Generating Electromagnetic Waves

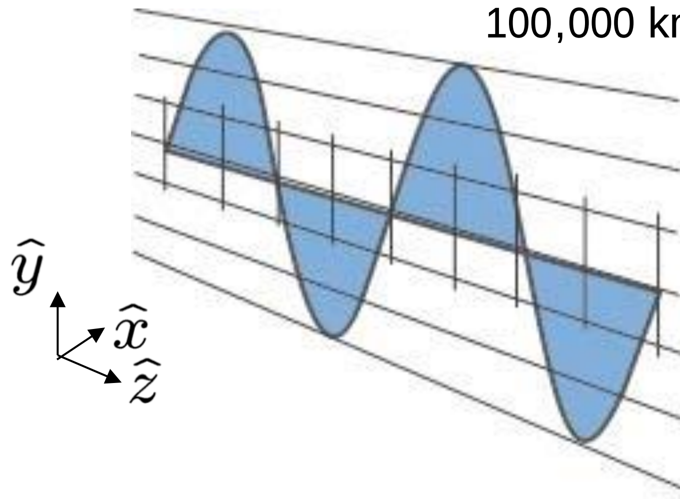
Parabolic antenna for communicating with spacecraft, Canberra, Australia

Outline

- Uniform Plane Waves
- Energy Transported by EM Waves (Poynting Vector)
- Current Sheet & Quasi-static Approximation
- Transient Analysis for Current Sheet Antenna

Uniform Plane Wave

$$E_y = A_1 cos(\omega t - kz)$$



Estimate the wavevector k?

Estimate the angular frequency ?

Estimate the magnitude of the electric field.

Estimate the magnitude of the magnetic field.

Coupling of Electric and Magnetic Fields

Maxwell's Equations couple H and E fields...

$$\oint_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}(\int_S \vec{B}\cdot d\vec{A})$$

$$\oint_C \vec{H}\cdot d\vec{l} = \oint_S \vec{J}\cdot d\vec{A} + \frac{d}{dt}\int \epsilon E\, dA$$

$$\frac{\partial E_y}{dz} = \frac{\partial B_x}{dt}$$

Source free
$J = 0$

$$\frac{\partial B_x}{\partial z} = \epsilon\mu\frac{\partial E_y}{\partial t}$$

$$\boxed{\frac{\partial^2 E_y}{\partial z^2} = \epsilon\mu\frac{\partial^2 E_y}{\partial t^2}}$$

The Wave Equation

Electromagnetic Energy Storage

Recall ...

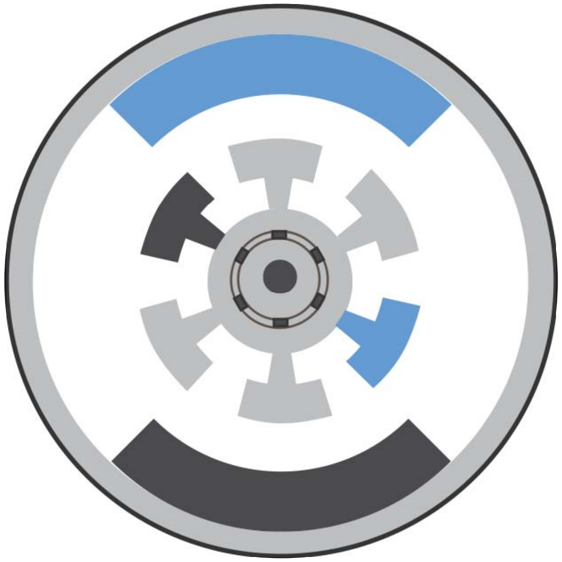
Magnetic

$$\frac{W_s}{V} = \frac{1}{2}\mu_o H \cdot H$$

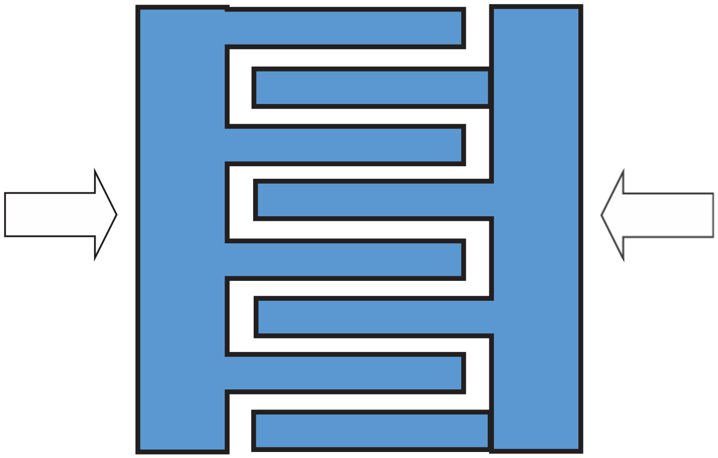
Electric

$$\frac{W_s}{V} = \frac{1}{2}\epsilon_o E \cdot E$$

Magnetic machine



Electric machine



Power Flow of a Uniform Plane Wave

$$\frac{P}{A} = c\left(\frac{W_E}{V} + \frac{W_M}{V}\right)$$

$$= c\frac{2W_M}{V}$$

$$= |\vec{E}| \cdot |\vec{H}|$$

$$\boxed{\frac{\vec{P}}{A} = \vec{E} \times \vec{H}}$$

Poynting Vector

$$Intensity = |\vec{E}| \cdot |\vec{H}| = \frac{E^2}{\eta} = \eta H^2$$

Euler's Formula

This gives us the famous identity known as Euler's formula:

$$e^{iy} = cos(y) + i * sin(y)$$

From this, we get two more formulas:

$$cos(y) = \frac{e^{iy} + e^{-iy}}{2} \qquad sin(y) = \frac{e^{iy} - e^{-iy}}{2i}$$

Exponential functions are often easier to work with than sinusoids, so these formulas can be useful.

