14.75: PROBLEM SET 3

Please include stata do-file code and output for all exercises.

(1) Use votingData.dta for the following.

    (a) Mathematically prove that the fixed effects estimation of

$$y_{it} = \gamma_0 + \alpha_i + \beta_0 x_{it} + \epsilon_{it}$$

    delivers a numerically identical coefficient to the regression of y_{it}'s deviation from its time average $y_{i\cdot} := T^{-1} \sum_t y_{it}$ on x_{it}'s deviation from its time average.
We accept any solution where you show that

$$y_{it} - y_{i\cdot} = \beta_0 \left(x_{it} - x_{i\cdot} \right) + \epsilon_{it} - \epsilon_i$$

    and therefore a fixed effects regression, which demeans at the i level delivers a numerically identical estimate.

    (b) As a man of science, Bruce Wayne has decided to donate campaign contributions to random candidates in each of 100 county elections for each of the last 10 years. However, his campaign contribution amounts may vary, systematically, with the county. That is, Bruce may favor giving contributions more in some counties than in others.

        (i) Regress the vote percent received by Wayne's party on the campaign donation. What is the effect of a one standard deviation increase in campaign donation on the share of votes received?

        (ii) Generate a scatter plot of vote percentage against campaign donations.

        (iii) Regress the vote percent received by Wayne's party on the campaign donation, including fixed effects. What is the effect of a one standard deviation increase in campaign donation on the share

        (iv) Discuss the difference, if any, between (i) and (iii). How is this possible?

        (v) Generate a scatter plot of vote percentage against campaign donations separately for two counties of your choosing. Compare these pictures to the one in (ii). Does this support your explanation in (iv)?

(2) There is a population of n individuals. They have differing earning potentials, supply one unit of labor, earning y_1 or y_0 depending on whether they are of higher earning potential (1) or lower earning potential (0). Every individual belongs to a caste, h or l. Let λ_{jc} be the fraction of population in potential j and caste c. Clearly $\sum_{j,c} \lambda_{jc} = 1$. Define $\lambda_{1l} + \lambda_{0l} =: \lambda_l$.

    (a) Write a condition such that the low caste types constitute a minority in the population.

    (b) Interpret the condition that

$$\frac{\lambda_{0l}}{\lambda_{1l}} > \frac{\lambda_{0h}}{\lambda_{1h}}.$$

    Why might we assume this in a model?

    (c) Individual income is taxed at rate τ, if it is taxed. There are two ways it can be redistributed.

        (i) Policy 0: $\tau = 0$, no redistribution.

        (ii) Policy 1: T is given to every individual. Let $\tau = 1$.

- Interpret what this means in terms of outcomes for high earning potential and low earning potential workers.

    (iii) Policy 2: $\delta_c = \delta$ is given to all low caste types and $\delta_c = 0$ to all high caste types. Let $\tau = 1$.

- Interpret what this means in terms of outcomes for high and low caste individuals.

(d) We will assume that the budget is balanced. So the transfers to all individuals sum up to the entire income in the economy. What does this mean for:
    (i) Policy 1? Express mathematically.
    (ii) Policy 2? Express mathematically.

(e) Under each of the 3 policies, payoffs are given by:

$$u_{jc} = y_j \text{ or } u_{jc} = T \text{ or } u_{jc} = \delta_c.$$

For each of the responses below, if there are multiple answers, depending on parameter values, please specify the cases and the threshold parameter values. That is, if a group will prefer policy 1 if, say $x \geq \bar{x}$ but policy 2 if $x < \bar{x}$, please note this clearly.
    (i) What policy do high earning potential, high caste prefer? Why?
    (ii) What policy do low earning potential, high caste prefer? Why?
    (iii) What policy do high earning potential, low caste prefer? Why?
    (iv) What policy do low earning potential, low caste prefer? Why?

(f) Let $y_1 = 1$ and $y_0 = 0$. Assume that a plurality always wins. If there are fewer low earnings potential low caste people than low earnings potential high caste people ($\lambda_{l0} < \lambda_{1h}$), and there are more low caste people than high caste people ($\lambda_l > \lambda_h$), which policy wins?

(3) Imagine that voters and politicians play the following game. A politician who is in office has to decide how much effort e to apply to affect the outcome of some policy. The policy output x has two states, indicated by $x_h = 1$ or $x_l = 0$ respectively. In particular, when the politician applies effort e, we have

$$x = \begin{cases} 1 & \text{with probability } e \\ 0 & \text{with probability } 1 - e \end{cases}.$$

Moreover, the politician dislikes applying effort, and faces a cost $c(e) = \frac{1}{2}ce^2$. Before the politician takes a decision, voters tell the politician that they will contribute h to the re-election campaign if $x = 1$ and l if $x = 0$. Both the voters and the politicians are risk-neutral. Finally, if the politician does not like the contract setup, he can walk away and just get some payoff m. The timing of the game is as follows:

- Voters offer a take-it-or-leave-it contract $\{h, l\}$ and politicians choose to accept or not. If not, his outside option is m.
- Then politicians apply hidden effort $e \in [0, 1]$.
- After this, x is realized.
- Voters pay $w(x) \in \{h, l\}$ and keep $x - w(x)$.
- Payoffs are

$$u_v(x) = x - w(x) \text{ and } u_p(w) = w - \frac{1}{2}ce^2.$$

(a) First, assume that voters cannot commit to paying the declared $w(x)$. This is because, after politicians apply their effort and the outcome x is realized, voters can change their mind. Then what is the equilibrium level of effort by the politician?

(b) Now suppose that voters can commit to reciprocating outcomes by paying $w(x)$. Therefore, voters solve the following problem.

(i) They wish to maximize their expected payoffs, which is the expectation of the outcome ($x = x_h$ or x_l) minus the payment to the campaign contribution (h or l). Write the expected payoffs of the voters $\pi(h, l, e)$.

(ii) One constraint the voters have to have in mind is that they cannot contribute negative amounts to the campaign. What does this mean for h and l? Write the mathematical statements.

(iii) Another constraint the voters have to have in mind is that they have to make sure that the politician wants to join the arrangement. Therefore, the expected payoffs to the politician less the cost of effort need to be at least as much as his outside option. Write the mathematical condition.

(iv) Finally, the voters need to keep in mind that the politician will choose an effort level that maximizes his own expected payoff.
   • What is this constraint?
   • Solve for $e(h, l)$, the optimal effort level as a function of h, l.

(v) Using the results of (iv), write the maximization problem that the voter faces subject to the two relevant constraints: that l has to be non-negative and that the politician finds it worthwhile to participate.

(vi) Now assume that $1/8c < m$ which means that the politician will be indifferent between participating in the process and taking his outside option. This implies that

$$\frac{(h - l)^2}{2c} + l = m.$$

   • Show that this allows us to write the program as

$$\max_{h,l} \pi(h, l) = \frac{h - l}{c} - \frac{(h - l)^2}{2c} - m.$$

   • Assume that $l > 0$. Then what does $h - l$ have to be? What is l? What is h? What is $e(h, l)$?
   • Assume that $l = 0$. Then show that $h = \sqrt{2cm}$. What is $e(\sqrt{2cm}, 0)$?

(vii) Interpretations:
   (A) Plot e on the y-axis and m on the x-axis where m goes from 0 to something just over $1/2c$. What does this curve look like? (Hint: focus on $1/8c$ and $1/2c$ as key points.)
   (B) In this model, if the politician has better outside options (that is, m increases), what happens to effort?

(c) Reflect on how commitment affects incentives of the politician, by comparing (a) and (b).

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