14.771 Development Economics: Microeconomic Issues and Policy Models
Fall 2008

1 The Family

- A family consists two people F and M with utility functions $U_F(\mathbf{q}, \mathbf{a}), U_M(\mathbf{q}, \mathbf{a})$, where $\mathbf{q} = (\mathbf{q}_F, \mathbf{q}_M, \mathbf{Q})$ is a vector of amounts of private consumption goods for the two people and the amount of public consumption goods $(\mathbf{Q})$ and $\mathbf{a} = (\mathbf{a}_F, \mathbf{a}_M, \mathbf{A})$ likewise, is a vector of actions that they each can take and a public action.

- Let $\widetilde{\mathbf{p}} = (\mathbf{p}, \mathbf{p}, \mathbf{P})$ be the vector of all the prices of consumption goods. Then the budget constraint for the family:

$$\widetilde{\mathbf{p}}\mathbf{q} = x = x_F(\mathbf{a}) + x_M(\mathbf{a}) + \boldsymbol{\phi}_F + \phi_M$$

  where ϕ_F and ϕ_M are the unearned incomes and $x_F(\mathbf{a})$ and $x_M(\mathbf{a})$ are the earned incomes assigned by the existing system of property rights to F and M. Does not mean M earns the income that is assigned to him.

- Special case 1: $\mathbf{a}$ is **pure investment**. In this case $\partial U_F(\mathbf{q}, \mathbf{a})/\partial \mathbf{a}$ and $\partial U_M(\mathbf{q}, \mathbf{a})/\partial \mathbf{a}$ are both zero. $\mathbf{a}$ only enters through $x_F(\mathbf{a})$ and $x_M(\mathbf{a})$:what is an example?

- Special case 2: **Multiplicative Separability:** $U_F(\mathbf{q}, \mathbf{a}) = U_F(\mathbf{q})\mathbf{g}_F(\mathbf{a})$

1.1 The Unitary Model

- $U_F(\mathbf{q}, \mathbf{a}) \equiv U_M(\mathbf{q}, \mathbf{a}) = U(\mathbf{q}, \mathbf{a})$, i.e identical preferences.

- The consumption decision: Maximize $U(q, \mathbf{a})$ over $\mathbf{q}$ subject to $\widetilde{\mathbf{p}}\mathbf{q} = x_F(\mathbf{a}) + x_M(\mathbf{a}) + \boldsymbol{\phi}_F + \phi_M$. Pareto Optimality by construction.

- FOC

$$
\begin{aligned}
\frac{\partial U(\ \mathbf{q}, \mathbf{a})}{\partial\ \mathbf{q}} &= \boldsymbol{\lambda}\widetilde{\mathbf{p}} \\
\frac{\partial U(\ \mathbf{q}, \mathbf{a})}{\partial \mathbf{a}} &= -\boldsymbol{\lambda}(x'_F(\mathbf{a}) + x'_M(\mathbf{a})) \\
x_F(\mathbf{a}) + x_M(\mathbf{a}) + \boldsymbol{\phi}_F + \phi_M &= \widetilde{\mathbf{p}}\mathbf{q}
\end{aligned}
$$

1.2 The fixed bargaining power collective model

- $U_F(\ q, \mathbf{a}) \neq U_M(\ q, \mathbf{a})$, i.e non-identical preferences.

- The family maximizes $U_F(\ q, \mathbf{a}) + \mu U_M(\ q, \mathbf{a})$, where μ is bargaining weight.

- The key assumption is that μ is independent of $x, \mathbf{a}, \mathbf{q}$. One possible scenario is that μ is chosen first, then all the other decisions are taken.

- Given that μ is a constant, the decision taken by the family decision must be Pareto efficient. Why?

