14.23 Spring 2003
Problem Set 1
Model Solutions

1. Regulating a monopoly.

   (a) (9 points) Regulated $P = MC$. In equilibrium price must equal public's willingness to pay for the next kilowatt hour, so

$$6 - Q = 1.25 + 0.75Q \quad (1)$$
$$\Longrightarrow \quad Q^* = 2.71, \ P^* = 6 - Q^* = 3.29. \quad (2)$$

   Consumer surplus: $CS^* = Q^*(6 - P^*)\frac{1}{2} = 3.67$.

   Producer surplus is now positive despite the regulation $P = MC$. All but the last kilowatt hour are produced with lower cost then the output price, so the firm is making profits. They are equal to the area of the triangle between output price line and the marginal cost line: $PS^* = Q^*(P^* - 1.25)\frac{1}{2} = 2.76$.

   (b) (9p) Monopolist maximizes profits by equating marginal revenue with marginal cost. Total revenue is $TR(Q) = PQ = (6-Q)Q$ so marginal revenue is $MR(Q) = \frac{\partial}{\partial Q}TR(Q) = 6 - 2Q$.

$$MR = MC \quad \Longleftrightarrow \quad (3)$$
$$6 - 2Q = 1.25 + 0.75Q \quad (4)$$
$$\Longrightarrow \quad Q^m = 1.73, \ P^m = 6 - Q^m = 4.27 \quad (5)$$

   Consumer surplus: $CS^m = Q^m(6 - P^m)\frac{1}{2} = 1.49$

   Producer surplus : $PS^m = Q^m(P^m - 1.25)\frac{1}{2} + Q^m(P^m - MC(Q^m))$ $= 1.12 + 2.99 = 4.11$.

   The producer surplus includes the cost savings for inframarginal units (those produced below marginal cost) and the revenue derived from having price above marginal cost.

   (c) (9p) The deadweight loss is the difference in social surplus under the two cases:

   $DWL = CS^* + PS^* - (CS^m + PS^m) = 6.43 - 5.6 = 0.83$.

2. Two identical firms and quantity-setting.

   (a) (6p) Cournot competition. Profits of firm 1 depend on the output of both firms:

$$\Pi_1(q_1, q_2) = Pq_1 - C(q_1) \tag{6}$$
$$= (100 - 3(q_1 + q_2))\, q_1 - \left(10q_1 + q_1^2\right). \tag{7}$$

   Taking the output of firm 2 as given, firm 1 maximizes profits. The first order condition is

$$\frac{\partial}{\partial q_1}\Pi_1(q_1, q_2) = 100 - 6q_1 - 3q_2 - (10 + 2q_1) = 0 \tag{8}$$
$$\implies q_1^* = \frac{1}{8}(90 - 3q_2) \equiv R(q_2) \tag{9}$$

   The solution gives the optimal output for firm 1, given the output of firm 2 ("the reaction function"). Since firms are identical, the reaction functions are the same for both firms and both firms produce the same level of output in equilibrium:

$$q = R(q) \implies 8q = 90 - 3q \tag{10}$$
$$\implies q^* = \frac{90}{11} = 8.18. \tag{11}$$

   Total supply is then $Q^* = 2q^* = 16.36$
   and market price $P^* = 100 - 3 \times 16.36 = 50.9$.
   Profits of each firms are $\Pi^* = P^*q^* - C(q^*) = 416.5 - 148.7 = 267.8$.

   (b) (6p) Collusion. Now firms agree on the level of output that maximizes joint profits. They set total output like a monopolist would, i.e. set $MR = MC$, and then divide the monopoly profits.
   Marginal revenue is $MR(Q) = \frac{\partial}{\partial Q}((100 - 3Q)Q) = 100 - 6Q$.
   Marginal cost for the pair of firms is not the same as for a single firm! The cost function $C(q) = 10q + q^2$ exhibits increasing marginal cost: $MC(q) = \frac{\partial}{\partial q}C(q) = 10 + 2q$ is increasing in q. Therefore it is always cheaper to divide the production of any amount of output equally between the two firms: $2C(\frac{Q}{2}) < C(Q)$. Marginal cost under collusion is therefore

$$MC^m(Q) = \frac{\partial}{\partial Q}\left(2C(\frac{Q}{2})\right) = \frac{\partial}{\partial Q}2\left(10\frac{Q}{2} + (\frac{Q}{2})^2\right) \tag{12}$$
$$= 10 + Q. \tag{13}$$

