14.451 Midterm
Spring 2004

You have **1.5 hours**. You must answer **all** questions. No books or notes are allowed. **Good luck!**

Question 1   [60 points/100]

Consider the following variation of the standard neoclassical growth model. A fraction μ of the population are 'hand-to-mouth' consumers (type 1). They simply consume any labor income they earn each period, they never save or borrow, and they never own any assets whatsoever. The remaining fraction $1 - \mu$ are 'savers' (type 2). They have preferences and choices as in the standard neoclassical model. There is no exogenous technological change and no population growth and total population is normalized to 1.

The preferences for the savers are standard:

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t^2\right) \tag{1}$$

for some $\beta < 1$, and u twice continuously differentiable, increasing and strictly concave with the Inada condition $\lim_{c \to 0} u'(c) = \infty$.

Technology is given by the production function $Y_t = K_t^{\alpha} L_t^{1-\alpha}$. Labor, L_t, is supplied inelastically by both types of agents with total labor supply normalized to 1 – the hand-to-mouth agents supply μ and the savers $1 - \mu$. The resource constraint is,

$$C_t + K_{t+1} = Y_t + (1 - \delta) K_t$$

where $C_t \equiv \mu c_t^1 + (1 - \mu)c_t^2$ is aggregate consumption and c_t^1 and c_t^2 represents consumption of hand-to-mouth consumers and savers, respectively. It follows that the resource constraint is

$$\mu c_t^1 + (1 - \mu)c_t^2 + K_{t+1} = K_t^{\alpha} + (1 - \delta) K_t. \tag{2}$$

Finally, the wage rate is denoted with w_t and the interest rate with r_t.

(a) Characterize the equilibrium w_t and r_t. Next, show that in equilibrium the hand-to-mouth agents absorb some fraction of total output. That is,

$$\mu c_t^1 = \lambda Y_t = \lambda K_t^{\alpha} \tag{3}$$

Determine the equilibrium λ.

(b) Derive the Euler condition that characterizes the optimal behavior of the savers. Derive the conditions that characterize the general equilibrium. Characterize the steady state value of K.

(c) How does the steady state level of capital with hand-to-mouth workers compare with the steady state level in the standard neoclassical growth model? Explain.

(d) Consider now the optimal allocation from the perspective of the savers (not the perspective of the whole society). In particular, consider a dictator who is interested only in the welfare of savers and solves the following problem: Maximize (1) subject to the resource constraint and the fact the hand-to-mouth consumers absorb a fraction λ of output, that is, subject to (2) and (3). Derive the Euler condition for this problem.

(e) Does the Euler condition characterizing the savers' optimal allocation coincide with the one characterizing the competitive equilibrium? Does the savers' optimal and the competitive allocations coincide? If yes, why yes. If not, why not.

(f) Consider a proportional tax on capital income, the proceeds of which are distributed lump-sum across only the savers. (The tax might be negative, in which case the lump-sum transfer is also negative.) The tax rate is denoted with τ and is constant over time. What is the value of τ that implements the savers' optimal allocation as a competitive equilibrium? Would the optimal τ be different if the lump-sum transfer was across all agents, not just the savers?

Question 2   [40 points/100]

(a) The Solow growth model can help explain persistent cross-country differences in both the levels of output and the rates of investment.

(b) Consider the neoclassical growth model with exogenously fixed labor. Briefly describe the response of aggregate consumption, investment, and output to the arrival of news that TFP will stay the same for the next two years but will then increase and stay higher for ever after.

(c) Since it is a kind of rent-seeking activity, industrial espionage necessarily reduces growth.

(d) Since knowledge is non-rival, the models of endogenous technological change and knoweledge creation, although they can explain the existence of long-run growth, can not explain persistent differences in productivity levels across countries.

Good Luck!