The following property of exponentials is still valid for complex z:

$$e^{z_1 + z_2} = e^{z_1} e^{z_2}$$

Using the formulas on this page, we can prove many common trigonometric identities. Proofs are presented in the text.

Modern Version of Steinmetz' Analysis

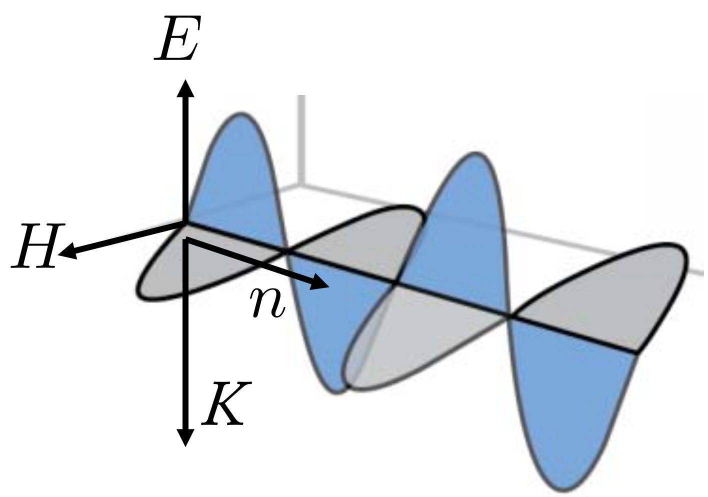
1. Begin with a time-dependent analysis problem posed in terms of real variables.
2. Replace the real variables with variables written in terms of complex exponentials; $e^{j\omega t}$ is an eigenfunction of linear time-invariant systems.
3. Solve the analysis problem in terms of complex exponentials.
4. Recover the real solution from the results of the complex analysis.

How Are Uniform EM Plane Waves Launched?

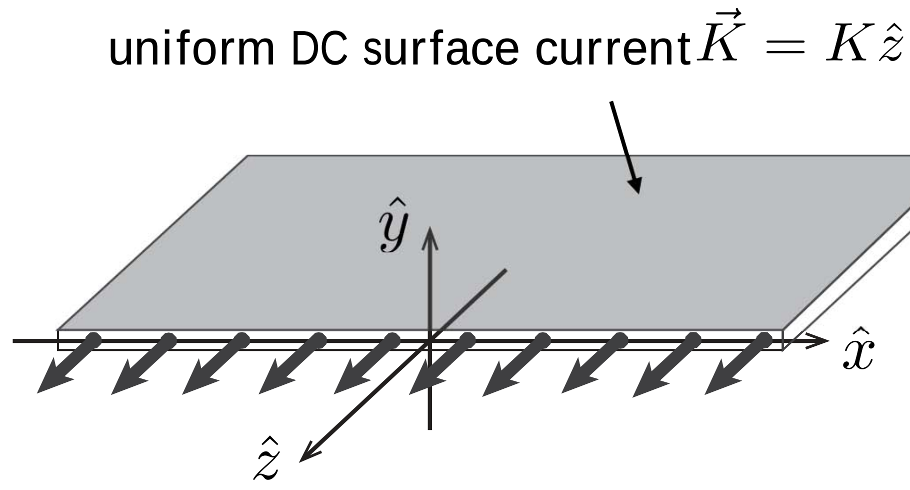
Generally speaking, electromagnetic waves are launched by time-varying charge distributions and currents.

Man-made systems that launch waves are often called antennas.

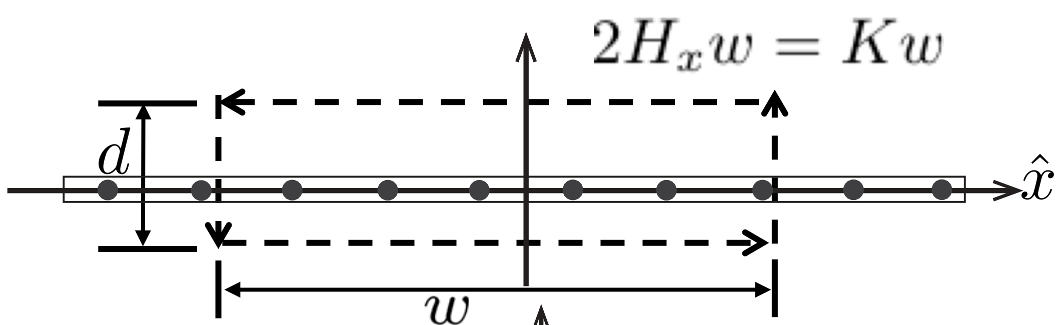
Uniform plane waves are launched by current sheets.

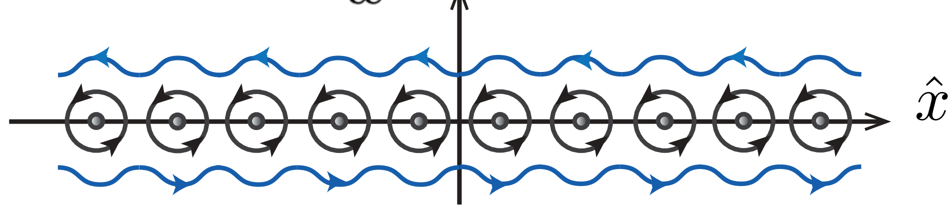


Static Magnetic Field from Current Sheet



$$\boxed{\oint_C \vec{H}\cdot d\vec{l} = \oint_S \vec{J}\cdot d\vec{A} + \frac{d}{dt}\int \epsilon E \, dA}$$