   Setting $MR = MC$:

$$100 - 6Q = 10 + Q \tag{14}$$
$$Q^m = \frac{90}{7} = 12.86 \tag{15}$$
$$P^m = 100 - 3Q^m = 61.42 \tag{16}$$

Each firm will produce half of output so $q^m = 0.5 \times 12.86 = 6.43$ and make profits $\Pi^m = P^m q^m - C(q^m) = 394.9 - 105.6 = 289.3$, which is more than in the Cournot competition case above.

(c) (6p)

What are the profits of a cheating firm? It maximizes (this period's) profits, taking it as given that the other firm produces the collusive output q^m. Therefore the optimal level output for the cheater is given by the reaction function:

$$q^{ch} = R(q^m) = \frac{1}{8}(90 - 3 \times 6.43) = 8.84. \tag{17}$$

Total output is then $Q^{ch} = q^{ch} + q^m = 15.3$ and output price $P^{ch} = 100 - 3 \times Q^{ch} = 54.2$. The profits of the cheater are

$$\Pi^{ch} = P^{ch} q^{ch} - C(q^{ch}) = 479 - 166.5 = 312.5. \tag{18}$$

If either firm cheats than the collusion breaks down and both firms will only make the Cournot profits in the future. So a cheater faces a trade-off between a gain $\Pi^{ch} - \Pi^m$ this period and loss $\Pi^m - \Pi^*$ every period in the future.

If the firm cheats then the present value of its profits are

$$\Pi^{ch} + \frac{\Pi^*}{1+r} + \frac{\Pi^*}{(1+r)^2} + \cdots = \Pi^{ch} + \frac{\Pi^*}{r}. \tag{19}$$

If the firm colludes the present value of profits is

$$\Pi^m + \frac{\Pi^m}{1+r} + \frac{\Pi^m}{(1+r)^2} + \cdots = \Pi^m + \frac{\Pi^m}{r}. \tag{20}$$

Cheating results in a higher present value of profits if

$$\Pi^{ch} + \frac{\Pi^*}{r} > \Pi^m + \frac{\Pi^m}{r} \tag{21}$$

$$\Longrightarrow \Pi^{ch} - \Pi^m > \frac{\Pi^m - \Pi^*}{r}. \tag{22}$$

Cheating would be attractive if

$$r^* > \frac{\Pi^m - \Pi^*}{\Pi^{ch} - \Pi^m} = \frac{289.3 - 267.8}{312.5 - 289.3} = \frac{21.5}{23.2} = 0.93. \tag{23}$$

Thus cheating would only be tempting if discount rate is above a whopping 93% (in that case, knowing that cheating is attractive, firms would not collude in the first place). The length of periods in the model is not necessarily a year: it is the time that it takes for the cheated firm to react. Only a very long reaction time could justify this discount rate.

(d) (6p) The firm moving first ("the Leader") knows how the other firm ("the Follower") will react to its output. It maximizes profits, knowing that the follower will find it optimal to react according to the reaction function $R()$.

$$\Pi_1(q_1, R(q_1)) = \left(100 - 3\left(q_1 + \left(\frac{1}{8}(90 - 3q_1)\right)\right)\right) q_1 - \left(10q_1 + q_1^2\right) \tag{24}$$

This can be simplified to $\Pi_1(q_1, R(q_1)) = \frac{1}{8}(450 - 23q_1) q_1$. Solving the maximization yields

$$\begin{aligned}
\frac{\partial}{\partial q_1}\Pi_1(q_1, R(q_1)) &= 450 - 23q_1 = 0 && (25)\\
q^L &= \frac{23}{450} = 9.78 && (26)\\
&\implies q^F = R(q^L) = 7.58 && (27)\\
Q &= q^L + q^F = 17.36 && (28)
\end{aligned}$$

$$\begin{aligned}
P &= 100 - 3 \times 17.36 = 47.9 && (29)\\
\Pi^L(q^L, q^F) &= 275.1 && (30)\\
\Pi^F(q^F, q^L) &= 229.9 && (31)
\end{aligned}$$

It is better to be a the leader than the follower. This is a general result when firms compete by setting output. The follower in effect faces a smaller "residual demand" left over from the leader. Notice that $\Pi^m > \Pi^L > \Pi^* > \Pi^F$, i.e. even the leader would be better off colluding, whereas the follower is worse off than in simultaneous quantity setting.

