14.771 Development Economics: Microeconomic Issues and Policy Models
Fall 2008

# 14:771: Recitation Handout #6

Consumption Insurance and Separability

# Models of Consumption Insurance

Implicit in the Duflo and Udry (2003) paper is the idea that members in a household who engage in economic production should insure one another's consumption. The purpose of this section is to go over a few different models of consumption insurance within a household so that you can better understand these concepts in this paper.

## Consumption Insurance with Perfect Commitment

First, let's assume that at time 0, a risk averse husband and a wife can write a contract mapping income realizations into consumption allocations and savings decisions, and that they have no problem commiting to this contract at future dates. Furthermore, let's assume that income and savings decisions are perfectly observable to both husband and wife. There is surplus from mutual insurance, and there are many different ways that this surplus can be allocated - what we would like to do is trace out the set of efficient contracts. In order to trace out this frontier, let's maximize the husband's utility subject to promising the wife ex ante expected utility $U_W$. As we vary $U_W$, we trace out the Pareto frontier.

In order to make the problem more tractable, let's assume that there are a finite number of income realizations $y_i^s$, $i = f, m$ where $s \in \{1, ...S\}$ are the states of nature. Assume the state of nature follows a Markov process with transition probability from state $s$ to $r$ given by $\pi_{sr}$. There is some initial distriubtion over period 0 states given by $\pi_s^0$. To keep things simple, let's assume that households cannot borrow or save - they must consume all their income in the period it is realized (this is in the spirit of Townsend (1994)). Denote a history $s^t = \{s_1, s_2, ..., s_t\}$ as the collection of states realized up to time $t$. The whole set of possible histories at time $t$ (all possible $s^t$s) is denoted by $S^t$. The corresponding probability of each history is given by $\pi(s^t) = \pi_{s_1}^0 \cdot \pi_{s_1 s_2} \cdot \pi_{s_2 s_3} \cdot ... \cdot \pi_{s_{t-1} s_t}$. Then for any allocation $c^m(s^t), c^f(s^t)$ we can write expected utility of agent $i$ as

$$U^i = \sum_{t=0}^{\infty} \beta^t \sum_{s^t \in S^t} \pi(s^t)\, u^i\left(c^i(s^t)\right)$$

So now let's turn to the maximization problem. We write

$$\max_{c^m(s^t),\, c^f(s^t)} \sum_{t=0}^{\infty} \beta^t \sum_{s^t \in S^t} \pi(s^t)\, u^m\left(c^m(s^t)\right) \text{ s.t.}$$

$$c^m(s^t) + c^f(s^t) \leq y^m(s^t) + y^f(s^t) \ \ \forall s^t$$

$$U^f = \sum_{t=0}^{\infty} \beta^t \sum_{s^t \in S^t} \pi(s^t)\, u^f\left(c^f(s^t)\right) \geq \bar{U}^f$$

We can solve the problem using either a value function or Lagrangian approach. I'll do it using a Lagrangian. Denote the multiplier on the budget constraint $\lambda(s^t)$ and the multiplier on utility promise constraint $\gamma$. The first order conditions for this problem are given by:

$$\begin{aligned}\left[c^m\left(s^t\right)\right] &: \beta^t\pi\left(s^t\right)u^{m\prime}\left(c^m\left(s^t\right)\right)=\lambda\left(s^t\right)\\ \left[c^f\left(s^t\right)\right] &: \gamma\beta^t\pi\left(s^t\right)u^{f\prime}\left(c^f\left(s^t\right)\right)=\lambda\left(s^t\right)\end{aligned}$$

Using the first order conditions on consumption for the husband and wife, we see that

$$\frac{u^{m\prime}\left(c^m\left(s^t\right)\right)}{u^{f\prime}\left(c^f\left(s^t\right)\right)}=\gamma$$

Clearly, the budget constraint must bind, so we can substitute in to see:

$$\frac{u^{m\prime}\left(c^m\left(s^t\right)\right)}{u^{f\prime}\left(y^m\left(s^t\right)+y^f\left(s^t\right)-c^m\left(s^t\right)\right)}=\gamma$$

assuming both utility functions are strictly increasing and concave, we can use the quotient rule to see if the left hand side is monotonic:

$$u^{f\prime}\left(c^f\left(s^t\right)\right)u^{m\prime\prime}\left(c^m\left(s^t\right)\right)+u^{m\prime}\left(c^m\left(s^t\right)\right)u^{f\prime\prime}\left(c^f\left(s^t\right)\right)<0$$

Since this is strictly decreasing, we conclude that given a total income realization $y^m\left(s^t\right)+y^f\left(s^t\right)$, there is a unique allocation of consumption between husband and wife. This implies that consumption allocations will be independent of the distribution of income between husband and wife conditional on total income. This will go through if we allow the household to borrow and save at a market interest rate $R$.

