

Problem Set #5

14.41 Public Economics

DUE: Dec 3, 2010

1 Tax Distortions

This question establishes some basic mathematical ways for thinking about taxation and its relationship to the marginal rate of substitution between goods. Consider individuals that have preferences

$$u(c_1, c_2, L)$$

over two goods, c_1, c_2 , and leisure, L . Let p_1, p_2 be the (before-tax) prices of goods c_1 and c_2 . Let w be the (before-tax) wage. The agent has an endowment of non-labor wealth of m . As usual, we assume that u is continuously differentiable in (c_1, c_2, L) . Let $u_1(c_1, c_2, L)$ denote the marginal utility of c_1 , $u_2(c_1, c_2, L)$ denote the marginal utility of c_2 , and $u_L(c_1, c_2, L)$ denote the marginal utility of leisure.

1. Write the agent's budget constraint assuming no taxes and that the individual is endowed with 1 unit of leisure so that $l + L = 1$, where l is the amount of time spent working for a wage.
2. Derive the first-order conditions for the agent's maximization problem by placing a Lagrange multiplier, λ , on the budget constraint. Derive two equations that combined with the budget constraint characterize the solution to the maximization problem, (c_1^*, c_2^*, L) . One condition should be for the marginal rate of substitution between c_1 and c_2 and another for the MRS between c_1 and L . Explain the intuition of both equations. [Note: you do not (and cannot in general) solve for the solution explicitly; however, you should notice that you have 3 equations and 3 unknowns, so that given any well-specified function u one could solve the system of equations. Also, note that one could also write a condition for the MRS between c_2 and L ; however this would be redundant given the other two equations].

Now, suppose that the government levies a lump-sum tax of τ on all individuals, so that their net non-labor wealth is now $m - \tau$ (Assume for simplicity that this tax is used to finance things for which individuals have no utility).

3. Set up the new budget constraint (assuming the agents never receive the money paid to the government and the money is used to finance things for which individuals have no utility). Derive the first order conditions and provide the analogous two conditions for the marginal rates of substitutions (for c_1 vs c_2 and for c_1 vs L). Explain the intuition of both equations. Is the MRS distorted? (i.e. are they different than they would be in the absence of taxation?) Why or why not?

Now, suppose that instead of the lump-sum tax, the government institutes a tax on c_1 of τ_1 , so that individuals must now pay $p_1 + \tau_1$ per unit of c_1 .

4. Set up the new budget constraint (assuming the agents never receive the money paid to the government and the money is used to finance things for which individuals have no utility). Derive the first order conditions and provide the analogous two conditions for the marginal rates of substitutions (for c_1 vs c_2 and for c_1 vs L). Explain the intuition of both equations. Is the MRS distorted? (i.e. are they different than they would be in the absence of taxation?) Why or why not?

Now, suppose that, in addition to the tax on c_1 , the government institutes a tax on c_2 of τ_2 and on labor earnings, τ_w .

5. Set up the new budget constraint assuming the agents never receive the money paid to the government. Derive the first order conditions and provide the analogous two conditions for the marginal rates of substitutions (for c_1 vs c_2 and for c_1 vs L). Explain the intuition of both equations. Is the MRS distorted? Why or why not?

Now, consider a conceptually different, but mathematically similar economy. Suppose there are two time periods, 1 and 2. In the first time period, agents can consume and work. In the second time period, agents can only consume. Agents are endowed with non-labor wealth of m in the first period. Agents can save and/or borrow at a gross interest rate of R so that savings of s in period 1 yield Rs in period 2. Assume that the price of consumption in both periods is 1 (in terms of money within the period). Also, assume the wage is equal to 1, $w = 1$. Agents utility functions are, as before, given by $u(c_1, c_2, L)$.

