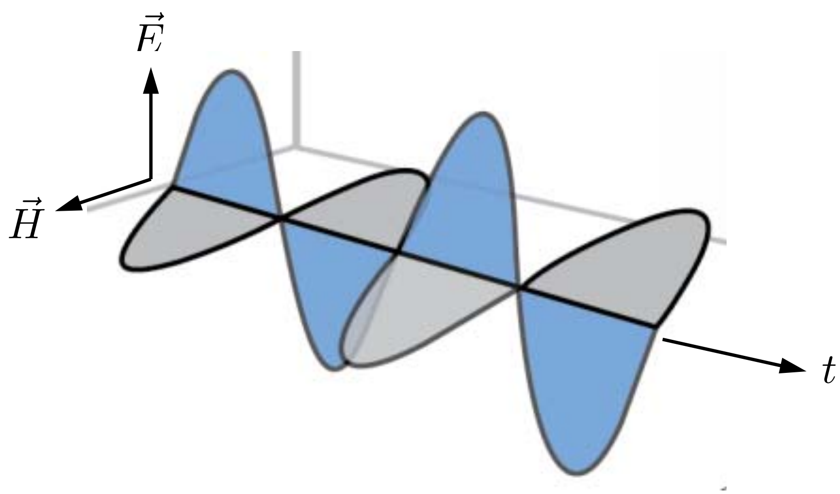
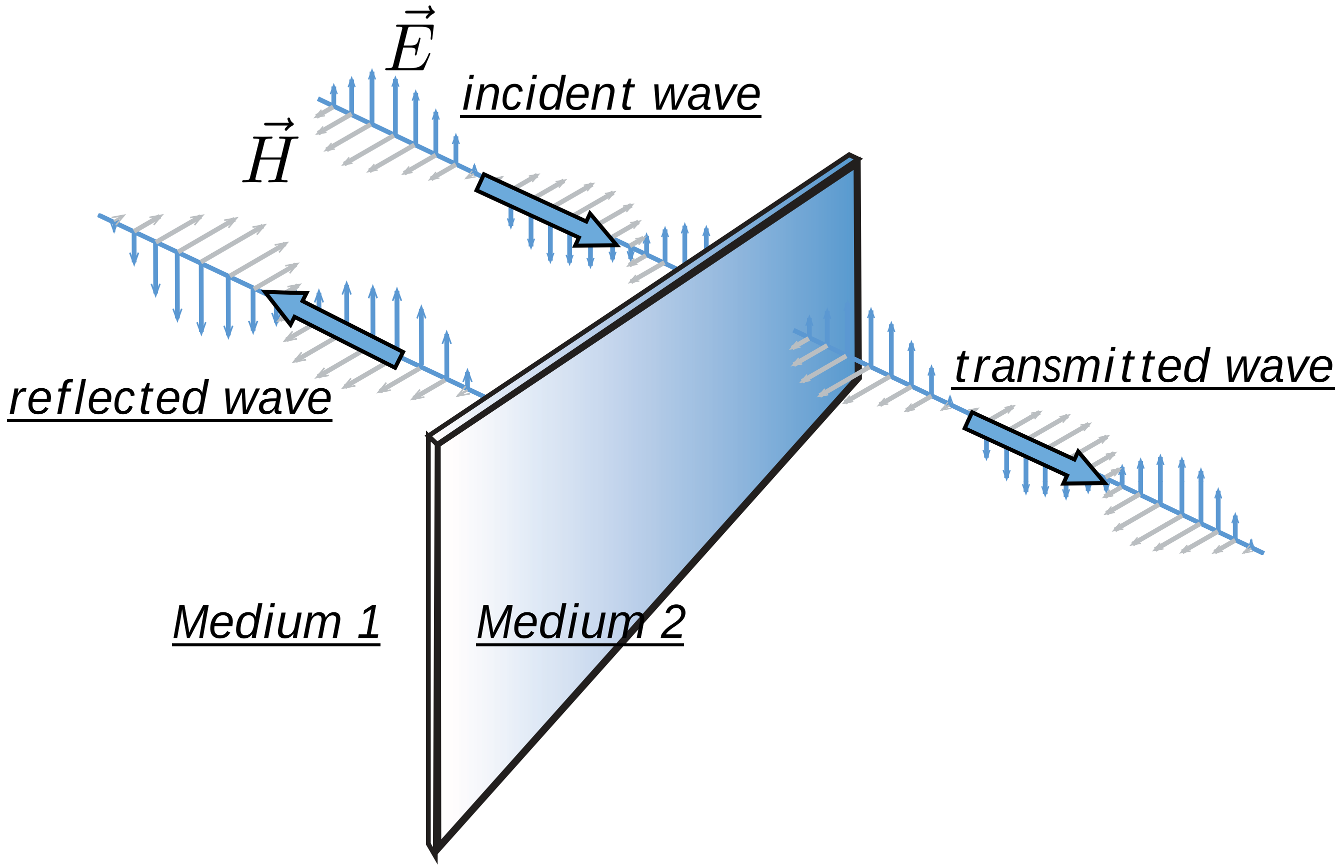


