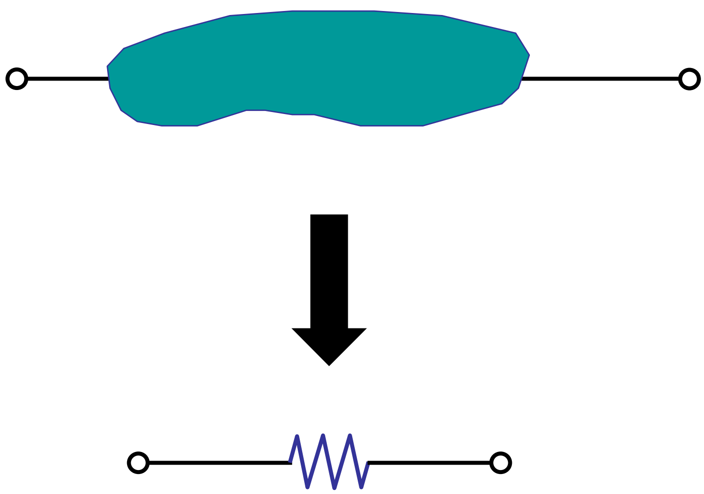
The Digital Abstraction

Review

- Discretize matter by agreeing to observe the lumped matter discipline



Lumped Circuit Abstraction

- Analysis tool kit: KVL/KCL, node method, superposition, Thévenin, Norton (remember superposition, Thévenin, Norton apply only for linear circuits)

Today

Discretize value ⟹ Digital abstraction

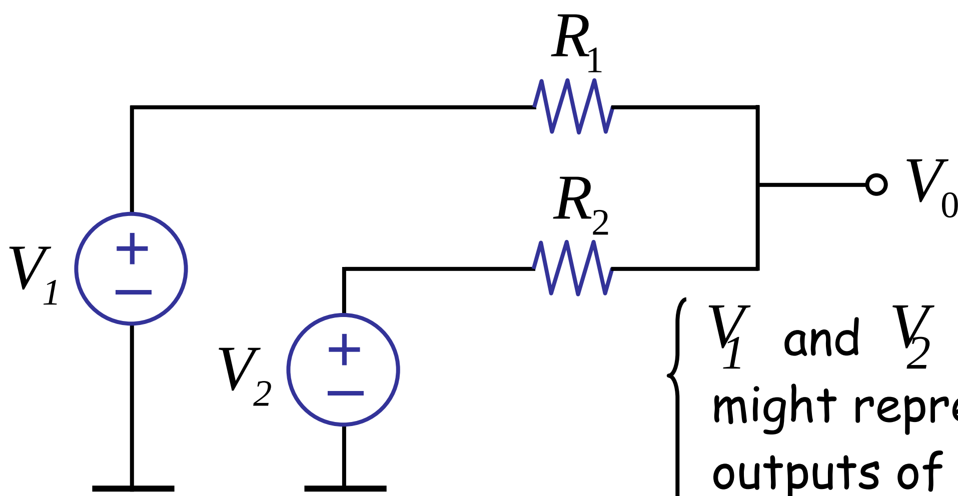
Interestingly, we will see shortly that the tools learned in the previous three lectures are sufficient to analyze simple digital circuits

Reading: Chapter 5 of Agarwal & Lang

But first, why digital?

In the past ...

Analog signal processing



V_1 and V_2 might represent the outputs of two sensors, for example.

By superposition,

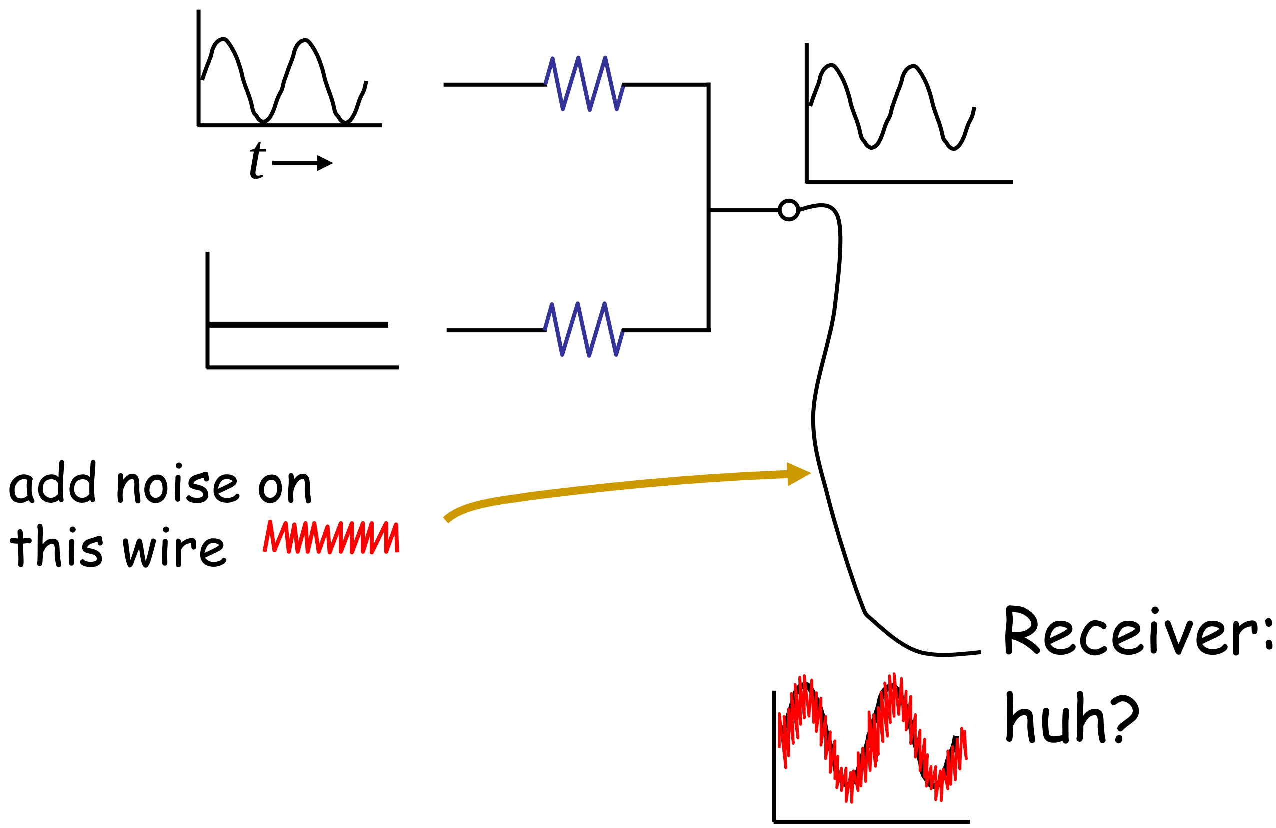
$$V_0 = \frac{R_2}{R_1 + R_2} V_1 + \frac{R_1}{R_1 + R_2} V_2$$