3. Two firms producing imperfect substitutes.

Notice that the firms have identical cost functions but different demand functions: a price reduction by Firm 2 reduces the demand for the output of Firm 1 by more than vice versa.

(a) (9p) Demand for the product of Firm 1 ("your firm") depends on the prices of both firms. Profits are

$$\begin{aligned}
\Pi_1(P_1, P_2) &= P_1Q_1 - C(Q_1) && (32)\\
&= P_1Q_1 - (3 + 2Q_1) && (33)\\
&= (P_1 - 2)\,Q_1 - 3 && (34)\\
&= (P_1 - 2)(24 - 6P_1 + 3P_2) - 3 && (35)\\
&= 36P_1 - 6P_1^2 + 3P_1P_2 - 6P_2 - 51 && (36)
\end{aligned}$$

Firm 1 chooses P_1 taking P_2 as given. The first order condition is

$$\begin{aligned} \frac{\partial}{\partial P_1}\Pi_1(P_1,P_2) &= 0 \Longrightarrow 36 - 12P_1 + 3P_2 = 0 && (37) \\ &\Longrightarrow P_1^* = 3 + \frac{1}{4}P_2 \equiv R_1(P_2). && (38) \end{aligned}$$

The reaction function of Firm 1 is now the price that it will set, given P_2. Since the products have slightly different demand functions, they also have different profit and reaction functions.

$$\begin{aligned} \Pi_2(P_2,P_1) &= P_2Q_2 - (3 + 2Q_2) && (39) \\ &= (P_2 - 2)(24 - 6P_2 + 2P_1) - 3 && (40) \\ &= 36P_2 - 6P_2^2 + 2P_1P_2 - 4P_1 - 51 && (41) \end{aligned}$$

Firm 2 first order condition is

$$\begin{aligned} \frac{\partial}{\partial P_2}\Pi_2(P_2,P_1) &= 0 \Longrightarrow 36 - 12P_2 + 2P_1 = 0 && (42) \\ &\Longrightarrow P_2^* = 3 + \frac{1}{6}P_1 \equiv R_2(P_1). && (43) \end{aligned}$$

We see that firms react to price increases by their competitor by increasing their own price. Furthermore, Firm 1 is more responsive than Firm 2: it will raise its price by $\frac{1}{4}$ Dollars to every dollar increase by Firm 2, whereas Firm 2 will respond by only $\frac{1}{6}$ Dollars. This is because the demand for the product of Firm 2 is less responsive to the output of Firm 1 than vice versa.

In equilibrium neither firm wants to change their price given the price of its competitor. Therefore $P_1 = R_1(P_2)$ and $P_2 = R_2(P_1)$ must hold simultaneously.

$$\begin{aligned} P_1 &= R_1(R_2(P_1)) && (44) \\ &\Longrightarrow P_1 = 3 + \frac{1}{4}\left(3 + \frac{1}{6}P_1\right) && (45) \\ &\Longrightarrow P_1^* = \frac{90}{23} = 3.91 && (46) \\ &\Longrightarrow P_2^* = R_2(P_1^*) = \frac{84}{23} = 3.65 && (47) \end{aligned}$$

Plugging in the numbers into the profit functions yields

$$\begin{aligned} \Pi_1(P_1^*,P_2^*) &= 18.96 && (48) \\ \Pi_2(P_2^*,P_1^*) &= 13.38. && (49) \end{aligned}$$

