Problem Set 5

Problem 1: Suppose there is a unit mass of consumers who can purchase a product at some cost c. Each consumer i has a type v_i drawn independently from a continuous distribution F on $[0, 1]$. The good exhibits network effects. In particular, if a fraction x of the population purchases, then the value of purchasing to consumer i is

$$u_i(v_i, x) = v_i h(x) - c.$$

The value of not purchasing is zero.

(a) Suppose h is nonnegative, continuous, and strictly increasing on $[0, 1]$. Derive the best response correspondence for this game—that is, fixing $h(x)$, what fraction of the population $\hat{x}$ wishes to purchase? Prove that an equilibrium exists.

(b) Suppose $h(x) = x$ and the cumulative distribution function F for v_i takes the form

$$F(v) = \left(\frac{\gamma + \beta}{1 + \beta}\right)\left(\frac{(1 + \beta)e^{-\alpha/v}}{\gamma + \beta e^{-\alpha/v}}\right)$$

for some $\alpha > 0$ and $0 < \gamma < \beta$. Derive the best response correspondence. For what parameter values does there exist an equilibrium in which consumers purchase the product?

Problem 2: Consider another market with network effects. There is a unit mass of potential consumers who can purchase a product at some fixed price p with $0 < p < \frac{3}{16}$. Each consumer i has a private value v_i drawn independently from a uniform distribution on $[0, 1]$. If a fraction x of the population purchases the product, the consumer's payoff from purchasing is

$$u_i = v_i g(x) - p,$$

where

$$g(x) = \begin{cases} x & \text{if } x \leq \frac{1}{4} \\ \frac{1}{2} - x & \text{if } x > \frac{1}{4} \end{cases}$$

(a) Intuitively describe the kind of externalities $g(x)$ is capturing. Can you give a real life example that would fit?

(b) Characterize the set of equilibria.

(c) Which equilibria are stable? Why?

(d) Is social welfare maximized in any of the equilibria? Explain.

Problem 3: Consider the local network game with strategic substitutes from the lecture slides. Each player i chooses an action $x_i \geq 0$ and earns the payoff

$$U_i(x_i, x_{-i}, \delta, G) = b\left(x_i + \delta \sum_{j \neq i} g_{ij} x_j\right) - k x_i.$$

Assume b is such that $b'(1) = k$. Suppose G is a circle graph with four players. Compute the set of equilibria.

Problem 4: Consider the following version of the Prisoner's dilemma game:

	C	D
C	$(2, 2)$	$(-1, 6)$
D	$(6, -1)$	$(0, 0)$

In an infinitely repeated version of this game, with discount rate δ, can you construct a subgame perfect equilibrium in which the players trade off cooperating and defecting on one another? That is, on the equilibrium path, in period 1 the action profile is (C, D), in period 2 it is (D, C), in period 3 it is (C, D), and so on. How high must δ be for this to be an equilibrium? How does welfare in this equilibrium compare to the equilibrium with cooperation in every period (when this is an equilibrium)?

Problem 5: Consider the following game:

	A	B	C
A	$(3, 3)$	$(0, 4)$	$(-2, 0)$
B	$(4, 0)$	$(1, 1)$	$(-2, 0)$
C	$(0, -2)$	$(0, -2)$	$(-1, -1)$

(a) In a one-shot play of this game, what are the pure strategy equilibria?

(b) Suppose the game is played twice with no discounting ($\delta = 1$). What is the highest welfare that can be obtained in a subgame perfect equilibrium, and what is the equilibrum?