If $R_1 = R_2$,

$$V_0 = \frac{V_1 + V_2}{2}$$

The above is an "adder" circuit.

Noise Problem



... noise hampers our ability to distinguish between small differences in value — e.g. between 3.1V and 3.2V.

Value Discretization

Restrict values to be one of two

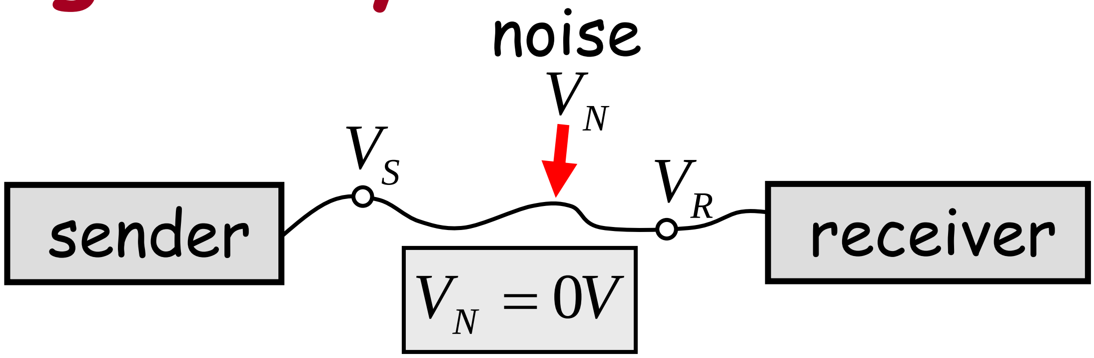
HIGH	LOW
5V	0V
TRUE	FALSE
1	0

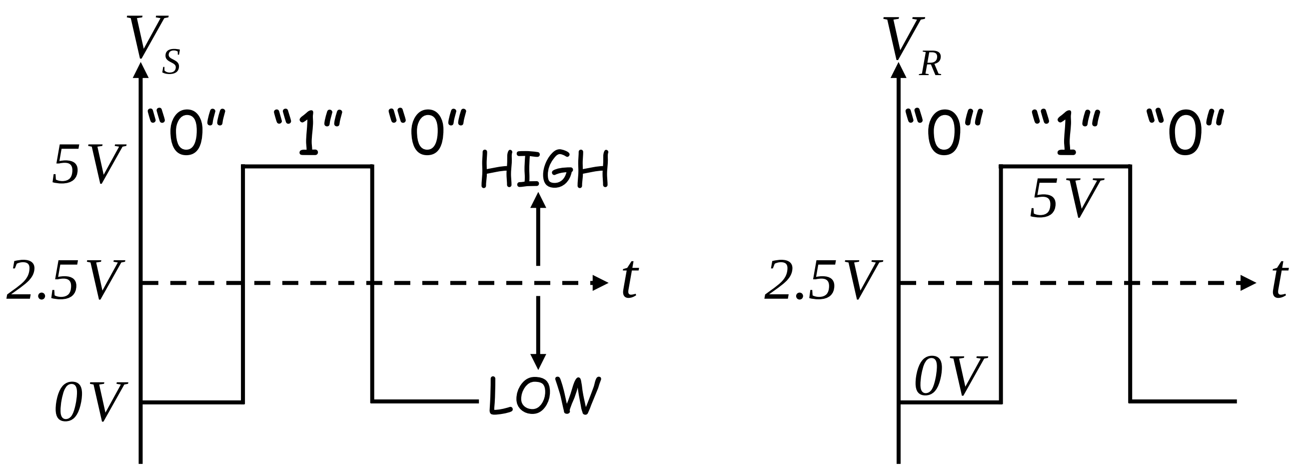
...like two digits 0 and 1

Why is this discretization useful?