6. Denote the agents net savings/borrowing position in the first period by s . Write the agents' two budget constraints, one for each time period. Then, combine these budget constraints into a single budget constraint over c_1 , c_2 , and L . Show how this economy relates to the economy described in parts 1 and 2 by providing prices for p_1 , p_2 , and w for the general economy (parts 1 & 2) that make this economy mathematically equivalent (you should normalize $p_1 = 1$).
7. Suppose the government institutes a tax on savings of τ (and no other taxes). What is the agent's marginal rates of substitution between c_1 and c_2 and between L and c_1 ? Explain the intuition of both equations. Is the MRS distorted? (i.e. are they different than they would

be in the absence of taxation?) Why or why not? [Note: no derivations should be necessary - just apply the results from parts 1-4]

8. Suppose the government institutes a tax on labor earnings of τ (and no other taxes). Solve for the agent's marginal rate of substitution between c_1 and c_2 , and between L and c_1 . Explain the intuition of both equations. Is the MRS distorted? Why or why not? [Note: no derivations should be necessary - just apply the results from parts 1-4]

2 Firm Taxation

Consider an economy populated by a set of individuals who each supply labor, l , and capital, k , to a competitive market of firms at before-tax prices w and r . Each firm has an identical production function which produces $F(k, l) = k^\alpha l^{1-\alpha}$ units of output for any inputs k, l . Assume the output good is the numeraire so that w and r are in terms of the output good. For the first part of the problem, assume that the capital stock is supplied inelastically, so that $k = \bar{K}$ for any set of prices. Also, assume labor is supplied according to a supply function $l(w) = bw$ where $b > 0$.

1. Write the firm's maximization problem and solve for the demand for l as a function of w and \bar{K} , $l(w, \bar{K})$. Solve for the equilibrium labor quantity, l^* , the equilibrium wage w^* , and the competitive price of capital, r^* , that equates demand with the inelastic supply at \bar{K} .
2. Suppose now the government institutes a labor tax of τ which requires agents to pay a tax to the government so that their after-tax wage is $\tilde{w} = w(1 - \tau)$. Discuss graphically and mathematically what happens to the agent's labor supply function as a result of the tax.
3. Solve for the new equilibrium allocation of labor, l^* , the equilibrium before-tax wage w^* , the equilibrium after-tax wage \tilde{w}^* , and the competitive price of capital r^* . How do these relate to your solution in part 1? Why?
4. What is the net change in gross labor earnings (i.e. pre-tax wage x labor)? What is the net change in total capital earnings (interest rate * capital)? If capital earnings changed, explain why.

Now, suppose that instead of a tax on labor, the government institutes a tax on capital, so that individuals' return on capital is given by $\tilde{r} = r(1 - \tau)$; for every r that owners of capital receive from the firm, they must pay τr to the government.

5. Discuss the effect of this tax. Solve for the equilibrium l^* , the equilibrium wage w^* , the equilibrium after-tax return on capital r . What is the deadweight loss of the tax? Why? [Note: you are not required to make any unnecessary/duplicate calculations if you don't need to].

Now, suppose that capital is no longer supplied inelastically. Rather, let's make the polar opposite assumption. Let's assume that firms have access to an infinite amount of capital at a world price of \hat{r} . The interpretation Owners of capital may invest their capital either in the firm in this economy, or in the world market.

6. Without doing any math, discuss the impact of imposing a capital tax. How much of the tax would be paid by capital owners? How much by labor owners?
7. Again without doing any math, discuss the impact of imposing a labor tax. How much of the tax would be paid by capital owners? How much by labor owners? Why?

3 Empirical Evaluation

Barack Obama is back from his trip overseas and is considering a change in the tax code to help reduce the deficit. He liked your advice from problem set 3 and decided to give you a call back. He asks you a couple of questions about what would happen under various changes to the tax code. For each question, he asks you to do two things:

- Discuss briefly what economic theory predicts and what existing studies may have shown on this question. How confident are we in these predictions/results?
- Discuss a potential empirical method that would allow you to answer my question if you had access to any reasonable amount of data that could potentially be required. Discuss the potential limitations of your approach¹.

He asks you the following questions:

1. "I'm considering increasing the tax on savings (interest income), but am worried that this might decrease the amount of savings. How much would this tax increase reduce savings?"
2. "I'm considering reducing EITC credits to the poor, but am worried this might decrease their labor supply. To what extent does changing the level of EITC benefits affect labor supply?"
3. "I'm considering raising the highest tax bracket by from 35% to 40% to help balance the budget. But, I'm worried that if I raise the tax rate this will decrease the amount of income that people report (either through illegal evasion, or because the rich choose to work less in the face of higher taxation). How much will this increase lead to a decline in taxable income amongst the rich?"
4. "I'm considering raising the benefits provided to single mothers with children. But, how much will this increase the number of single mothers?"

¹President Obama explicitly mentions that some of his advice he received last time failed to mention the potential limitations/qualifications of the results. So he reminds you to include a brief discussion of the potential limitations of your proposed approach.

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