- *When did this plane wave turn on?*
- *Where is there no plane wave?*

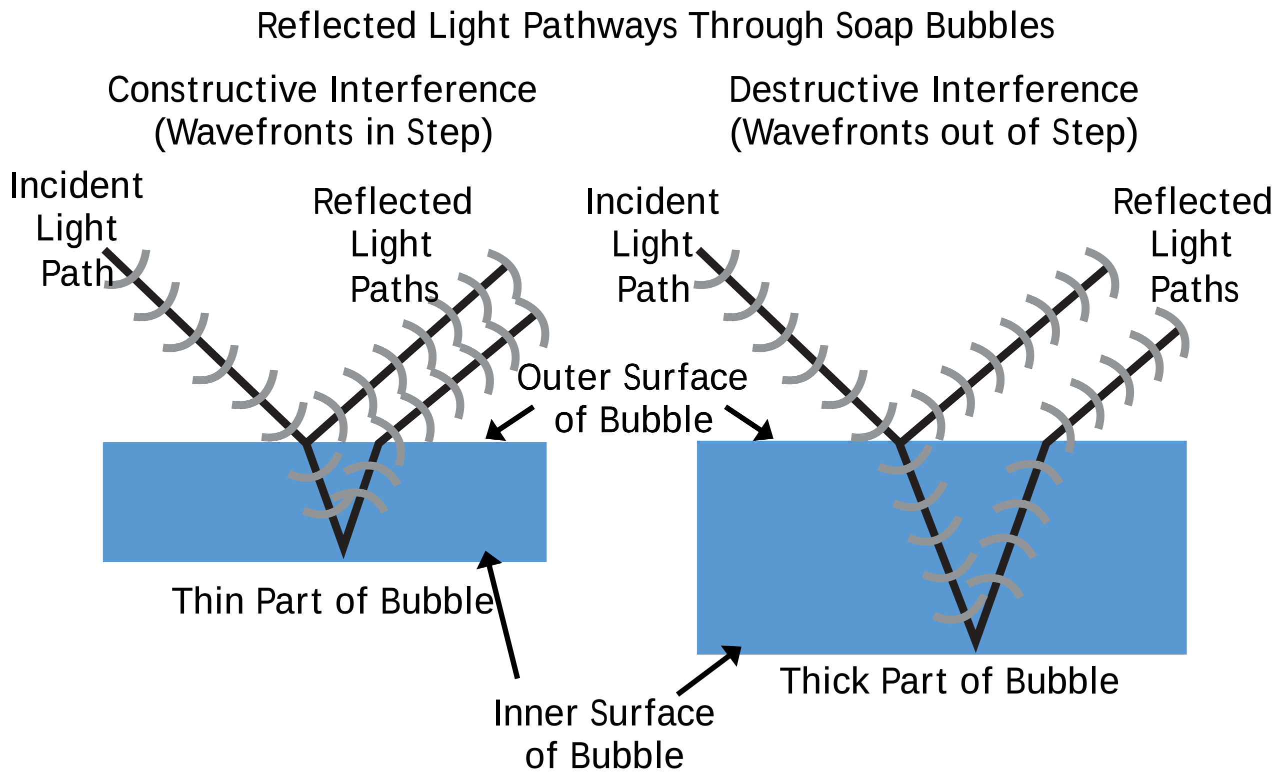
Superposition Example: Reflection



How do we get a wavepacket (localized EM waves) ?

Superposition Example: Interference

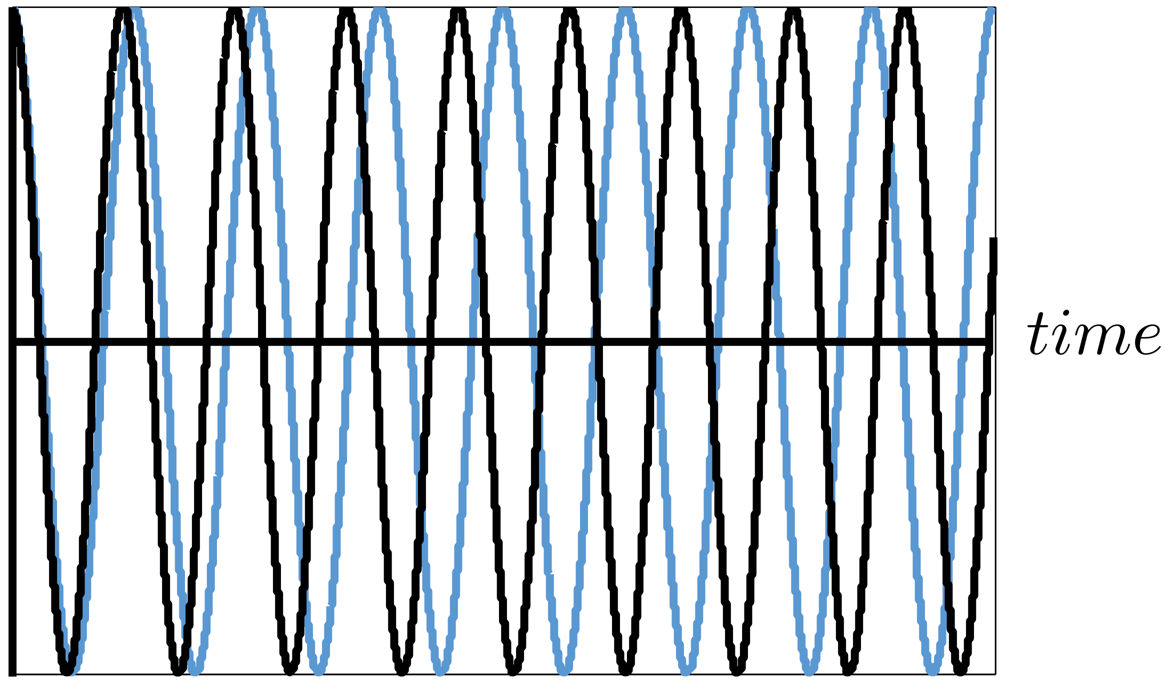
Image by Ali T http://www.flickr.com/photos/77682540@N00/2789338547/ on Flickr.



What if the interfering waves do not have the same frequency (ω, k) ?