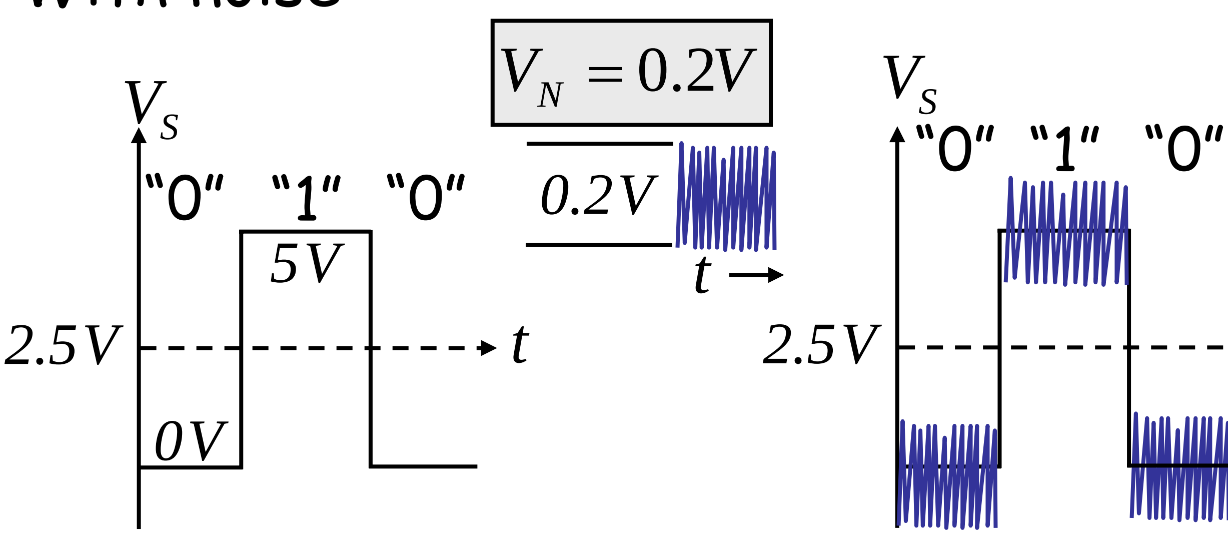
(Remember, numbers larger than 1 can be represented using multiple binary digits and coding, much like using multiple decimal digits to represent numbers greater than 9. E.g., the binary number 101 has decimal value 5.)

Digital System





With noise

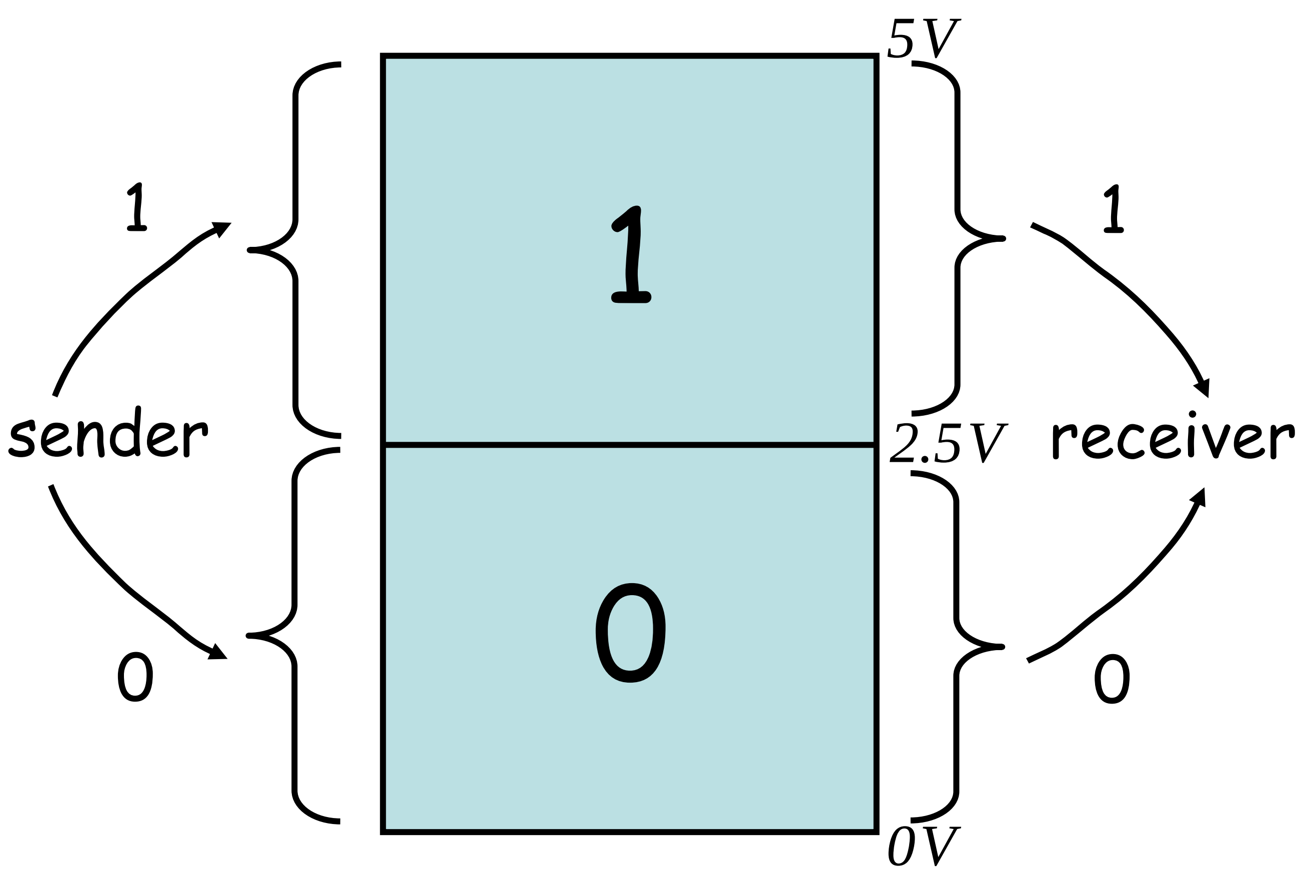


Digital System

Better noise immunity
Lots of "noise margin"