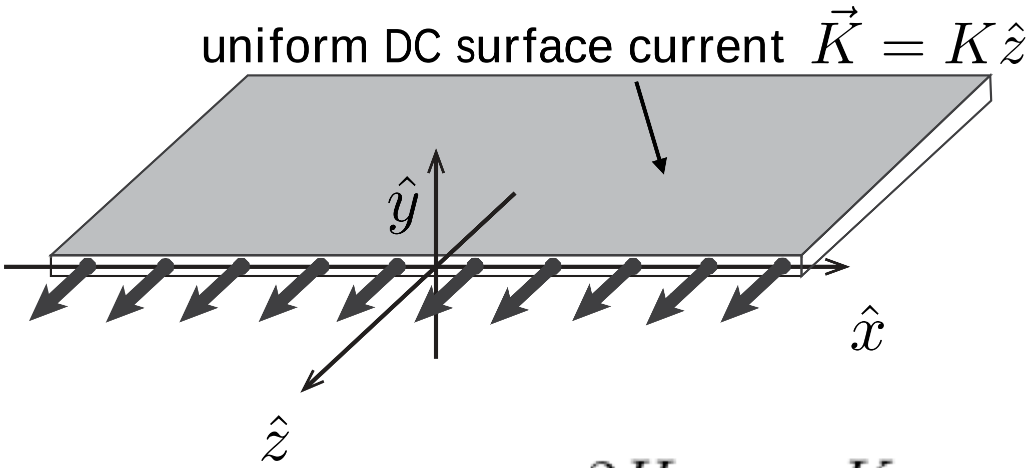


$$\vec{H} = \boxed{}$$

What is the $\vec{E}$-field?

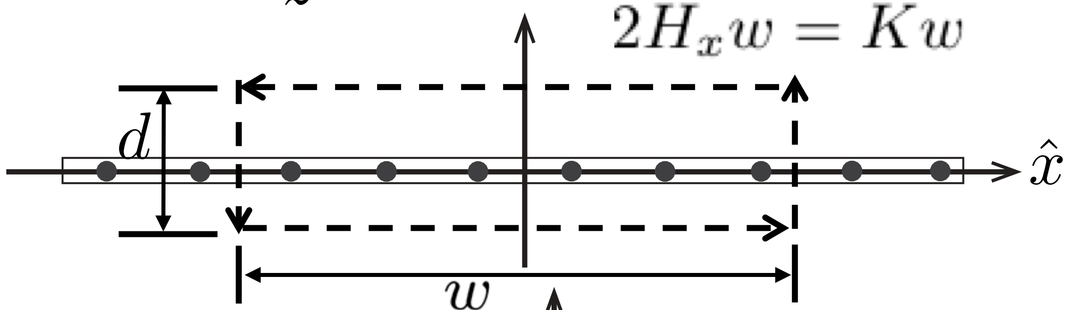
<u>*Magnetic Field Above/Below a Sheet of Current*</u>

... flowing in the $\hat{z}$ direction with current density K

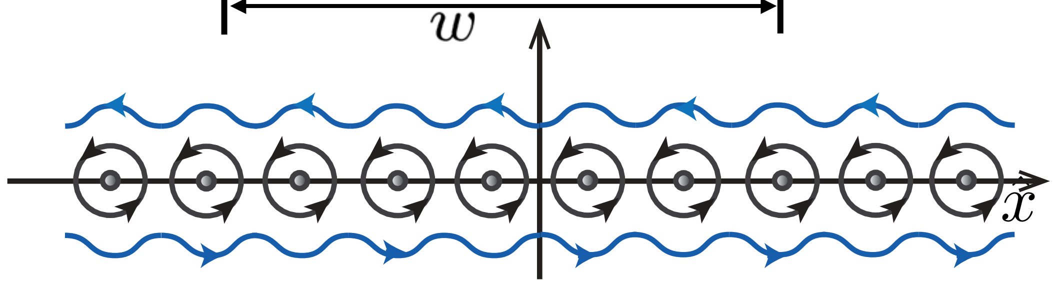


As seen "end on", the current sheet may be thought of as a combination of parallel wires, each of which produces its own H field. These H fields combine, so that the total H field above and below the current sheet is directed in $-\hat{x}$ and $\hat{x}$ direction, respectively.

$$2H_x w = Kw$$



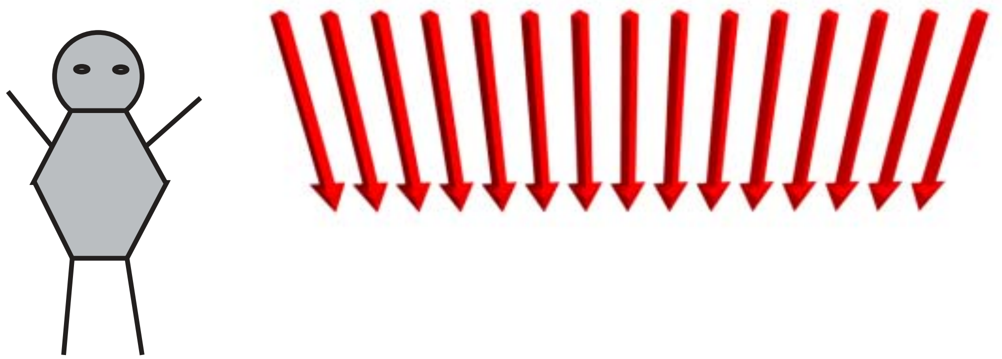
$$\vec{H} \cdot \vec{dl} \begin{cases} H_x\, dx & on\ AB \\ 0 & on\ BC \\ (-H_x)(-dx) & on\ CD \\ 0 & on\ DA \end{cases}$$



