14.54 Fall 2016
Recitation: HO Model

Consider an economy (Home) with 2 goods, Cloth (C) and Food (F). All consumers have same Cobb-Douglas utility function:

$$U\left(D_C, D_F\right) = \left(D_C\right)^{1-\alpha}\left(D_F\right)^{\alpha} \tag{1}$$

with $\alpha \in (0,1)$. The production technologies are:

$$F_F\left(K_F, L_F\right) = \left(K_F\right)^{\beta_F}\left(L_F\right)^{1-\beta_F} \tag{2}$$
$$F_C\left(K_C, L_C\right) = \left(K_C\right)^{\beta_C}\left(L_C\right)^{1-\beta_C} \tag{3}$$

with β_F, $\beta_C \in (0,1)$ and $\beta_F > \beta_C$.

1. The objective of this first exercise is to demonstrate the Stolper-Samuelson Theorem when Equations (2)-(3) hold.

(a) Let a_{Ki} and a_{Li} denote the amount of capital and labor used in one unit of good $i = C, F$. Show that:

$$\frac{a_{Ki}}{a_{Li}} = \frac{\beta_i}{(1-\beta_i)}\frac{w}{r} \tag{4}$$

(b) Using Equation (4), show that:

$$a_{Li} = \left[\frac{\beta_i}{(1-\beta_i)} \cdot \frac{w}{r}\right]^{-\beta_i}$$
$$a_{Ki} = \left[\frac{\beta_i}{(1-\beta_i)} \cdot \frac{w}{r}\right]^{-\beta_i+1}$$

(c) Let p_i denote the price of good $i = C, F$. Show that:

$$p_i = \beta_i^{-\beta_i}\left(1-\beta_i\right)^{\beta_i - 1} r^{\beta_i} w^{1-\beta_i} \tag{5}$$

(d) Show that an decrease in $p = p_C/p_F$ increases the real return of capital and decreases the real return of labor [Stolper-Samuelson Theorem].

2. The objective of this second exercise is to demonstrate the Rybczynski Theorem when Equations (2)-(3) hold.

(a) Let Q_C and Q_F denote the output of good C and F. Show that

$$Q_C = \frac{L - \frac{a_{LF}}{a_{KF}}K}{a_{KC}\left(\frac{a_{LC}}{a_{KC}} - \frac{a_{LF}}{a_{KF}}\right)} \tag{6}$$

$$Q_F = \frac{\frac{a_{LC}}{a_{KC}}K - L}{a_{KF}\left(\frac{a_{LC}}{a_{KC}} - \frac{a_{LF}}{a_{KF}}\right)} \tag{7}$$

(b) Using Equation (4), show that an increase in K raises Q_F and lowers Q_C [Rybczynski Theorem].

3. The objective of this last exercise is to demonstrate the Heckscher-Ohlin Theorem when Equations (1)-(3).

(a) Using Equation (1), show that

$$\frac{p_C Q_C}{p_F Q_F} = \frac{1-\alpha}{\alpha}$$

(b) Using Equation (5), show that

$$\frac{\beta_F^{\beta_F}\left(1-\beta_F\right)^{1-\beta_F}}{\beta_C^{\beta_C}\left(1-\beta_C\right)^{1-\beta_C}}\left(\frac{w}{r}\right)^{\beta_F-\beta_c}\frac{Q_C}{Q_F} = \frac{1-\alpha}{\alpha} \tag{8}$$

(c) Using Equations (6), (7), and (8), show that labor-abundant countries tend to produce disproportionate amount of Cloth [Heckscher-Ohlin Theorem].