For "1": noise margin $5V$ *to* $2.5V = 2.5V$
For "0": noise margin $0V$ *to* $2.5V = 2.5V$

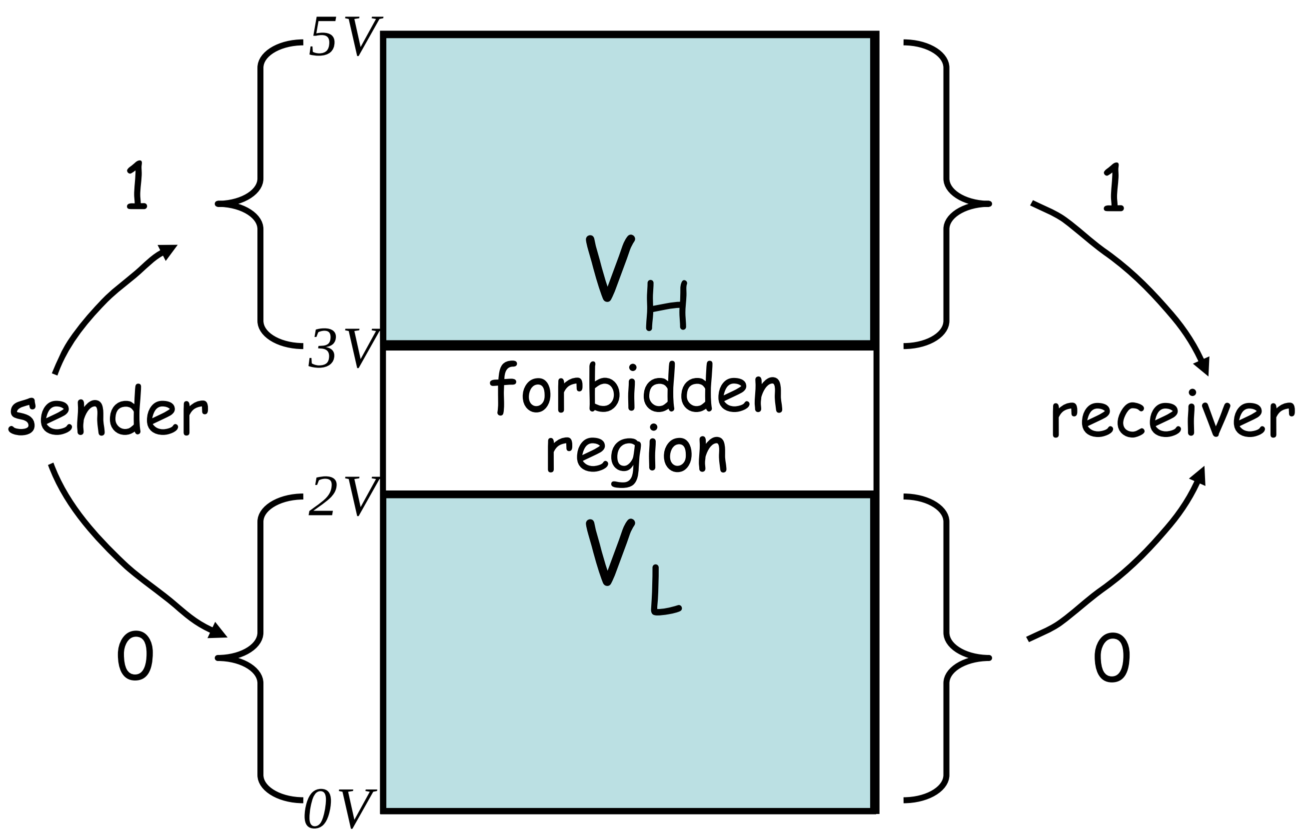
Voltage Thresholds and Logic Values



But, but, but …
What about 2.5V?