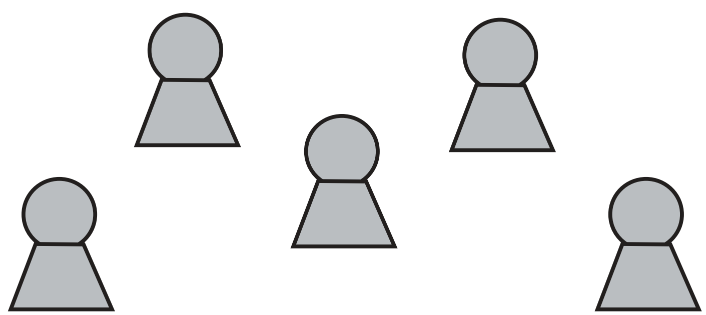
In between the wires the fields cancel

$$\vec{H} = \begin{cases} \dfrac{-\hat{x}K}{2} & y > 0 \\ \dfrac{\hat{x}K}{2} & y < 0 \end{cases}$$

Transient Magnetic Field from Current Sheet



- Current into slide turned on at $t = 0$



Turn on current (red) at $t = 0$

For now lets note that $\vec{H}$ can't propagate further than distance ct_o away from the current sheet in time t_o

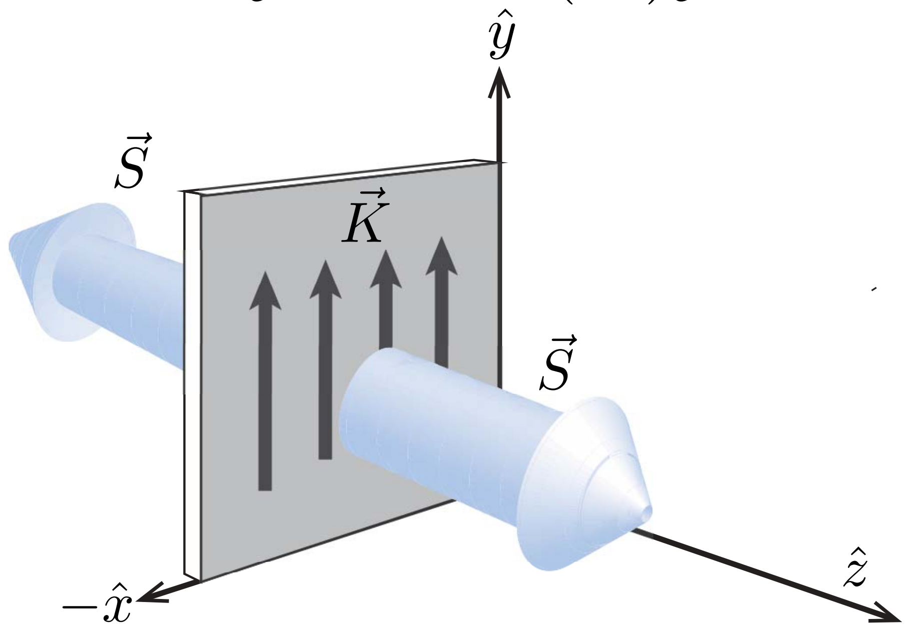
Don't worry about the shape of $\vec{H}$ yet.

$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{A} \cdot + \frac{d}{dt}\int_S \epsilon_o \vec{E}\, d\vec{A}$$

Which way does $\vec{E}$ point ?

Time-Varying Magnetic Field from Current Sheet

$$\vec{K} = K_s \hat{y} = K_o cos(\omega t) \hat{y}$$



Close to the sheet (quasi-static) ...

$$H_x(z_o = 0^+) = +\tfrac{1}{2} K_0 \cos(\omega t)$$

$$\frac{\partial E_y}{\partial z} = \frac{\partial B_x}{\partial t}$$

$$E_y(\Delta z) = -\left(\Delta z \omega\right) \tfrac{\mu_o}{2} K_0 \sin(\omega t)$$

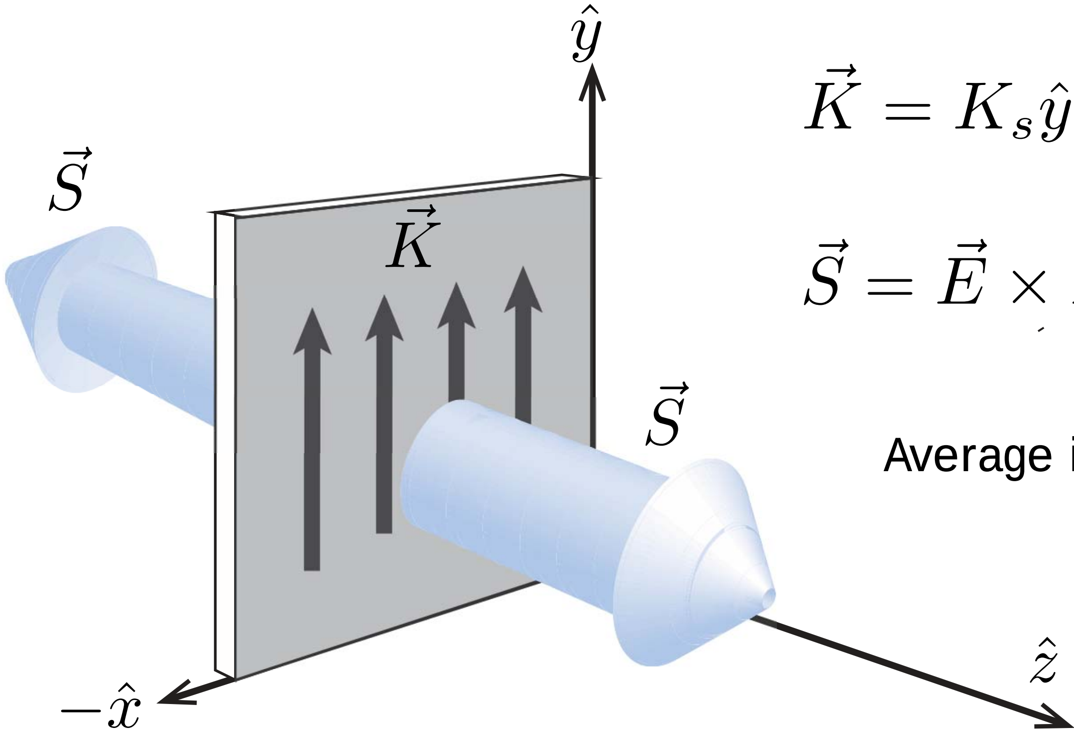
$$\frac{\partial B_x(z_o)}{\partial z} = \epsilon_o \mu_o \frac{\partial E_y}{\partial t}$$