Two waves at different frequencies

... will constructively interfere and destructively interfere at different times

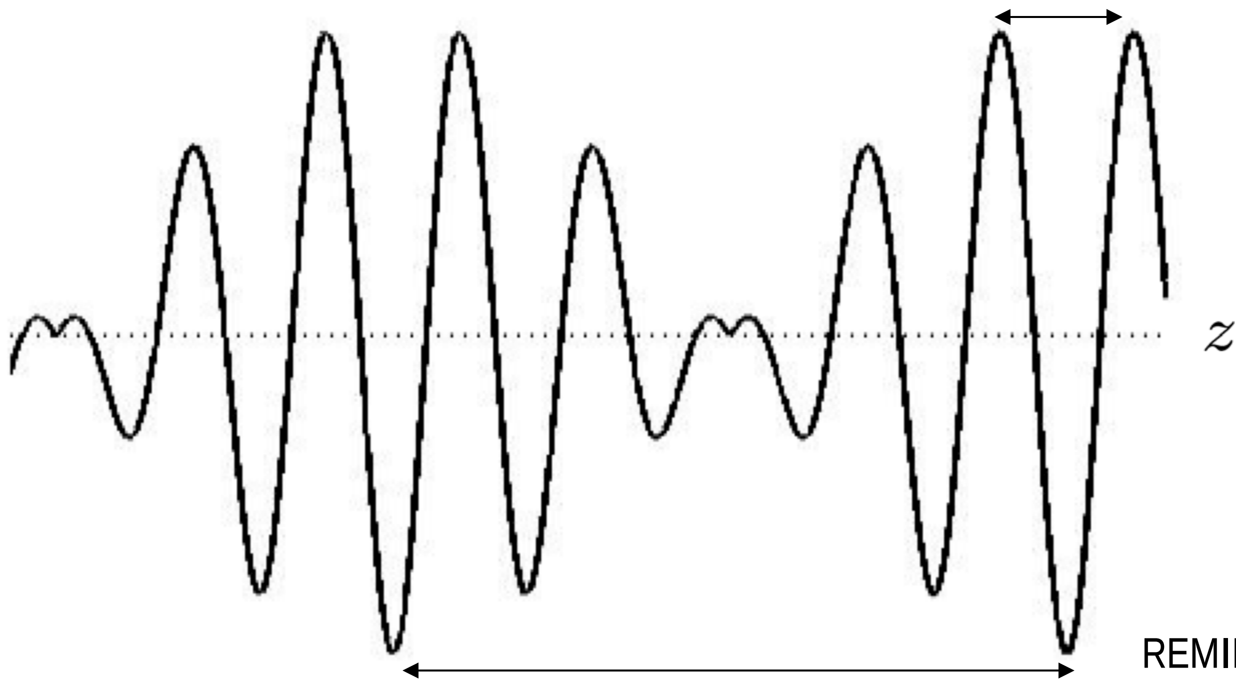


In Figure above the waves are chosen to have a 10% frequency difference. So when the slower wave goes through 5 full cycles (and is positive again), the faster wave goes through 5.5 cycles (and is negative).

Wavepackets: Superpositions Along Travel Direction

$$\vec{E} = \vec{E}_o \left(e^{j(\omega_1 t - k_1 z)} + e^{j(\omega_2 t - k_2 z)} \right) \hat{y}$$

$$\frac{4\pi}{|k_2 + k_1|}$$



$$\frac{4\pi}{|k_2 - k_1|}$$

REMINDER:

$$k = \frac{n}{c}\omega$$

SUPERPOSITION OF TWO WAVES OF DIFFERENT FREQUENCIES
(hence different k's)

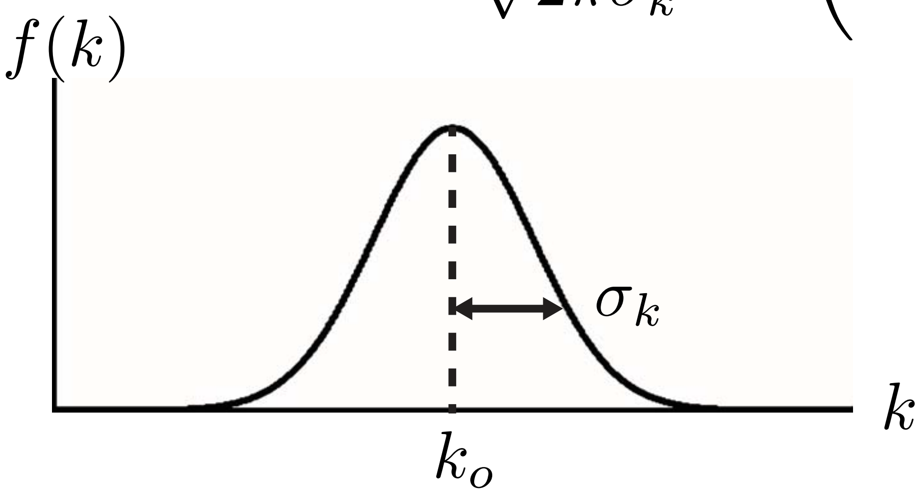
Wavepackets: Superpositions Along Travel Direction

WHAT WOULD WE GET IF WE SUPERIMPOSED WAVES OF MANY DIFFERENT FREQUENCIES ?