- The family's decision: Maximize $U_F(\ q, \mathbf{a}) + \mu U_M(\ q, \mathbf{a})$ over $q, \mathbf{a}$ subject to $\tilde{\mathbf{p}}\ q = x_F(\mathbf{a}) + x_M(\mathbf{a})$. FOC

$$\begin{aligned}
\frac{\partial U_F(\ \mathbf{q}, \mathbf{a})}{\partial\ \mathbf{q}} + \boldsymbol{\mu}\frac{\partial U_M(\ \mathbf{q}, \mathbf{a})}{\partial\ \mathbf{q}} &= \boldsymbol{\lambda}\tilde{\mathbf{p}} \\
\frac{\partial U_F(\ \mathbf{q}, \mathbf{a})}{\partial \mathbf{a}} + \boldsymbol{\mu}\frac{\partial U_M(\ \mathbf{q}, \mathbf{a})}{\partial \mathbf{a}} &= -\boldsymbol{\lambda}(x'_F(\mathbf{a}) + x'_M(\mathbf{a})) \\
x_F(\mathbf{a}) + x_M(\mathbf{a}) + \boldsymbol{\phi}_F + \phi_M &= \tilde{\mathbf{p}}\mathbf{q}
\end{aligned}$$

- Notice that this is formally identical to the unitary model, with

$$U_F(\ \mathbf{q}, \mathbf{a}) + \mu U_M(\ \mathbf{q}, \mathbf{a}) = U_C(\ \mathbf{q}, \mathbf{a})$$

- These two models are not easy to distinguish unless we can observe individual choice behavior outside the usual context of family decision-making (i.e. if you offer choices to one of the members without telling the other).

1.3 Testable Implications of the Model

1.3.1 Income pooling tests

Recall the FOC

$$\begin{aligned}\frac{\partial U_F(\mathbf{q},\mathbf{a})}{\partial \mathbf{q}}+\boldsymbol{\mu}\frac{\partial U_M(\mathbf{q},\mathbf{a})}{\partial \mathbf{q}} &= \boldsymbol{\lambda}\widetilde{\mathbf{p}} \\ \frac{\partial U_F(\mathbf{q},\mathbf{a})}{\partial \mathbf{a}}+\boldsymbol{\mu}\frac{\partial U_M(\mathbf{q},\mathbf{a})}{\partial \mathbf{a}} &= -\boldsymbol{\lambda}(x_F'(\mathbf{a})+x_M'(\mathbf{a})) \\ x_F(\mathbf{a})+x_M(\mathbf{a})+\boldsymbol{\phi}_F+\phi_M &= \widetilde{\mathbf{p}}\mathbf{q}\end{aligned}$$

- Suppose there are different families with the same $x_F(\mathbf{a})$ and $x_M(\mathbf{a})$,and the same total income, but in some of them ϕ_F is large and in others ϕ_M is large.

- Then if they have the same bargaining power, same production technology and same preferences, and face the same prices, they will make the same choices.

- How would we actually test this?

- One problem with this is that we do not usually observe production and utility functions.

- Under what conditions is this not a problem?

  – The windfall test of income pooling

- What do you if windfall shocks are not available?

  1. Strong Separability:

$$\partial U_F(\mathbf{q}, \mathbf{a})/\partial q_i = \frac{\partial u_F(\mathbf{q})}{\partial q_i} g(\mathbf{a}),$$
$$\partial U_M(\mathbf{q}, \mathbf{a})/\partial q_i = \frac{\partial u_M(\mathbf{q})}{\partial q_i} g(\mathbf{a})$$

  note $g(\mathbf{a})$ does not have F or an M subscript.

Then

$$\begin{aligned}\frac{\partial u_F(\mathbf{q})}{\partial \mathbf{q}} + \boldsymbol{\mu}\frac{\partial u_M(\mathbf{q})}{\partial \mathbf{q}} &= \boldsymbol{\lambda}\frac{\widetilde{\mathbf{p}}}{g(\mathbf{a})} \text{ and} \\ x_F(\mathbf{a}) + x_M(\mathbf{a}) + \boldsymbol{\phi}_F + \phi_M &= \widetilde{\mathbf{p}}\mathbf{q}\end{aligned}$$

can be solved to get $\mathbf{q}(x_F(\mathbf{a}) + x_M(\mathbf{a}) + \boldsymbol{\phi}_F + \phi_M)$.

