6.004 Computation Structures

Spring 2009

M A S S A C H U S E T T S I N S T I T U T E O F T E C H N O L O G Y
DEPARTMENT OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE

6.004 Computation Structures
Spring 2009

Quiz #1: February 20, 2009

Name	*Athena login name*	*Score*
Solutions		Avg: 21.4

Problem 1 (6 points): **Quickies and Trickies**

(A) You are trying to guess a card picked at random from a standard 52-card deck. Sam tells you the card is a spade; Nora tells you it's not an ace; Rita tells you it's a seven. What is the ***total*** amount of information about the card given by Sam, Nora, and Rita? You may give your answer as either a number or a formula.

Bits of info (number or formula): $\log_2(52)$

(B) Give the four-bit two's complement representation of -6.

4-bit 2's complement representation of -6: 1010

(C) Let *w* be an eight-bit code word. How many 8-bit code words are there whose Hamming distance from *w* is exactly one?

Number of 8-bit code words distance 1 from *w*: 8

(D) In Lecture 4, you saw that a 2^k -input XOR could be implemented as a k-level tree of 2-input gates. Can every 2^k-input, 1-output combinational function be implemented as a k-level tree of (not necessarily identical) 2-input 1-output gates?

Circle one: **YES** **NO**

(E) The propagation delay specified for an inverter is less than its contamination delay

Circle one: **NEVER** **SOMETIMES** **ALWAYS**

(F) A single CMOS gate, consisting of an output node connected to a single pullup circuit containing zero or more PFETs and a single pulldown circuit containing zero or more NFETs (as described in lecture), computes **F(A, B, C, D)**.

It is observed that **F(1,0,1,1) = 1**. What can you say about the value of **F(1,0,0,0)?**

(circle one): F(1,0,0,0) = 1 … 0 … can't tell

Problem 2. (5 points): **Variable-length Codes**

GradeStore, Inc makes storage devices designed to hold only letter grades issued in college courses. Each grade is one of A, B, C, D, F, or X (for dropped courses), and the grades are distributed according to the following table:

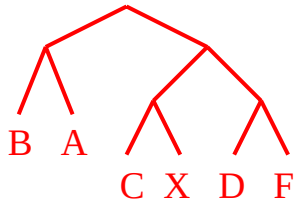
Grade	*Probability of occurrence*
A	26%
B	34%
C	11%
D	10%
F	8%
X	11%

(A) (1 point) How many bits would be required to store each grade using a fixed-length code?

Number of bits: 3

(B) (3 points) Devise a Huffman code to optimize the storage of grades.

The correct Huffman tree is



The specific answers at right are just one of the many possible encodings from this tree.

Encoding for A: 10

Encoding for B: 11

Encoding for C: 010

Encoding for D: 000

Encoding for F: 001

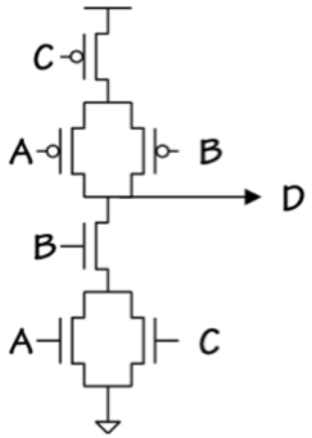
Encoding for X: 011

A well-heeled customer wants GradeStore to customize their encoding to include a seventh grade, M, to identify students whose work was incomplete due to their being hit by a meteor. This is a very rare event, and GradeStore expects its probability to be on the order of 0.00000000007%.

(C) (1 point) How many bits would Huffman encoding allocate to the representation of an M grade?

Bits for encoding of M grade: 4

Problem 3. (7 points): **Saga of Helena Handbasket**



Helena Handbasket, who barely passed 6.004, has been hired to design CMOS gates for Hapless Logic, Inc. Remembering something about PFETs in pullups and NFETs in pulldowns, her first design was a 3-input device whose circuit is shown to the right.

Helena's intent was that these devices compute some useful 3-input Boolean function D=F(A,B,C); unfortunately, the devices don't seem to work as planned. To make matters worse, she had 1,000,000,000 of the devices fabricated (thinking that the order to the fab must, of course, be in binary rather than decimal). The defective devices, known within HLI as the Gates of Helena, have become a subject of ridicule.