$$H_x(\Delta z) - H_x(0) = -\left(\frac{\Delta z \omega}{c}\right)^2 \tfrac{1}{2} K_0 \cos(\omega t)$$

$$H_x(\Delta z) = +\tfrac{1}{2} K_0 \cos(\omega t) \left(1 - \left(\frac{\Delta z \omega}{c}\right)^2\right) = +\tfrac{1}{2} K_0 \cos(\omega t - kz)$$

$$H_x(z) = +\tfrac{1}{2} K_0 \cos(\omega t - kz) \quad \vec{E}_y = -\frac{\mu_0 c}{2} K_0 \cos(\omega t - kz)$$

Time-Varying Magnetic Field from Current Sheet



$$\vec{K} = K_s\hat{y} = K_o cos(\omega t)\hat{y}$$

$$\vec{S} = \vec{E} \times \vec{H} = \frac{\eta_0 K_0^2}{4} cos^2(\omega t - kz)\hat{z}$$

Average intensity...

$$I = \left\langle \vec{S} \right\rangle \bullet \hat{n} = \frac{\eta_0 K_0^2}{8}$$

Why do we care about plane waves?

Examples of the Wireless Energy Transfer

An electric toothbrush uses traditional magnetic induction to recharge its batteries, avoiding the need for exposed electrical contacts.

A microwave oven utilizes microwave radiation to cook food.

MRI machines use "magnetic resonance imaging" to produce diagnostic images of soft tissue.

Nikola Tesla's Wardenclyffe tower built on Long Island, NY in 1904. This tower was intended to implement Tesla's vision of transmitting power and information around the world. The tower was destroyed in 1917.

On June 5, 1975 NASA JPL Goldstone demonstrated directed radiative microwave power transmission successfully transferring 34kW of electrical power over a distance of 1.5km at 82% efficiency.

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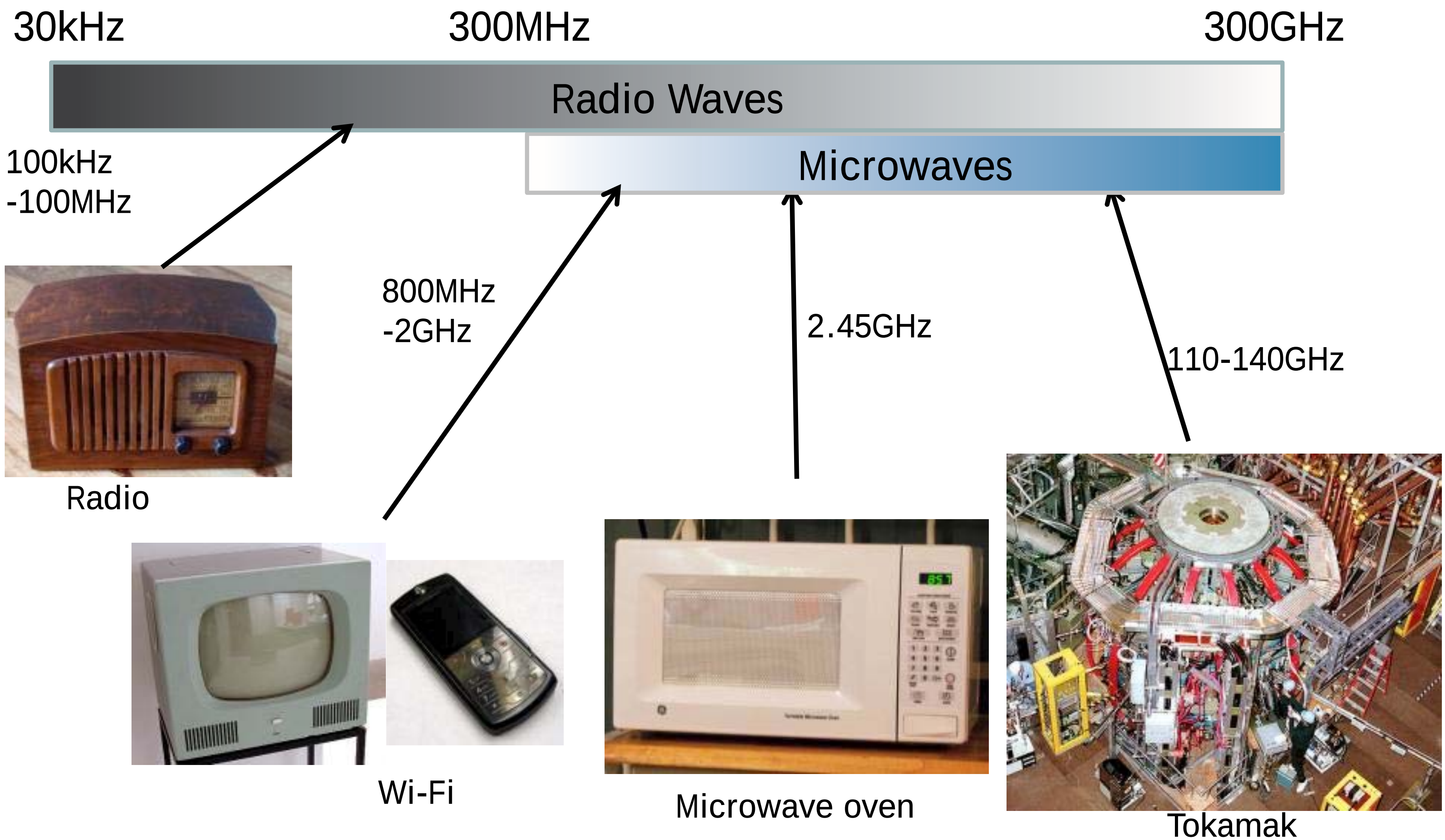
Wireless Energy Transfer



