

Given the CMOS circuit with pull-down shown here, we can generate the corresponding function that this CMOS circuit represents which is $\text{NOT}(F) = (A+B)C + D$.

So $F = ((A+B)C + D)$ the whole thing negated. This function can be expressed as a truth table which describes the output that the function produces for any set of inputs.

All of the possible input combinations are enumerated, and then the value of F is specified for each of those combinations. For this function, F, for example, if $A=0$ $B=0$ $C=0$ and $D=0$, then $(A+B)C = 0$ that ORed with $D = 0$, and finally the entire thing is negated, so $F = 1$. For $A=0$ $B=0$ $C=0$ and $D=1$, $(A+B)C = 0 + D=1$ gives us 1 and the whole thing negated is $F=0$.

In the same way, we can complete the rest of the truth table, and we get.

1 0 1 0 0 0 1 0 0 0 1 0 0 0 Given a truth table, one can be asked if the function F that this truth table defines, can be implemented using a single CMOS gate.

To figure that out, we first convert the truth table to a logical equation for F.

Given this truth table, the function F can be expressed as a sum of products by listing all terms that make $F=1$ and ORing them together. For this truth table $F = \text{NOT}(A) \text{NOT}(B) \text{NOT}(C) + \text{NOT}(A) B \text{NOT}(C) + A \text{NOT}(B) \text{NOT}(C)$. In order to determine if this function can be implemented as a single CMOS circuit, the function F needs to be manipulated through logic simplifications in order to see if $\text{NOT}(F)$ can be written as a function of just the inputs A, B, and C (without any negations of those inputs) together with ANDs and ORs.

We start with our function for F. Factoring out $\text{NOT}(C)$, that equals $\text{NOT}(C) (\text{NOT}(A) \text{NOT}(B) + \text{NOT}(A) B + A \text{NOT}(B))$. Rearranging the terms and repeating the first one continues to produce the same function. So $F = \text{NOT}(C) (\text{NOT}(A) \text{NOT}(B) + A \text{NOT}(B) + \text{NOT}(A) B + \text{NOT}(A) \text{NOT}(B))$. Noting that $A X + \text{NOT}(A) X = X$, this reduces to: $F = \text{NOT}(C) (\text{NOT}(B) + \text{NOT}(A))$. We are almost there, we now want to convert F to $\text{NOT}(F)$ by negating the entire thing, so $\text{NOT}(F) = \text{NOT} (\text{NOT}(C) (\text{NOT}(B) + \text{NOT}(A)))$.

To simplify this we make use of De Morgan's Law which states that $\text{NOT}(AB) = \text{NOT}(A) + \text{NOT}(B)$.

And that $\text{NOT}(A+B) = \text{NOT}(A) \text{AND} \text{NOT}(B)$. So $\text{NOT}(F) = C + \text{NOT}(\text{NOT}(B) + \text{NOT}(A))$, and finally $\text{NOT}(F) = C + (B A)$. Now that we have found that $\text{NOT}(F) = C + BA$, we know that F can be implemented as a single CMOS gate because we were able to express $\text{NOT}(F)$ as a function that consists purely of our non-negated inputs, A, B, and C, together with ANDs and ORs. The pull-down for this circuit would basically implement $\text{NOT}(F)$ so it would be series(A,B) in parallel with C.

The pull-up would be the inverse of that which is parallel(A,B) in series with C.