In this case $\mathbf{q}$ depends only on total family income, $x_F(\mathbf{a}) + x_M(\mathbf{a}) + \boldsymbol{\phi}_F + \phi_M$. *The intra-family distribution of income does not matter*.

What do we need to know to do this?

2. The Ratio Test

Using separability + purely private preferences and

$$\begin{aligned}U_F(\mathbf{q}, \mathbf{a}) &= u_F(\mathbf{q}_F) g_F(\mathbf{a}), \\ U_M(\mathbf{q}, \mathbf{a}) &= u_M(\mathbf{q}_M) g_M(\mathbf{a}).\end{aligned}$$

No spillovers, no family public consumption goods.

In this case by separability the FOC reduces to

$$\begin{aligned} g_F(\mathbf{a})\partial U_F(\,\mathbf{q}_F)/\partial\,\mathbf{q}_F &= \boldsymbol{\lambda}\mathbf{p} \\ g_M(\mathbf{a})\boldsymbol{\mu}\partial U_M(\,\mathbf{q}_M)/\partial\,\mathbf{q}_M &= \boldsymbol{\lambda}\mathbf{p} \\ x_F(\mathbf{a}){+}x_M(\mathbf{a}) + \boldsymbol{\phi}_F + \phi_M &= \widetilde{\mathbf{p}}\,\mathbf{q} \end{aligned}$$

Therefore $\frac{\partial U_F(\,\mathbf{q}_F)/\partial q_{iF}}{\partial U_F(\,\mathbf{q}_F)/\partial q_{jF}} = \frac{p_i}{p_j} = \frac{\partial U_M(\,\mathbf{q}_M)/\partial q_{iM}}{\partial U_M(\,\mathbf{q}_M)/\partial q_{jM}}$,which implies that the marginal rates of substitution between any two goods is independent of who has bargaining power, as long as there is efficient bargaining. Used for tests of efficiency.

How do we implement this test? Hint: Assume that both U_F and U_M are CRRA. Then derive the "ratio" test.

how robust is this test?

is there a more robust test?

Note that this test works as long as there are a subset of goods for which separability is a reasonable assumption.

3. The investment test

The investment model.

Recall the FOCs in the bargaining model

$$
\begin{aligned}
\frac{\partial U_F(\mathbf{q},\mathbf{a})}{\partial \mathbf{q}}+\boldsymbol{\mu}\frac{\partial U_M(\mathbf{q},\mathbf{a})}{\partial \mathbf{q}} &= \boldsymbol{\lambda}\tilde{\mathbf{p}} \\
\frac{\partial U_F(\mathbf{q},\mathbf{a})}{\partial \mathbf{a}}+\boldsymbol{\mu}\frac{\partial U_M(\mathbf{q},\mathbf{a})}{\partial \mathbf{a}} &= \boldsymbol{\lambda}(x'_F(\mathbf{a})+x'_M(\mathbf{a})) \\
x_F(\mathbf{a})+x_M(\mathbf{a})+\boldsymbol{\phi} &= \tilde{\mathbf{p}}\,\mathbf{q}
\end{aligned}
$$

- In the investment model $\frac{\partial U_F(\mathbf{q},\mathbf{a})}{\partial \mathbf{a}}, \frac{\partial U_M(\mathbf{q},\mathbf{a})}{\partial \mathbf{a}}$ are zero. Hence it must be true that

$$x'_F(\mathbf{a})+x'_M(\mathbf{a})=\mathbf{0}$$

This is what Chris Udry, for example, tests.