$$\vec{E} = \vec{E}_o \int_{-\infty}^{+\infty} f(k)\, e^{+j(\omega t - kz)} dk$$

LET'S SET THE FREQUENCY DISTRIBUTION as GAUSSIAN

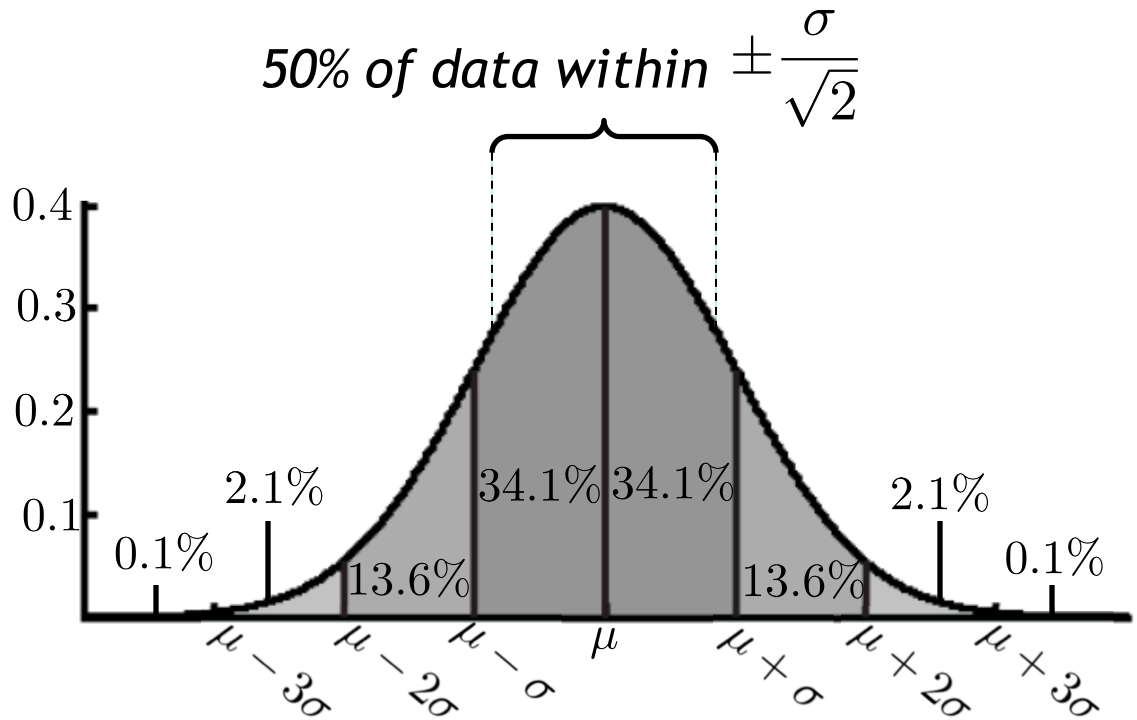
$$f(k) = \frac{1}{\sqrt{2\pi\sigma_k}} exp\left(-\frac{(k-k_o)^2}{2\sigma_k^2}\right)$$



REMINDER:

$$k = \frac{n}{c}\omega$$

Reminder: Gaussian Distribution



$$f\left(x\right) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

μ specifies the position of the bell curve's central peak
σ specifies the half-distance between inflection points

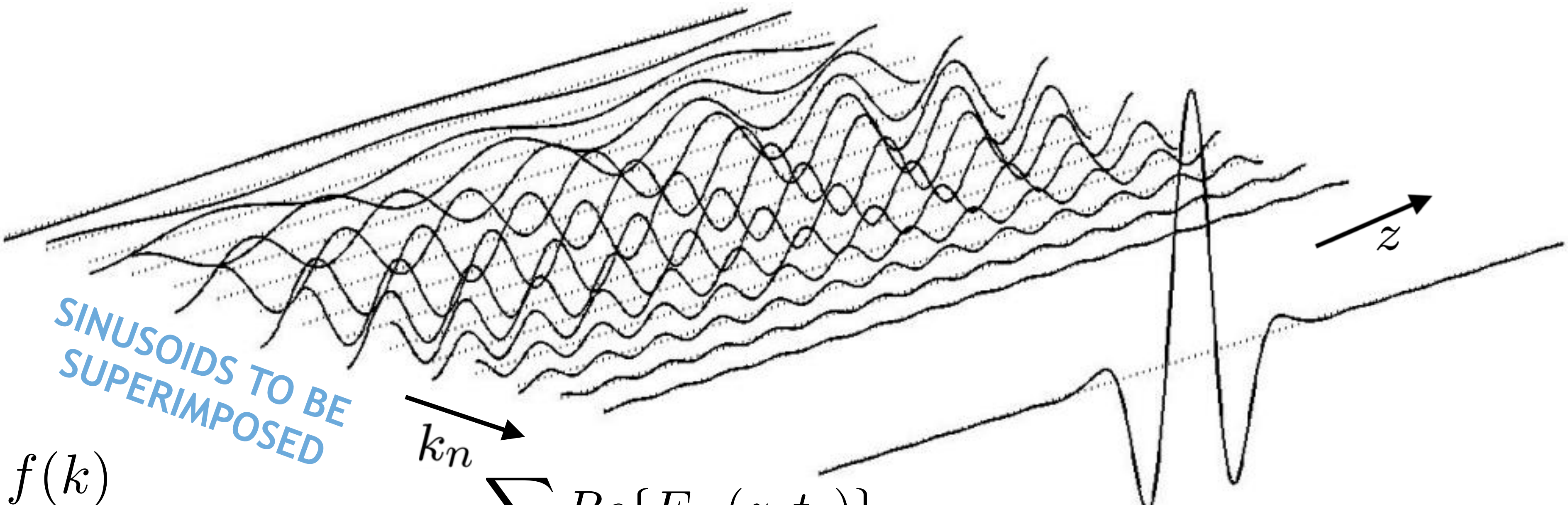
FOURIER TRANSFORM OF A GAUSSIAN IS A GAUSSIAN

$$\mathcal{F}_x\left[e^{-ax^2}\right](k) = \sqrt{\frac{\pi}{a}} e^{-\frac{\pi^2 k^2}{a}}$$

