14.13 Lecture 8

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1 Bounded Rationality

Three reasons to study:

- Hope that it will generate a unified framework for behavioral economics
- Some phenomena should be captured: difficult-easy difference. It would be good to have a metric for that
- Artificial intelligence

Warning – a lot of effort spend on bounded rationality since Simon and few results: there are many attempts but none is developed in cumulative fashion.

Three directions:

- Analytical models
  - Don't get all the fine nuances of the psychology, but those models are tractable.

- Process models, e.g. artificial intelligence
  - Rubinstein (*Modelling bounded rationality*, MIT Press) direction. Suppose we play Nash, given your reaction function, my strategy optimizes on both outcome and computing cost. Rubinstein proves some existence theorems. But it is very difficult to apply his approach.

- Psychological models
  - Those models are descriptively rich, but unsystematic, and often hard to use.

Human - computer comparison (see Kurzweil, *The Age of Spritual Machine*)

- Human mind 10^{15} operations per second
- Computer 10^{12} operations per second
- Moore's law: every 1.5 years computer power doubles
- Thus, every 15 years computer power goes up 10^3
- If we believe this, then in 45 years computers can be 10^6 more powerful than humans
- Of course, we'll need to understand how human think

1.1 Analytical models

- Bounded Rationality as noise. Consumer sees a noisy signal $\tilde{q} = q + \sigma\varepsilon$ of quantity/quality q, noise $\sigma\varepsilon$ has standard deviation σ and mean 0.

- Bounded Rationality as imperfect monitoring of the state of the world. People don't think about the variables all the time. They look up variable k at times $t_1, ..., t_n$.

- Bounded Rationality as adjustment cost. Let θ denote the state of the world.
  - Now I am doing a_0 and $\kappa =$ cost of decision/change
  - I change my decision from a_0 to $a^* = \arg\max u(a, \theta_t)$ iff

$$u(a^*, \theta_t) - u(a_0, \theta_t) > \kappa$$

1.1.1 Model of Bounded Rationality as noise

- Random utility model – Luce (psychologist) and McFadden (econometrician who provided econometric tools for the models)
  - n goods, $i = 1, ..., n$.
  - Imagine the consumer chooses

$$\max_i q_i + \sigma_i \varepsilon_i$$

  - What's the demand function?

- Definition. The Gumbel distribution G is

$$F(x) = P(\varepsilon < x) = e^{-e^{-x}}$$

and have density

$$f(x) = F'(x) = e^{-e^{-x}-x}.$$

- If ε has the Gumbel distribution then $E\varepsilon = \gamma > 0$, where $\gamma \simeq 0.577$ is the Euler constant.

- Proposition 1. Suppose ε_i are iid Gumbel. Then

$$P\left(\max_{i=1,..,n} \varepsilon_i + q_i \leq \ln n + q_n^* + x\right) = e^{-e^{-x}}$$

with q_n^* defined as $e^{q_n^*} = \frac{1}{n} \sum e^{q_i}$.This means that

$$M_n = \max_{i=1,..,n} \varepsilon_i + q_i =^d \ln n + q_n^* + \eta$$

and η is a Gumbel.

Proof of Proposition 1.

- Call $I = P\left(\max_{i=1,..,n} \varepsilon_i + q_i \leq y\right)$.

- Then
$$I = P\left((\forall i)\, \varepsilon_i + q_i \leq y\right) = \Pi_{i=1}^{n} P\left(\varepsilon_i + q_i \leq y\right)$$

- Thus,
$$\ln I = \sum P\left(\varepsilon_i + q_i \leq y\right)$$
and
$$\ln P\left(\varepsilon_i + q_i \leq y\right) = \ln P\left(\varepsilon_i \leq y - q_i\right) = -e^{-(y-q_i)}.$$

- Thus

$$\ln I = \sum -e^{-(y-q_i)} = -e^{-y} \sum e^{q_i}$$

- Using

$$e^{q_n^*} = \frac{1}{n} \sum e^{q_i}$$

we have

$$\ln I = -e^{-y} n e^{q_n^*} = -e^{-[y - \ln n - q_n^*]}$$

which proves that I is a Gumbel. QED

Demand with noise

- Demand for good $n+1$ equals

$$D_{n+1}(q_1, ..., q_{n+1}) = P\left(\varepsilon_{n+1} + q_{n+1} > \max_{i=1,..,n} \varepsilon_i + q_i\right)$$

  where q_i is total quality, including the disutility of price.

- Proposition 2.

$$D_{n+1}(q_1, ..., q_{n+1}) = \frac{e^{q_{n+1}}}{\sum_{i=1}^{n+1} e^{q_i}}.$$

  In general,

$$D_j = P\left(\varepsilon_j + q_j > \max_{i \neq j} \varepsilon_i + q_i\right) = \frac{e^{q_j}}{\sum_{i=1}^{n+1} e^{q_i}}$$

Proof of Proposition 2.

- Observe that $\sum_{j=1}^{n+1} D_j = 1$.