1.4 The collective model with endogenous bargaining power: Browning-Chiappori

- Gets rid of the assumption that bargaining power is a constant

- $\mu = \mu(x_F(\mathbf{a}), x_M(\mathbf{a}), \boldsymbol{\phi}_F, \phi_M, \mathbf{p})$

- What are properties of the Slutsky Matrix when μ is a constant?

- What is the SR1 property?

- What is the intuition for it?

- What is being tested here?

1.5 What can these models really tell us?

An incomplete contract approach

- What is the alternative to the collective model with endogenous bargaining power?

- One possibility is that the intrafamily contract is not enforceable.

- For a certain good i the husband consumes $\lambda_M^i q^i$ while the wife consumes $(1-\lambda_M^i)q^i$ when the total amount of good i purchased is q^i, irrespective of the relative intensity of their preferences.

- Do we have Pareto Efficiency here?

- Do we have the SR1 property?

- To see that the SR1 property still holds, define a new utility function for each member of the family:

$$
\begin{aligned}
& U_F(q_F^1, ..q_F^{i-1}, (1-\lambda_M^i)q^i, ..q_F^n, q_M^1, ..q_M^{i-1}, \lambda_M^i q^i, ..q_M^n, \mathbf{Q}, \mathbf{a}) \\
= \; & W_F(q_F^1, ..q_F^{i-1}, q_F^{i+1}, ..q_F^n, q_M^1, ..q_M^{i-1}, \lambda_M^i q^i, ..q_M^n, q^i, \mathbf{Q}, \mathbf{a})
\end{aligned}
$$

and likewise

$$
\begin{aligned}
& U_M(q_F^1, ..q_F^{i-1}, \lambda_M^i q^i, ..q_F^n, q_M^1, ..q_M^{i-1}, \lambda_M^i q^i, ..q_M^n, \mathbf{Q}, \mathbf{a}) \\
= \; & W_M(q_F^1, ..q_F^{i-1}, q_F^{i+1}, ..q_F^n, q_M^1, ..q_M^{i-1}, \lambda_M^i q^i, ..q_M^n, q^i, \mathbf{Q}, \mathbf{a})
\end{aligned}
$$

- W_F and W_M are just two other utility functions with an additional public good q^i.

- So the SR1 property will continue to hold.

- How about the ratio test?

1.5.1 An interesting incomplete contracts model (Maher-Wells)

- One good, investment model: $U_F = u(\ q_F), U_M = u(\ q_M)$.

- The budget constraint is that

$$q_F + q_M = x_F(a_F) + x_M(a_M).$$

- If μ is, as before, fixed, then the family will set ${x'}_F(a_F) = {x'}_M(a_M) = 0$ and then distribute consumption.

- Now let μ be dtermined after the investment is made but before consumption is chosen. Let $\mu(\frac{a_F}{a_M})$ and $\mu' < 0$. If the woman invests then she become more powerful. One reason may be that she can just walk off with her $x(a_F)$. i.e her outside option is $u(x(a_F))$ and the bargaining has to give her at least this.

1.5.2 The incomplete contract approach continued

- Maximizing $u(\ q_F) + \mu(a_F/a_M)u(\ q_M)$ subject to the budget constraint yields two functions

$$\begin{aligned} u_F^* &= u_F^*(a_F/a_M, x_F(a_F) + x_M(a_M)) \\ u_M^* &= u_M^*(a_M/a_F, x_F(a_F) + x_M(a_M) \end{aligned}$$

- Now suppose that F chooses a_F to maximize u_F^* and likewise for M. We assume non-cooperative behavior.

- Since u_F^* is increasing in a_F/a_M and u_M^* is increasing in a_M/a_F, both F and M will over-invest, i.e. $x_F' < 0$ and $x_M' < 0$.

- Maher-Wells give an example of delayed child-bearing.

- How can we distinguish this approach from the complete contract approach with more complex preferences?