Hmmm… create "no man's land" or forbidden region

For example,



"1" ➡ $V_H \rightarrow 5V$

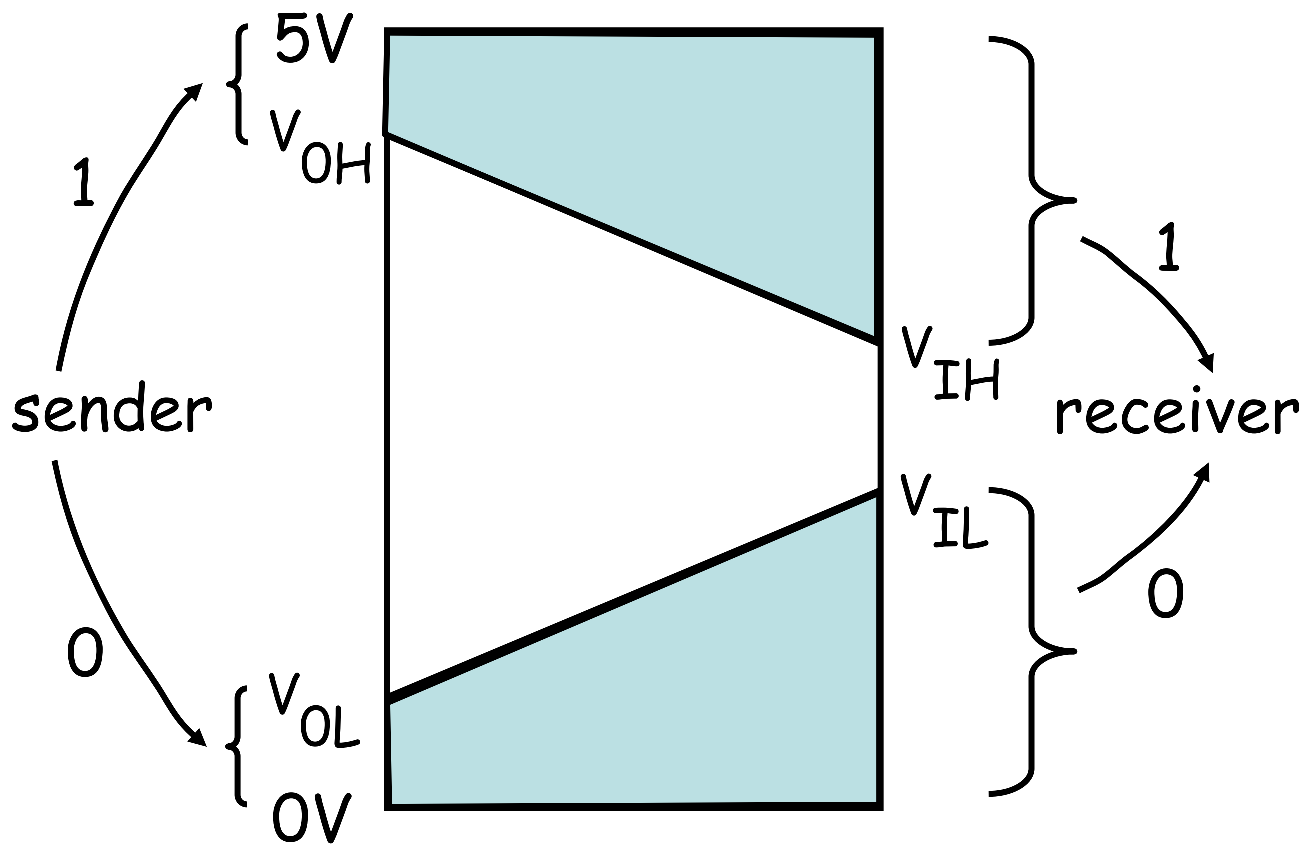
"0" ➡ $0V \rightarrow V_L$

But, but, but …

Where's the noise margin?

What if the sender sent 1: V_H ?

Hold the sender to tougher standards!

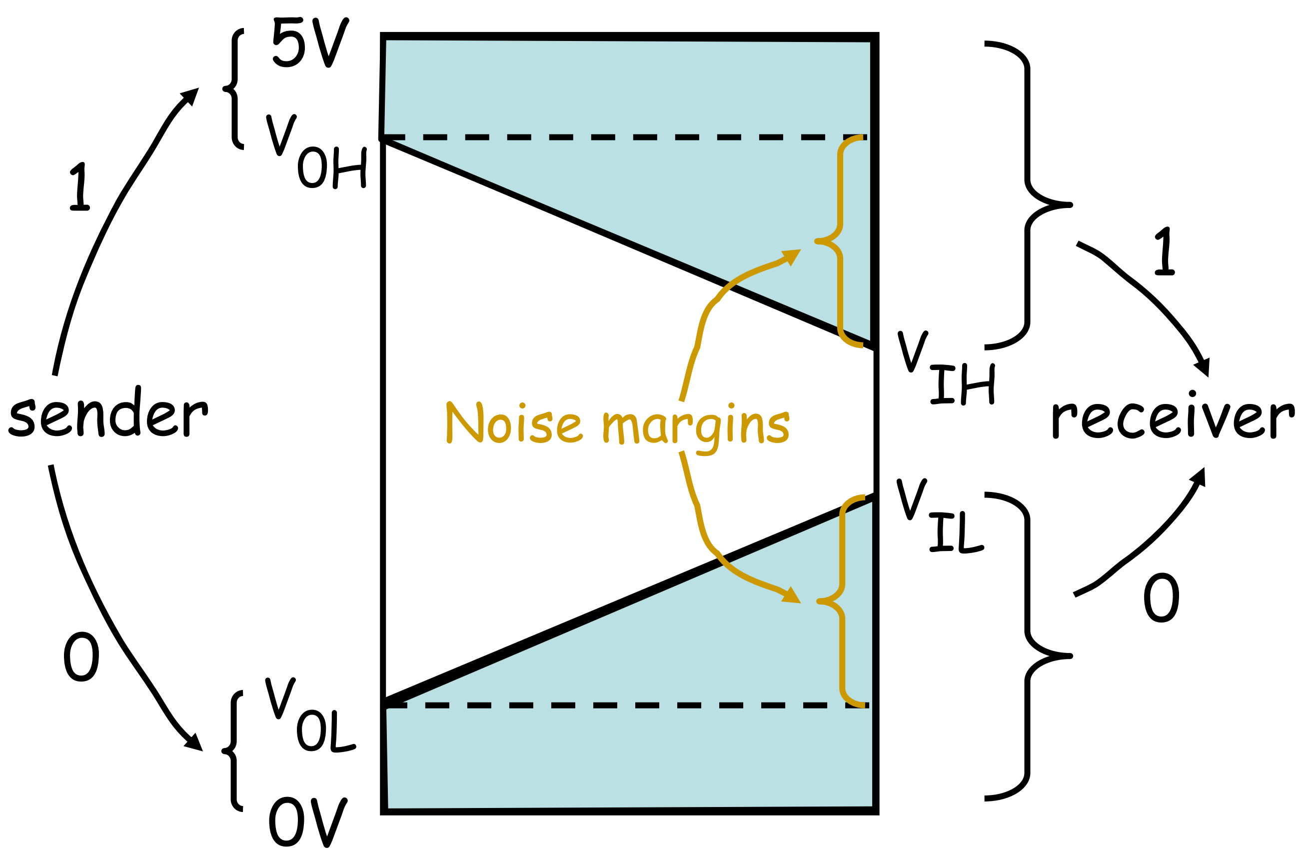


But, but, but …

Where's the noise margin?