Tokamak image by Sheliak (Princeton Plasma Physics Laboratory) <http://commons.wikimedia.org/wiki/File:NSTX.jpg> on Wikimedia, radio, TV and cellphone images are in the public domain

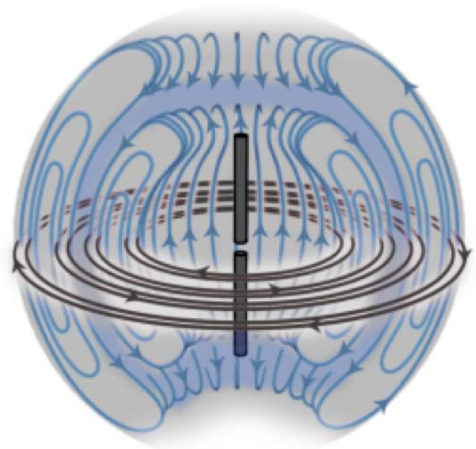
Cell Phones

30kHz 300MHz 300GHz

Radio Waves

Microwaves

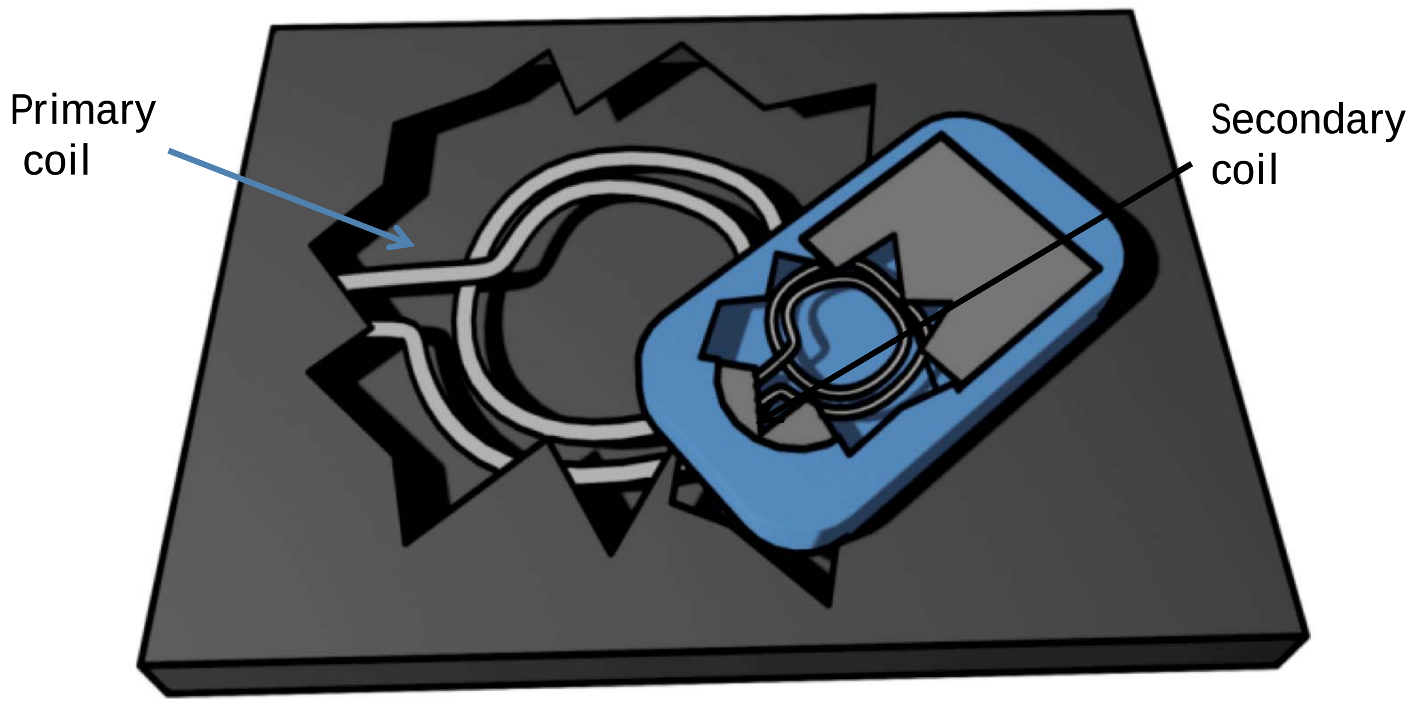
800MHz
-2GHz



Current / Planned Technologies	Band	Frequency (MHz)
SMR iDEN	800	806-824 and 851-869
AMPS, GSM, IS-95 (CDMA), IS-136 (D-AMPS), 3G	Cellular	824-849 and 869-894
GSM, IS-95 (CDMA), IS-136 (D-AMPS), 3G	PCS	1850–1915 and 1930–1995
3G, 4G, MediaFlo, DVB-H	700 MHz	698-806
Unknown	1.4 GHz	1392–1395 and 1432–1435
3G, 4G	AWS	1710–1755 and 2110–2155
4G	BRS/EBS	2496–2690