Gaussian Wavepacket in Space

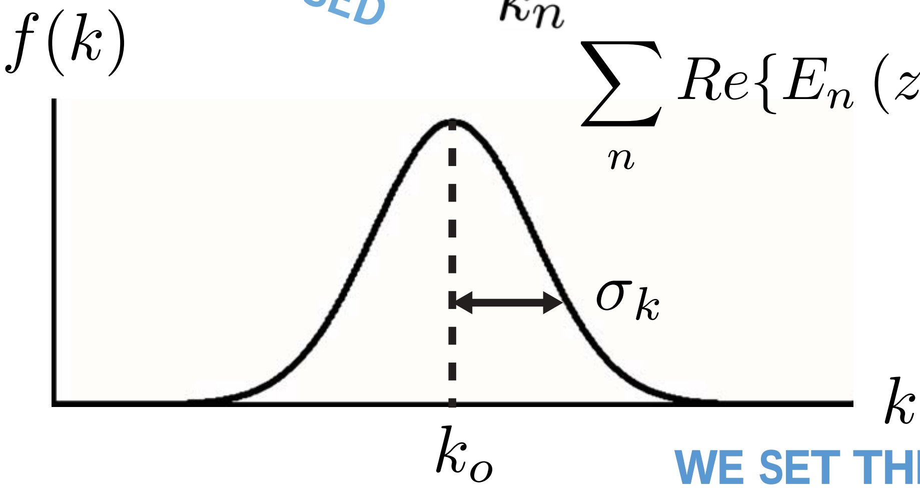
$$Re\{E_n(z,t_o)\}$$

$$t = t_o$$



$$\sum_n Re\{E_n(z,t_o)\}$$

SUM OF SINUSOIDS = WAVEPACKET



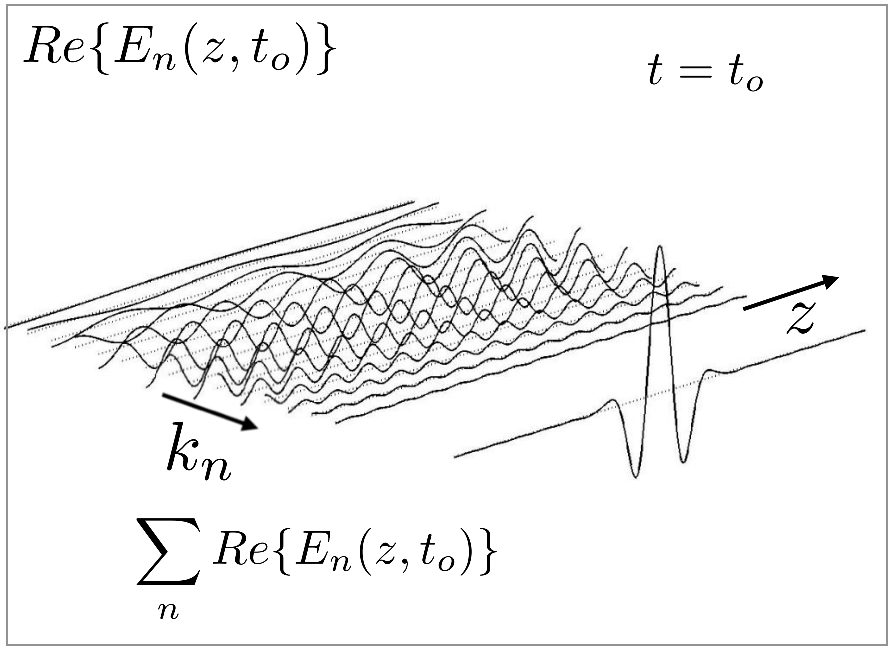
$$f(k) = \frac{1}{\sqrt{2\pi\sigma_k}} exp\left(-\frac{(k-k_o)^2}{2\sigma_k^2}\right)$$

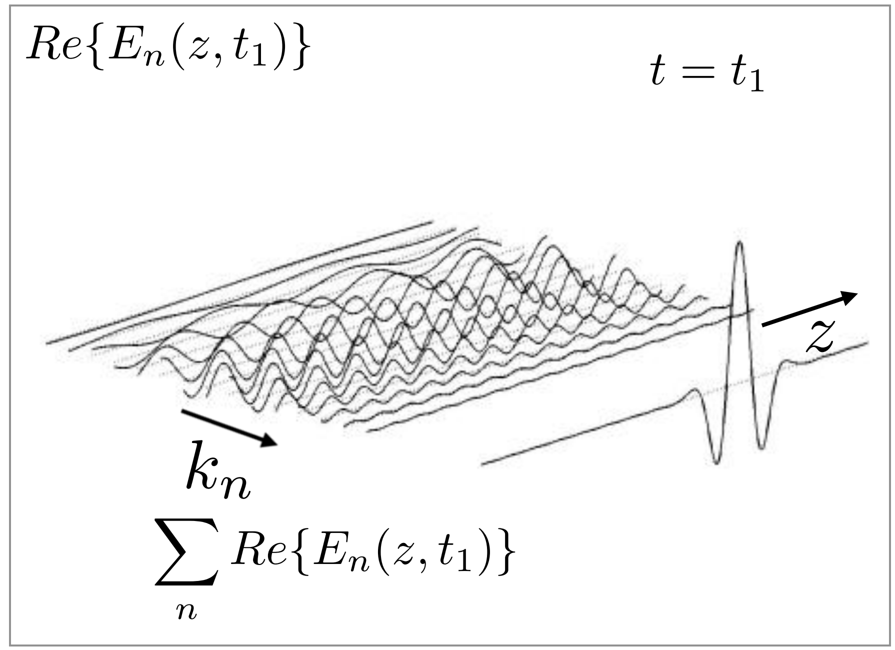
WE SET THE FREQUENCY DISTRIBUTION as GAUSSIAN

Gaussian Wavepacket in Time

$$E(z,t) = E_o exp\left(-\frac{\sigma_k^2}{2}(ct-z)^2\right)\cos(\omega_o t - k_o z)$$