- Note
$$D_{n+1}(q_1, ..., q_{n+1}) = P\left(\varepsilon_{n+1} > \max_{i=1,..,n} \varepsilon_i + q_i'\right)$$
where $q_i' = q_i - q_{n+1}$.

- Thus,
$$D_{n+1}(q_1, ..., q_{n+1}) = Ee^{-e^{-\left(\varepsilon_{n+1} - \ln n - q_n^*\right)}}$$

- Call $a = -\ln n - q_n^*$. Then

$$
\begin{aligned}
& D_{n+1}(q_1, ..., q_{n+1}) = E e^{-e^{-(\varepsilon_{n+1}+a)}} \\
& = \int e^{-e^{-(x+a)}} f(x)\, dx = \int e^{-e^{-(x+a)}} e^{-e^{-x}-x} dx \\
& = \int e^{-e^{-(x+a)}-e^{-x}-x} dx = \int e^{-e^{-x}(e^{-a}+1)-x} dx
\end{aligned}
$$

- Call $H = 1 + e^{-a}$ and re-write the above equation as

$$
\begin{aligned}
& D_{n+1}(q_1, ..., q_{n+1}) \\
& = \int e^{-e^{-(x-\ln H)}-x} dx \\
& = \int e^{-e^{-(x-\ln H)}-(x-\ln H)} e^{-\ln H} dx
\end{aligned}
$$

- Note that

$$\int_a^b e^{-e^{-y}-y}dy = \left[e^{-e^{-y}}\right]_a^b$$

- Thus

$$D_{n+1}\left(q_1, ..., q_{n+1}\right) = e^{-\ln H}\left[e^{-e^{-x-\ln H}}\right]_{-\infty}^{+\infty} dx = \frac{1}{H}$$

$$= \frac{1}{1+e^{-a}} = \frac{1}{1+e^{\ln n + q_n^*}} = \frac{1}{1+ne^{q_n^*}} = \frac{1}{1+\sum_{i=1}^{n} e^{q_i'}}$$

$$= \frac{1}{1+\sum_{i=1}^{n} e^{q_i - q_{n+1}}} = \frac{e^{q_{n+1}}}{e^{q_{n+1}} + e^{q_{n+1}}\sum_{i=1}^{n} e^{q_i - q_{n+1}}} = \frac{e^{q_{n+1}}}{\sum_{i=1}^{n+1} e^{q_i}}$$

QED

Demand with noise cont.

- This is called “discrete choice theory”.
  - It is exact for Gumbel.
  - It is asymptotically true for almost all unbounded distributions you can think off like Gaussian, lognormal, etc.

- Dividing total quality into quality and price components

$$D_1 = P\left(q_1 - p_1 + \sigma\varepsilon_1 > \max_{i=2,\ldots,n} q_i - p_i + \sigma\varepsilon_i\right)$$

where ε_i are iid Gumbel, $\sigma > 0$.

- Then

$$D_1 = P\left(\frac{q_1 - p_1}{\sigma} + \varepsilon_1 > \max_{i=2,\ldots,n} \frac{q_i - p_i}{\sigma} + \varepsilon_i\right) = \frac{e^{\frac{q_1 - p_1}{\sigma}}}{\sum_{i=1}^{n} e^{\frac{q_i - p_i}{\sigma}}}$$

- This is very often used in IO.

Optimal pricing. An application – example

- Suppose we have n firms, $n \gg 1$.

- Firm i has cost c_i and does

$$\max_{p_i} \left(p_i - c_i\right) D_i \left(p_1, ..., p_n\right) = \pi_i$$

- Denote the profit by π_i and note that

$$\ln \pi_i = \ln \left[(p_i - c_i) \frac{e^{\frac{q_1 - p_1}{\sigma}}}{\sum_{i=1}^{n} e^{\frac{q_i - p_i}{\sigma}}} \right]$$

$$= \ln (p_i - c_i) + \frac{q_i - p_i}{\sigma} - \ln \left(\sum_{j=1}^{n} e^{\frac{q_j - p_j}{\sigma}} \right)$$

and

$$\frac{\partial}{\partial p_i} \ln \pi_i = \frac{1}{p_i - c_i} - \frac{1}{\sigma} - \frac{-e^{-\left(\frac{q_i - p_i}{\sigma}\right)}}{\sum e^{\frac{q_j - p_j}{\sigma}}}$$

$$= \frac{1}{p_i - c_i} - \frac{1}{\sigma} + O\left(\frac{1}{n}\right)$$

- So
$$\frac{1}{p_i - c_i} - \frac{1}{\sigma} \simeq 0$$
and unit profits
$$p_i - c_i = \sigma$$

- Thus decision noise is good for firms' profits. See Gabaix-Laibson "Competition and Consumer Confusion"

- Evidence: car dealers sell cars for higher prices to women and minorities than to white men. Reason: difference in expertise. There is lots of other evidence of how firms take advantage of consumers. See paper by Susan Woodward on mortgage refinancing markets: unsophisticated people are charged much more than sophisticated people.