## Consumption Insurance with Limited Commitment

Now, suppose that ex ante commitment to a contract is not possible. That is, at any point in time, if autarky looks more appealing to the wife than staying in the contract, she can exit the contract. The idea is that if a wife gets a very good income realization and the husband gets a very poor one, then a contract under full commitment will require that she give a large amount of income to her husband. However, conditional on her good income realization, she may be better off walking out of the contract and staying in autarky for the rest of the period. Given this constraint on commitment, we must modify the second constraint of our maximization problem to:

$$\sum_{\tau=t}^{\infty}\beta^{\tau-t}\sum_{s^\tau\in S^\tau}\pi\left(s^\tau\right)u^f\left(c^f\left(s^\tau\right)\right)\geq\sum_{\tau=t}^{\infty}\beta^{\tau-t}\sum_{s^\tau\in S^\tau}\pi\left(s^\tau\right)u^f\left(y^f\left(s^\tau\right)\right)\ \forall\ s^t$$

Now we write the problem:

$$\begin{aligned}&\max_{c^m(s^t),\,c^f(s^t)}\sum_{t=0}^{\infty}\beta^t\sum_{s^t\in S^t}\pi\left(s^t\right)u^m\left(c^m\left(s^t\right)\right)\ \text{ s.t.}\\ &c^m\left(s^t\right)+c^f\left(s^t\right)\leq y^m\left(s^t\right)+y^f\left(s^t\right)\ \forall s^t\\ &\sum_{\tau=t}^{\infty}\beta^{\tau-t}\sum_{s^\tau\in S^\tau}\frac{\pi\left(s^\tau\right)}{\pi\left(s^t\right)}u^f\left(c^f\left(s^\tau\right)\right)\geq\sum_{\tau=t}^{\infty}\beta^{\tau-t}\sum_{s^\tau\in S^\tau}\frac{\pi\left(s^\tau\right)}{\pi\left(s^t\right)}u^f\left(y^f\left(s^\tau\right)\right)\ \forall\ s^t\end{aligned}$$

where we denote the multipliers $\lambda(s^t)$ and $\gamma(s^t)$ respectively. The first order conditions yield:

$$\begin{aligned}
\left[c^m\left(s^t\right)\right] &: \beta^t \pi\left(s^t\right) u^{m\prime}\left(c^m\left(s^t\right)\right) = \lambda\left(s^t\right) \\
\left[c^f\left(s^t\right)\right] &: \sum_{\tau=0}^{t} \beta^{t-\tau} \sum_{s^\tau \in S^\tau} \gamma\left(s^\tau\right) \frac{\pi\left(s^t\right)}{\pi\left(s^\tau\right)} u^{f\prime}\left(c^f\left(s^t\right)\right) = \lambda\left(s^t\right)
\end{aligned}$$

Equating these we see that

$$\begin{aligned}
\beta^t \pi\left(s^t\right) u^{m\prime}\left(c^m\left(s^t\right)\right) &= \sum_{\tau=0}^{t} \beta^{t-\tau} \sum_{s^\tau \in S^\tau} \gamma\left(s^\tau\right) \frac{\pi\left(s^t\right)}{\pi\left(s^\tau\right)} u^{f\prime}\left(c^f\left(s^t\right)\right) \\
\frac{u^{m\prime}\left(c^m\left(s^t\right)\right)}{u^{f\prime}\left(c^f\left(s^t\right)\right)} &= \sum_{\tau=0}^{t} \beta^{t-\tau} \sum_{s^\tau \in S^\tau} \frac{\gamma\left(s^\tau\right)}{\pi\left(s^\tau\right)}
\end{aligned}$$

Now the ratio marginal utilities explicitly depends on the history. Specifically, if the wife receives a series of good shocks, then the paricipation contraints more are more likely to bind (we will have more $\gamma\left(s^\tau\right)' s > 0$. This increases the right hand side of the equation, which implies that the husband must sacrifice some consumption in order to induce the wife to stay in the contract.

# Some optimal tax results

Before going into this paper, I want to summarize a few results from traditional optimal tax theory - I think these are useful to have in the back of your mind when thinking about constraints in developing countries.

- *Commodity Taxation* - there are several different results here. A common heuristic is that inelastically demanded commodities should be taxed more (what would this imply about the taxation of rice vs. luxury cars in a developing country?). However, a famous result states that is we have preferences of the form: $U = U\left(G\left(x\right), x_0\right)$, all goods within $G\left(.\right)$ should be taxed at the same rate. Generally it is assumed that $x_0$ is labor and $x$ is other consumption goods, which implies that commodity taxes should be uniform.

- *Labor Taxation* - Again, here are two famous results. The first states that if labor is perfectly observable, a nonlinear tax on labor dominates linear commodity taxation. This implies that we should not tax commodities at all, and that we should just tax labor. However, if we have moral hazard, labor taxation is somewhat more complicated. We can write a Mirlees problem very similar to what you will see/have seen in contract theory, which assumes that income, but not effort is observable, and we look for a truth-telling optimal tax structure. The most commonly cited result here is that as $Y \to \infty$, $MTR(Y) \to 0$. That is, marginal taxes for the richest should be zero.

- *Captial Taxation* - The famous result here is that long run taxes on capital should be zero (although at $t = 0$ we may want to levy a capital tax).

Gordon and Li results:

Here are some predictions from the Gordon and Li (2005) paper. I think there is a lot more work to do here, their model is pretty informal and not terribly rigorous, although some of these ideas are interesting:

- Poor countries may want to tax firms which benefit more from the banking sector, since they are less likely to switch to the informal sector. They argue that these firms are generally capital-intensive firms
- It may be efficient to use tariggs to shift domestic production towards highly taxed industries
- It may be optimal to allow for inflation, which "taxes" the informal sector more. With inflation, the country must allow steady depreciation of its currency in markets (that is, it cannot peg its currently).
- Governments should push multinationals out of sectors where it is easy to tax domestic producers, but encourage their entry into sectors in which it is difficult to tax domestic producers
- Red tape and bureaucracy may be efficient, if you let bureaucrats extract rent from the informal sector
- State ownership of banks may be a device to be sure that transactions of firms in the banking sector are observable