GAUSSIAN ENVELOPE

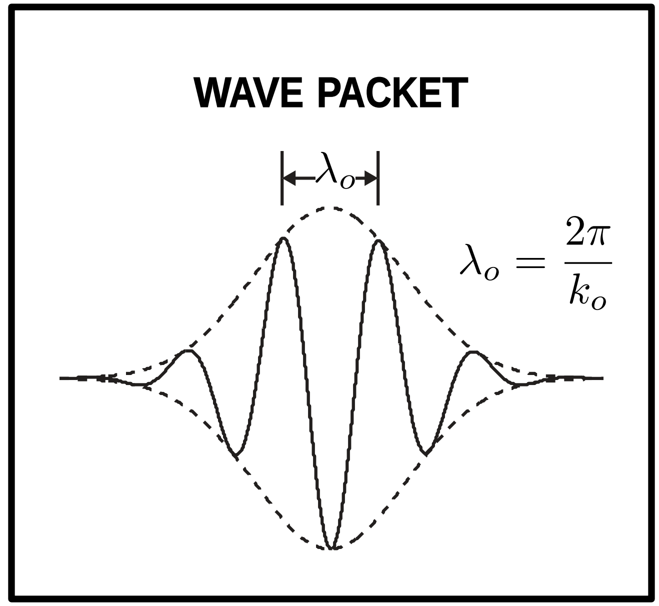




Gaussian Wavepacket in Space

$$E(z,t) = E_o exp\left(-\frac{\sigma_k^2}{2}(ct-z)^2\right)\cos(\omega_o t - k_o z)$$

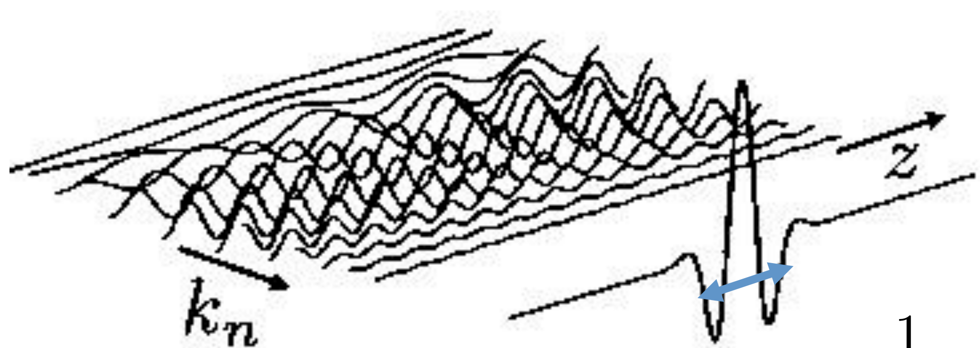
GAUSSIAN ENVELOPE



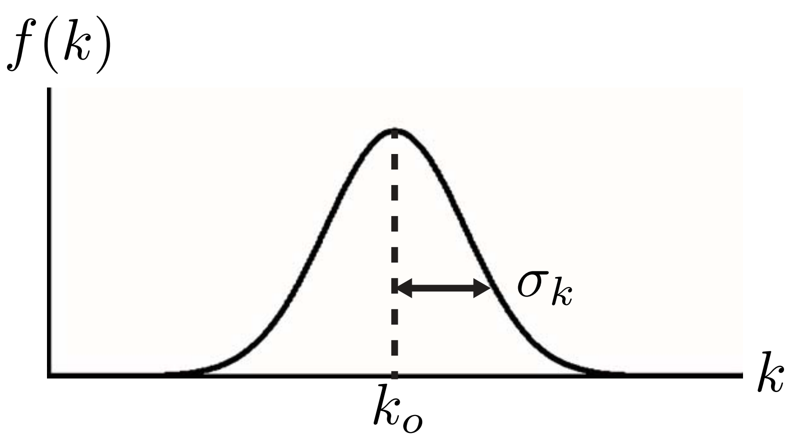
In free space ...

$$k = \frac{\omega}{c}$$

... this plot then shows the PROBABILITY OF WHICH k (or frequency) EM WAVES are MOST LIKELY TO BE IN THE WAVEPACKET



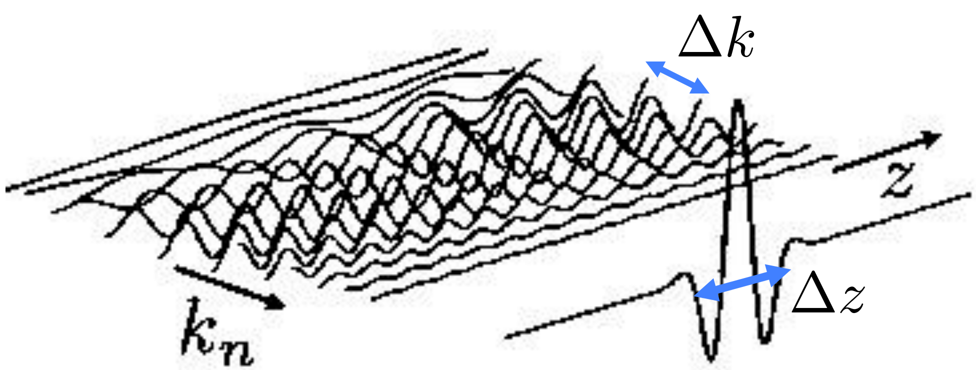
$$\Delta z = \frac{1}{\sqrt{2}\sigma_k}$$