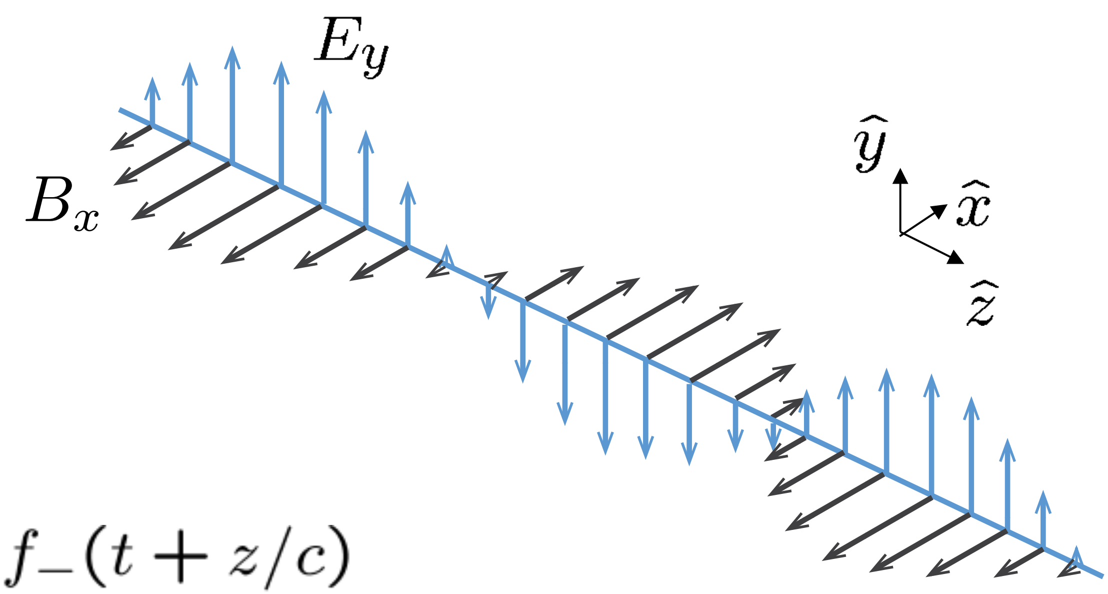
Table from http://en.wikipedia.org/wiki/Cellular_frequencies

Inductive Charging



Magnetic Field in a Uniform Electromagnetic Plane Wave

$$\frac{\partial^2 E_y}{\partial z^2} = \epsilon\mu \frac{\partial^2 E_y}{\partial t^2}$$



In free space ...

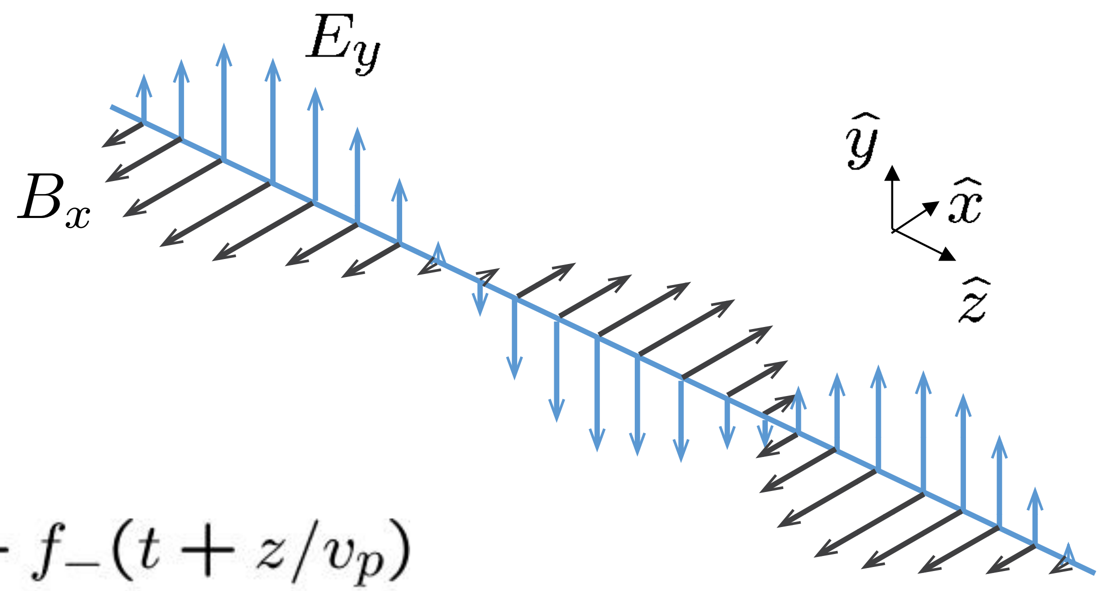
$$E_y = f_+(t - z/c) + f_-(t + z/c)$$

$$H_x = -\sqrt{\frac{\epsilon_o}{\mu_o}}\Big(f_+(t - z/c) - f_-(t + z/c)\Big)$$

... where $c = \dfrac{1}{\sqrt{\epsilon_o \mu_o}}$

Uniform Electromagnetic Plane Waves In Materials

$$\frac{\partial^2 E_y}{\partial z^2} = \epsilon\mu \frac{\partial^2 E_y}{\partial t^2}$$



Inside a material ...

$$E_y = f_+(t - z/v_p) + f_-(t + z/v_p)$$

$$H_x = -\sqrt{\frac{\epsilon}{\mu}}\left(f_+(t - z/v_p) - f_-(t + z/v_p)\right)$$

... where $\boxed{v_p = \frac{1}{\sqrt{\mu\epsilon}}}$ is known as the phase velocity