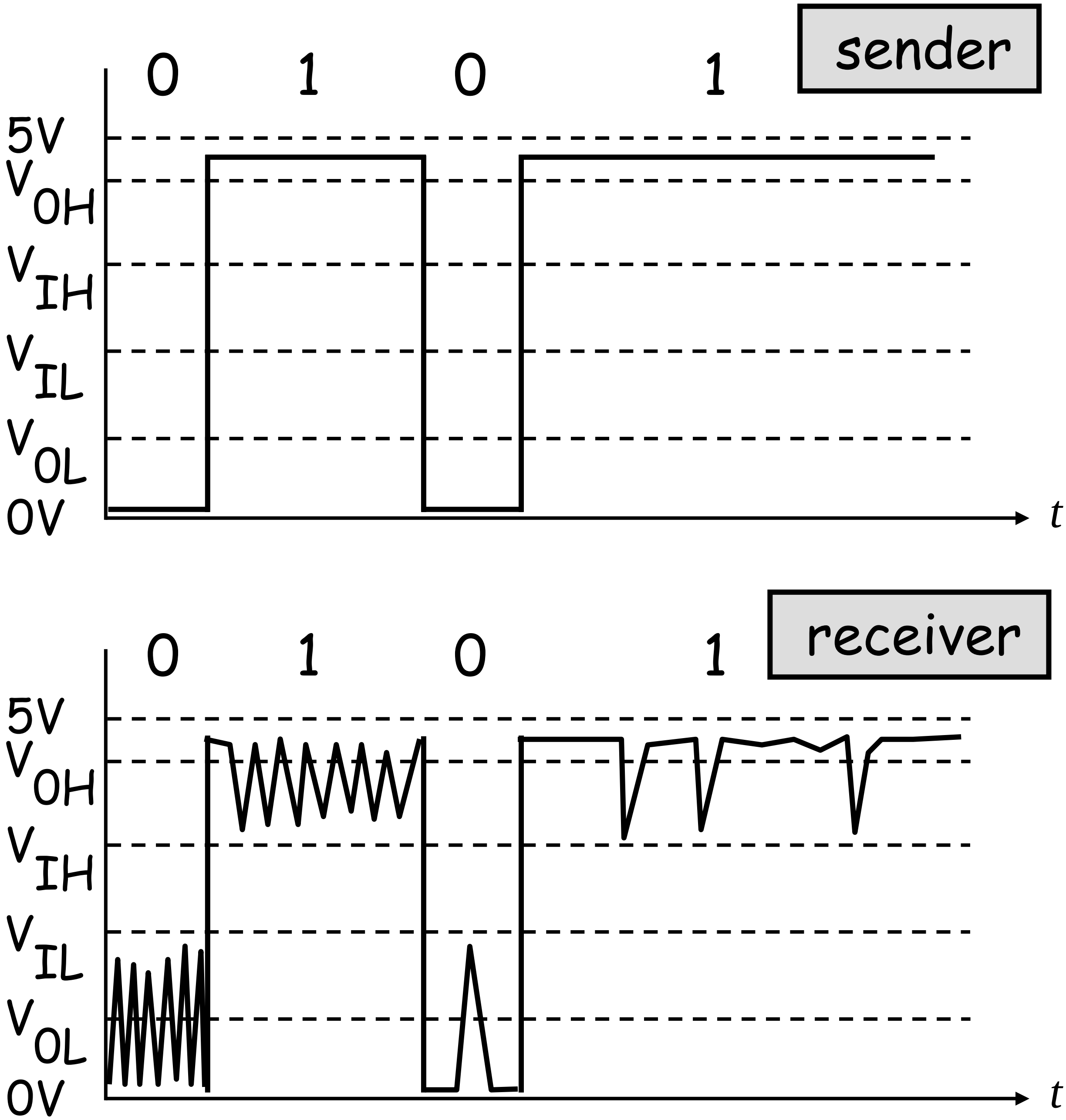
What if the sender sent 1: V_H ?

Hold the sender to tougher standards!



"1" noise margin: $V_{IH} - V_{OH}$

"0" noise margin: $V_{IL} - V_{OL}$



Digital systems follow static discipline: if inputs to the digital system meet valid input thresholds, then the system guarantees its outputs will meet valid output thresholds.

Processing digital signals

Recall, we have only two values —

1,0 ⇨ Map naturally to logic: T, F

⇨ Can also represent numbers

Processing digital signals

Boolean Logic

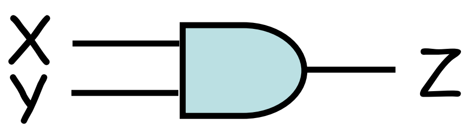
⇨ If **X** is true and **Y** is true
Then **Z** is true else **Z** is false.