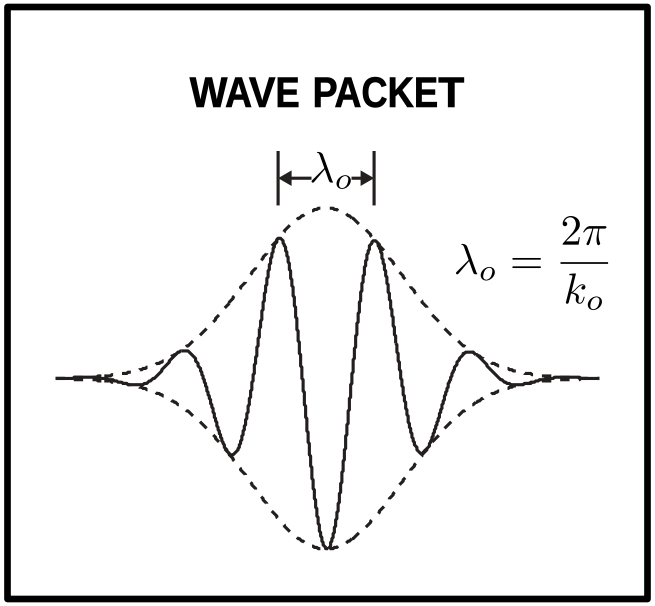


$$\Delta k \Delta z = 1/2$$

Gaussian Wavepacket in Time

$$E(z,t) = E_o exp\left(-\frac{\sigma_k^2}{2}(ct-z)^2\right)\cos(\omega_o t - k_o z)$$





UNCERTAINTY RELATIONS

$$\Delta z = \frac{c}{n}\Delta t$$

$$\Delta k = \frac{c}{n}\Delta \omega$$

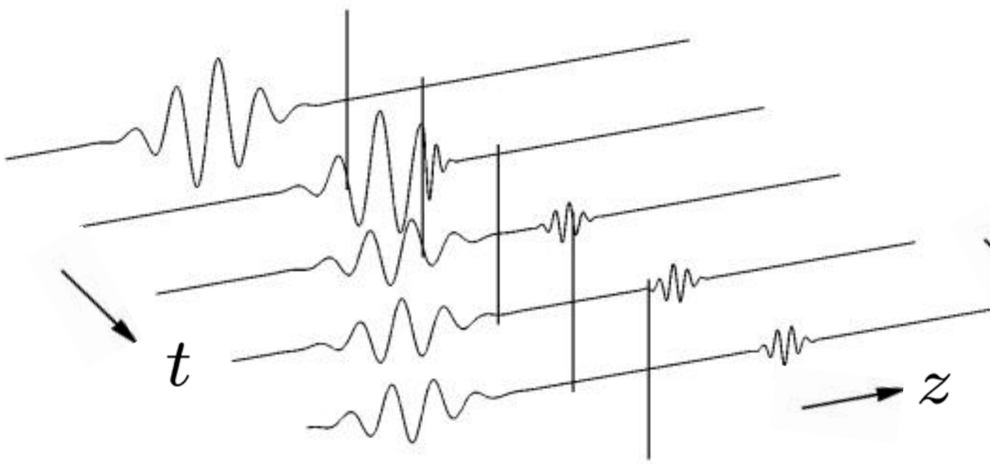
$$\Delta k \Delta z = 1/2$$

$$\Delta \omega \Delta t = 1/2$$

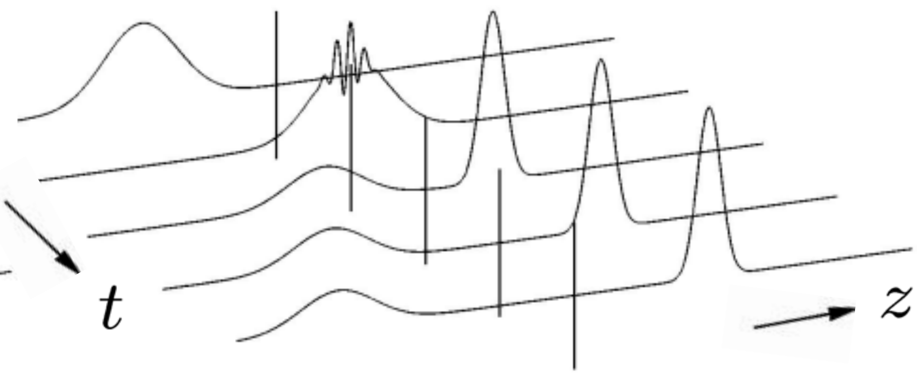
If you want to LOCATE THE WAVEPACKET WITHIN THE SPACE Δz you need to use a set of EM-WAVES THAT SPAN THE WAVENUMBER SPACE OF Δk = 1/(2Δz)