Index of Refraction

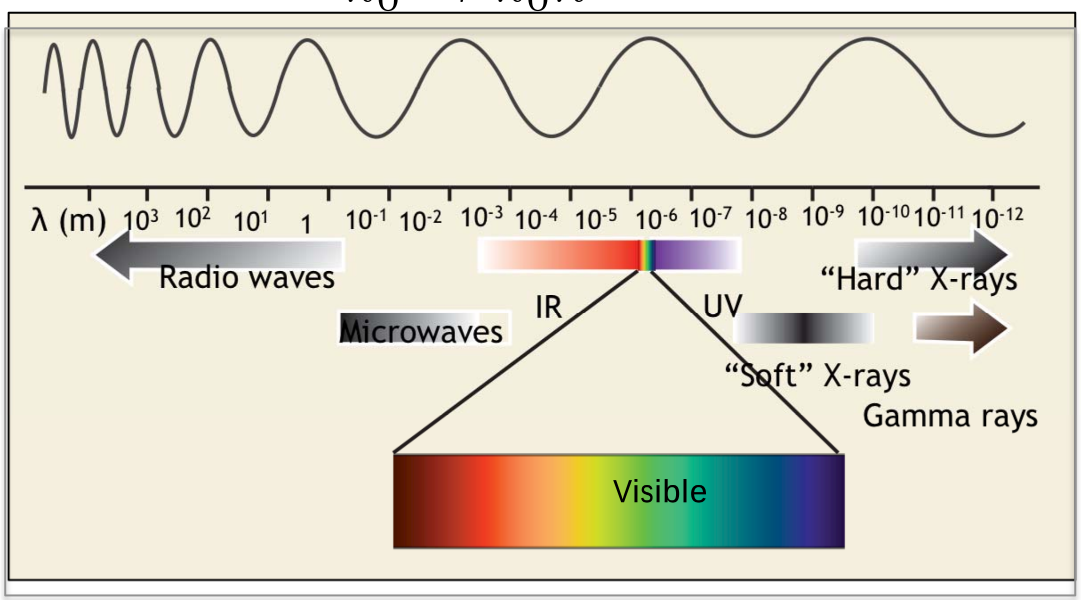
$$\nu\lambda = v_p = c/n$$

frequency wavelength

When propagating in a material,

$$c \rightarrow c/n$$
$$\lambda_0 \rightarrow \lambda_0/n$$
$$k_0 \rightarrow k_0 n$$

Material	n
Vacuum	1
Air	1.000277
Water liquid	1.3330
Water ice	1.31
Diamond	2.419
Silicon	3.96
at 5 x 10^{14} Hz	

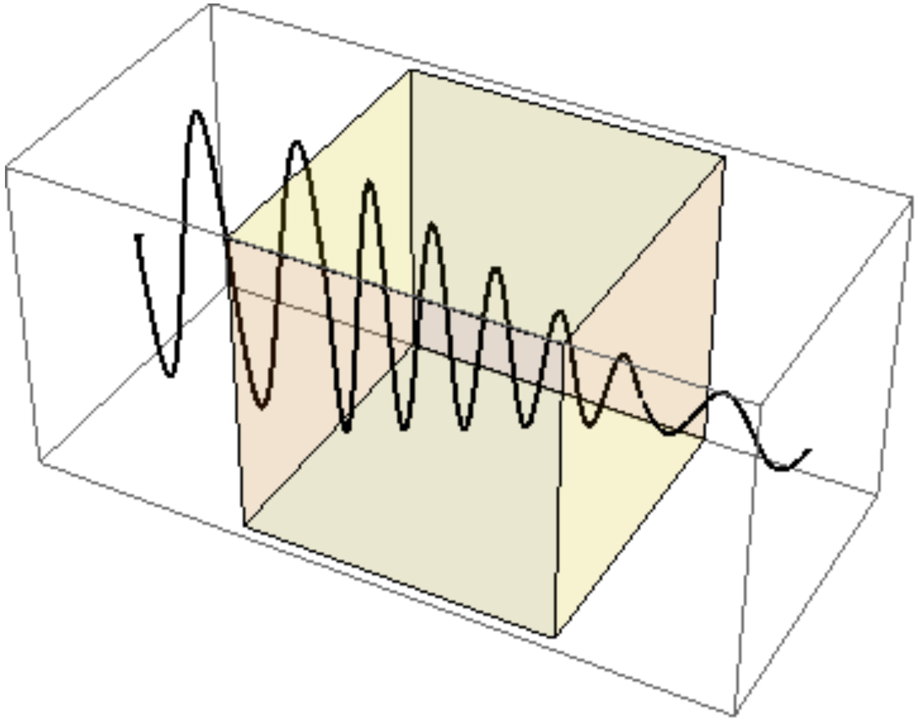


$$E(t,z) = Re\{\tilde{E}_0 e^{j(\omega t - k_0 n z)}\}$$

$$E(t,z) = Re\{\tilde{E}_0 e^{j(\omega t - k\ z)}\}$$

Absorption

Photograph by Hey Paul on Flickr.



Why are these stained glasses different colors?

$$E(t,z) = Re\{\tilde{E}_0 e^{-\alpha z/2} e^{j(\omega t - k_0 n z)}\}$$

Absorption coefficient

Refractive index

Tomorrow: lump refractive index and absorption into a complex refractive index $\tilde{n}$

Key Takeaways

Propagation velocity $v_p = \dfrac{1}{\sqrt{\mu\epsilon}}$ $\quad c = \dfrac{1}{\sqrt{\epsilon_o \mu_o}}$

Direction of propagation given by $\vec{E} \times \vec{H}$

Energy stored in electric field per unit volume at any instant at any point is equal to energy stored in magnetic field

Instantaneous value of the Poynting vector given by $E^2/\eta = \eta H^2 = E \cdot H$

When propagating in a material,

$$c \to c/n$$
$$\lambda_0 \to \lambda_0/n$$
$$k_0 \to k_0 n$$

$$E(t,z) = Re\{\tilde{E}_0 e^{-\alpha z/2} e^{j(\omega t - k_0 n z)}\}$$

Absorption coefficient

Refractive index

Antennas use time-varying current to send EM waves. Antennas receive EM waves because oscillating fields produce time-varying current.