Helena has brought you in as a consultant. Your first task is to figure out how badly Helena blew the design of her 3-input logic device – in particular, whether it drives the output D to a valid logic level for every combination of the inputs A, B, and C.

(A) (2 points) Are there logical (0/1) values of A, B, and C for which D is not driven at all? If so, give values for A, B, and C that leave D undriven; else write NONE.

A, B, C values for undriven D, or "NONE": B = 0 and C = 1 (A can be 0 or 1)

(B) (2 points) Are there logical values of A, B, and C for which D is pulled down and up simultaneously? If so, values for A, B, and C that cause such a conflict; else write NONE.

A, B, C values for output conflict, or "NONE": NONE

Nora Nanda, Helena's assistant, suggests that the devices might be salvaged by using them to compute useful functions of fewer than 3 inputs. She proposes that a two-input function of X and Y, for example, might be computed by connecting each of the three inputs A, B, and C to either X, Y, or the logical constants 0 (ground) or 1 (vdd), and reading the output on D.

(C) (1 point) Can Nora's approach be used to compute **NAND** of X and Y? If so, choose values (X, Y, 0, or 1) for each of A, B, and C such that D is $\overline{XY}$; else circle NO.

Choose A, B, C values or circle NO: A=X**; B=**Y**; C=**0**; or NO**

(D) (1 point) Can Nora's approach be used to compute **NOR** of X and Y? If so, choose values (X, Y, 0, or 1) for each of A, B, and C such that D is $\overline{X+Y}$; else circle NO.

Choose A, B, C values or circle NO: A=X**; B=**1**; C=**Y**; or NO**

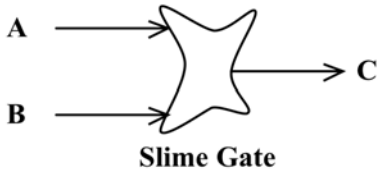
(E) (1 point) Can Nora's approach be used to compute $X+\overline{Y}$? If so, choose values (X, Y, 0, or 1) for each of A, B, and C such that D is $X+\overline{Y}$; else circle NO.

Choose A, B, C values or circle NO: A=____; B=____; C=____; or NO

Problem 4 (7 points): **Organic Logic**

Organic Logic, Inc, is a Cambridge startup that has developed an interesting device built using unidentified organic sludge from the depths of the Charles River; they would like to use it to perform logic functions. Their device, termed a Slime Gate, has two inputs **A** and **B**, and one output **C** (in addition to power and ground connections):

With a 3-volt power supply, they have noted that Slime Gates reliably behave as follows:

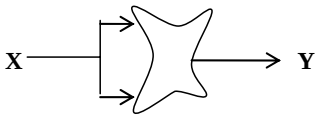


Slime Gate

- The output C is always in the range 0 volts < C < 3 volts.
- When both A and B have been at less than 1 volt for a nanosecond or more, the voltage at C is greater than 2.5 volts.
- When either or both of A and B have been at more than 2 volts for at least a nanosecond, C carries a voltage of less than 0.5 volts.

Aside from the above constraints, the voltage at C is generally unpredictable; it varies widely between individual Slime Gate devices.

As an O.L.I. consultant, you have proposed the circuit to the right as an inverter in the evolving family of Slime Gate logic:



Inverter

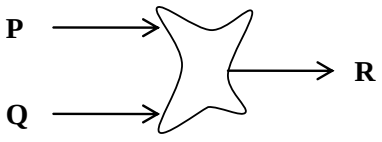
(A) (3 points) Give logic representation parameters yielding maximum noise margins and for which your diagram depicts a valid inverter.

V_{OL}: 0.5 V; V_{IL}: 1 V; V_{IH}: 2 V; V_{OH}: 2.5 V

(B) (2 points) Give appropriate specifications for propagation and contamination delays for this inverter:

t_{pd}: 1 ns; t_{cd}: 0 ns

(C) (1 point) Suppose the Slime Gate is used as a 2-input logic gate in this same family, as shown to the right. What, if any, function of P and Q is represented by the output R?



2-input gate

Boolean function of P and Q, or "None": $\overline{P + Q}$ (NOR)

(D) (1 point) Is the Slime Gate, based on the description above, a *lenient* device?

(Circle one:) **YES** / **NO**

END OF QUIZ!