⇨ Z = X AND Y

Z = X • Y

Boolean equation

X, Y, Z
are digital signals
"0" , "1"

⇨ 

AND gate

⇨ Truth table representation:

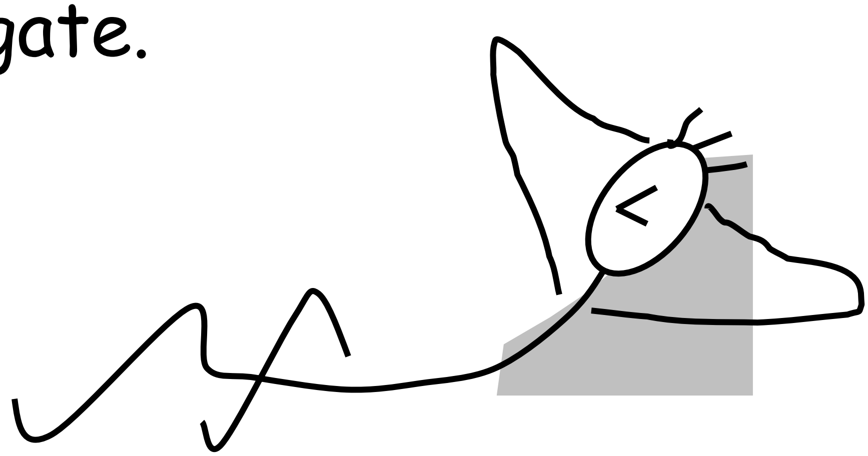
X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

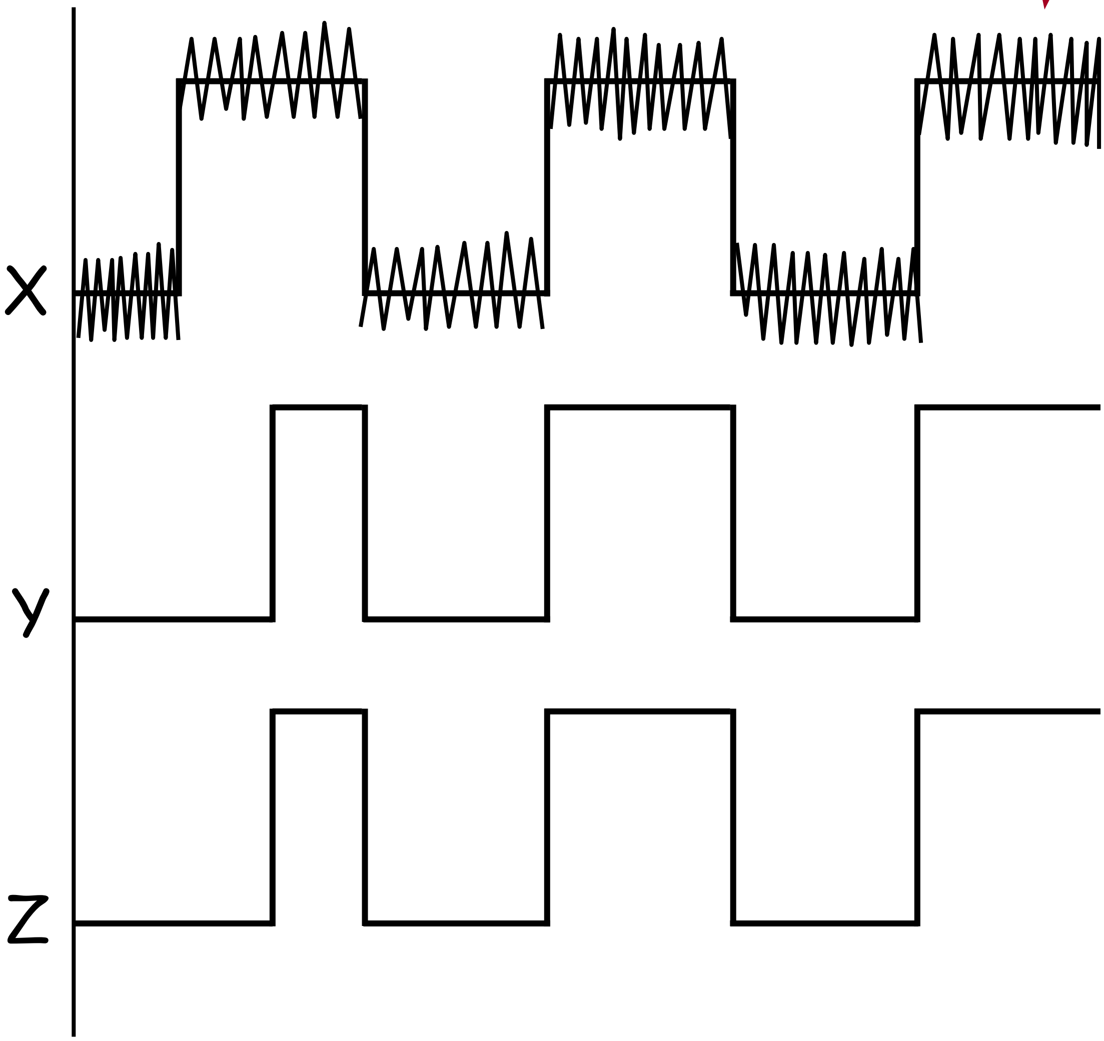
Enumerate all input combinations

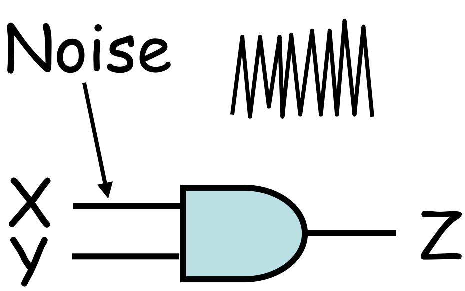
Combinational gate abstraction

- Adheres to static discipline
- Outputs are a function of inputs alone.