(b) (9p) Firm 1 sets price P_1knowing that Firm 2 will then react by setting its price at $R_2(P_1)$. Maximizing profits wrt P_1:

$$\Pi_1(P_1, R_2(P_1)) = (P_1 - 2)\left(24 - 6P_1 + 3\left(3 + \frac{1}{6}P_1\right)\right) - 3$$

$$= \quad 44(P_1 - \frac{11P_1^2}{2}) - 69 \tag{50}$$

$$\Longrightarrow \quad \frac{\partial}{\partial P_1}\Pi_1(P_1, R_2(P_1)) = 44 - 11P_1 = 0 \tag{51}$$

$$\Longrightarrow \quad P_1^L = 4. \tag{52}$$

Firm 2 follows and sets its price at

$$P_2^F = R_2(P_1^L) = 3 + \frac{1}{6} \times 4 = 3.67. \tag{53}$$

Again plug in the prices into the profit functions:

$$\Pi_1(P_1^L, P_2^F) = 19.0 \tag{54}$$

$$\Pi_2(P_2^F, P_1^L) = 13.67 \tag{55}$$

Both firms are better off than if they moved simultaneously, although the follower gains more. Here both the follower and the leader set a higher price than they did under simultaneous choice, which is *not* a general result.

(c) (9p) Firms collude and choose both prices so as to maximize joint profits $\Pi(P_1, P_2) = \Pi_1(P_1, P_2) + \Pi_2(P_2, P_1)$. The first order conditions are

$$\frac{\partial}{\partial P_1}\Pi(P_1, P_2) = 0 \tag{56}$$

$$\Leftrightarrow (36 - 12P_1 + 3P_2) + (2P_2 - 4) = 0 \tag{57}$$

$$\frac{\partial}{\partial P2}\Pi(P_1, P_2) = 0 \tag{58}$$

$$\Leftrightarrow (36 - 12P_2 + 2P_1) + (3P_1 - 6) = 0 \tag{59}$$

After simplification these become

$$32 - 12P_1 + 5P_2 = 0 \text{ and } 30 + 5P_1 - 12P_2. \tag{60}$$

The solution to this pair of equations is

$$P_1^{col} = 4.49,\ P_2^{col} = 4.37. \tag{61}$$

Both firms are setting their price higher than without collusion, as could be expected. If firms simply keep their own revenue and pay their own production cost, their profits are

$$\Pi_1(P_1^{col}, P_2^{col}) = 22.3 \tag{62}$$

$$\Pi_2(P_2^{col}, P_1^{col}) = 13.0 \tag{63}$$

But this means that the profits of Firm 2 are actually lower than without collusion! Lot of the gain of the price increase by Firm 2 comes through by increasing the revenue of Firm 1. Of course Firm 2 would have no reason to agree to collusion on these terms. But since joint profits are now higher than without collusion ($22.3+13 = 35.3 > 18.4+14.8 = 33.2$) Firm 1 could transfer some of its increased profits to Firm 2 and make collusion worth its while.

4. Excessive investment (e.g. into production capacity or future cost reductions) by an incumbent monopolist reduces its profits by 1 in case the competitor enters and by 5 in case the competitor does not enter. So if both firms moved simultaneously then the incumbent would never want to choose high investment. However, when the incumbent moves first it may be a good strategy by preventing the competitor from entering at all. The payoffs are

{incumbent,competitor}	Enter	Don't enter
High investment	$9, 5-K$	$20, 0$
Low investment	$10, 10-K$	$25, 0$

The incumbent moves first, in effect choosing the row of the payoff matrix. The incumbent can then anticipate the entry-decision of the competitor: it will enter if and only if it gets a positive payoff on the row that the incumbent chose. Parameter K can be interpreted as a cost of entry.

(a) (6p) Low entry cost: $K = 1$. Now the competitor will enter for sure regardless of incumbent's investment level. The incumbent should therefore choose low investment.

(b) (6p) Moderate entry cost: $K = 6$. Now the competitor would make negative profits if it enters after high investmentand positive profits after low investment by the incumbent. The incumbent will want to invest high.

(c) (6p) High entry cost: $K = 11$. Now entry is not profitable regardless of the investment level. The incumbent does not have to invest high because the competitor will not enter anyway.

(d) (4p) This question relates to part b), where a socially wasteful investment is optimal for the monopolist by deterring the entry of a competitor. Industries where fixed costs are relatively high compared to variable costs are good examples. The crucial features.of entry-deterring overinvestment are

i) it is irreversible (or very costly to reverse), because conditional on competitor entering the incumbent would not want to invest and would be better off withdraw the investment.

ii) it reduces the profits of a possible entrant, for example by reducing the post-entry market price.

iii) it reduces incumbent's profits (otherwise it wouldn't be overinvestment).

Telecommunications, automotive industry, airplane manufacturing give good examples.