Wavepacket Reflection

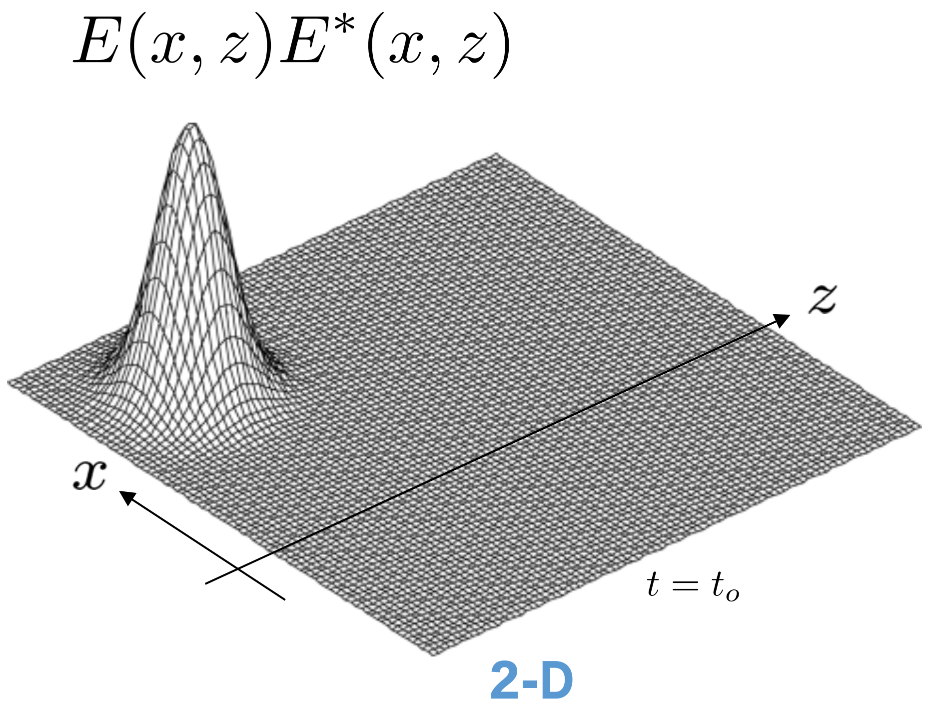
$E(z,t)$



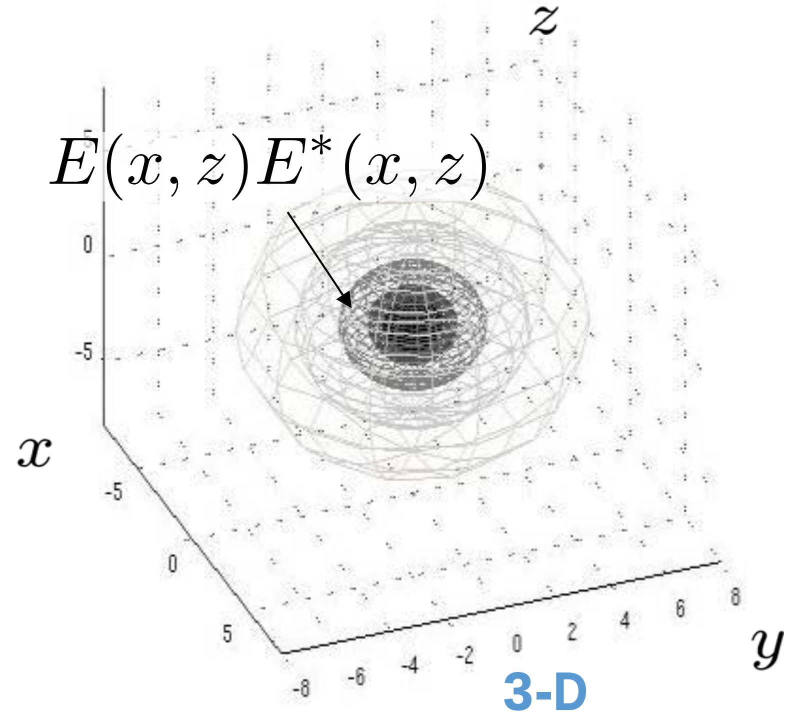
$E_c E_c^*$



Wavepackets in 2-D and 3-D



Contours of constant amplitude

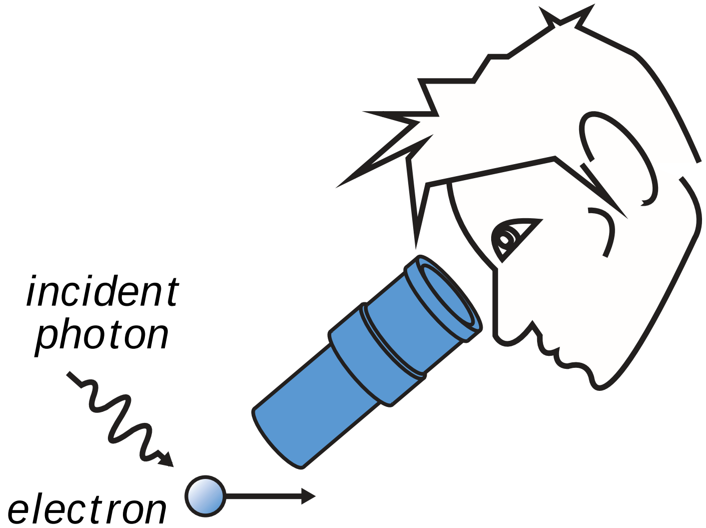


Spherical probability distribution for the magnitude of the amplitudes of the waves in the wave packet.

In the next few lectures we will start considering the limits of

Light Microscopes

and how these might affect our understanding of the world we live in



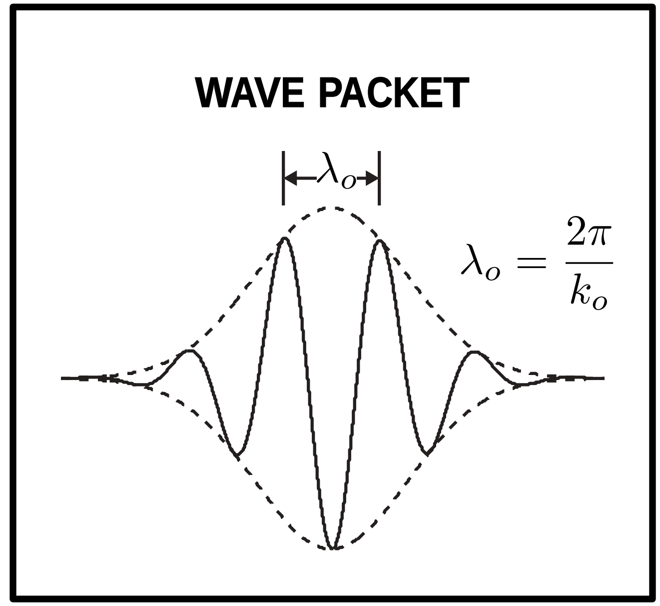
- Suppose the positions and speeds of all particles in the universe are measured to sufficient accuracy at a particular instant in time
- It is possible to predict the motions of every particle at any time in the future (or in the past for that matter)

Key Takeaways

GAUSSIAN WAVEPACKET IN SPACE

$$E(z,t) = E_o exp\left(-\frac{\sigma_k^2}{2}\left(ct - z\right)^2\right)\cos\left(\omega_o t - k_o z\right)$$

GAUSSIAN ENVELOPE



UNCERTAINTY RELATIONS

$$\Delta k \Delta z = 1/2$$

$$\Delta \omega \Delta t = 1/2$$