Digital logic designers do not have to care about what is inside a gate.

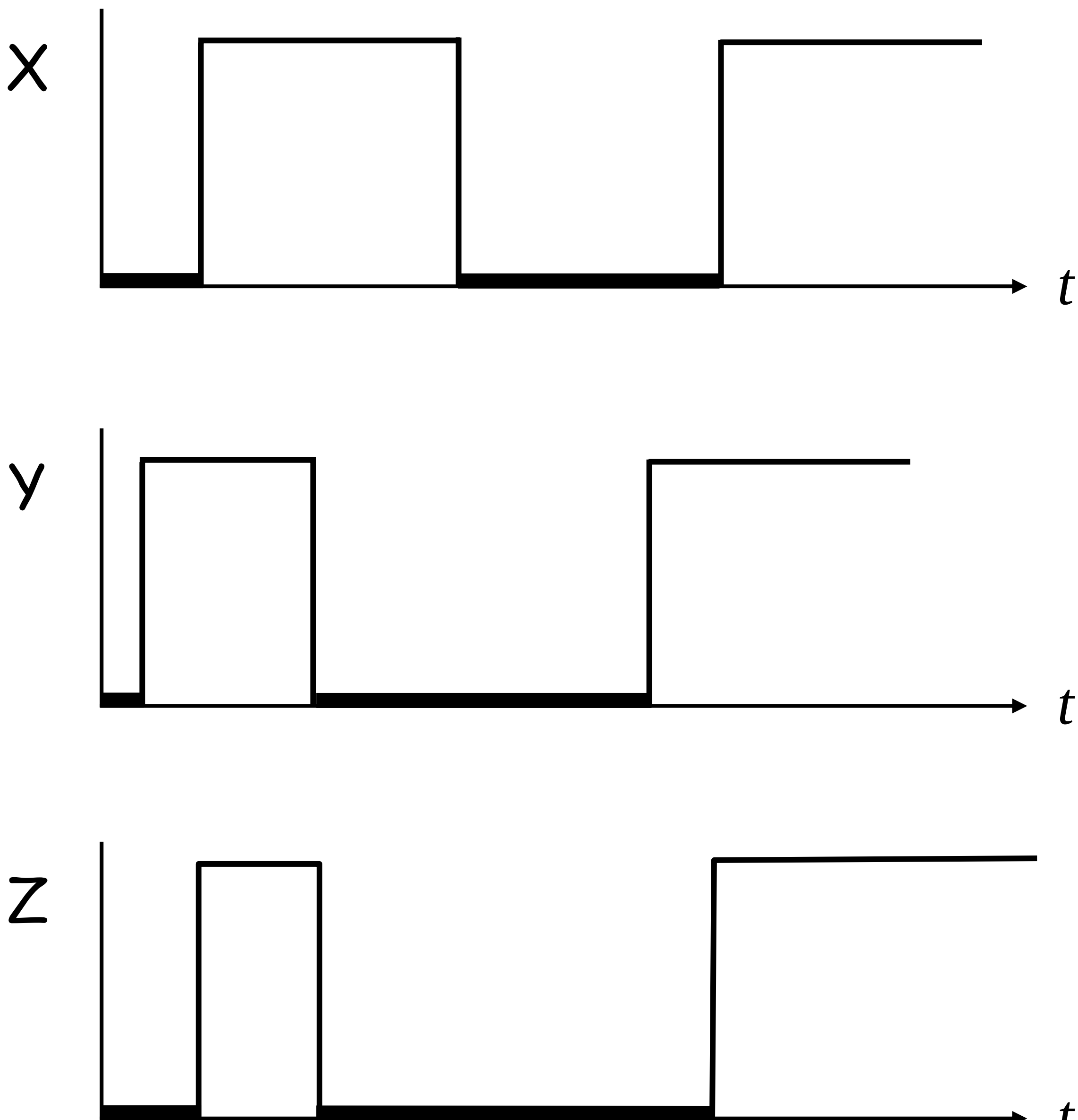






$$Z = X \cdot Y$$

Examples for recitation



$$Z = X \cdot Y$$

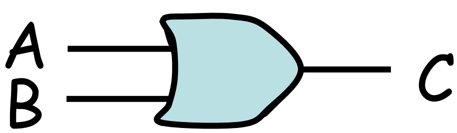
In recitation...

Another example of a gate

If (**A** is true) OR (**B** is true)
then **C** is true
else **C** is false

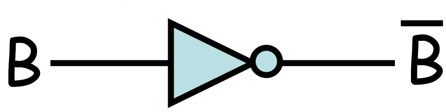
⇨ $C = A + B$ Boolean equation

(+ = OR)

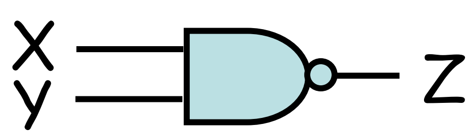
⇨ 

OR gate

More gates



Inverter



NAND

$$Z = \overline{X \cdot Y}$$

Boolean Identities

$$X \cdot 1 = X$$
$$X \cdot 0 = X$$
$$X + 1 = 1$$
$$X + 0 = X$$
$$\overline{1} = 0$$
$$\overline{0} = 1$$
$$AB + AC = A \cdot (B + C)$$

Digital Circuits

Implement: $\text{output} = A + \overline{B \cdot C}$