14.126 GAME THEORY

PROBLEM SET 3

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Question 1

Apply the forward-induction iterative elimination procedure described below to the following game. Two players, 1 and 2, have to play the Battle of the Sexes (BoS) game with the following payoff matrix

	A	B
A	3,1	ε, ε
B	ε, ε	1,3

where ε is a small but positive number. Before playing this game, player 1 first decides whether to burn a util; if he does so, his payoffs decrease by 1 at each action profile in BoS. Then player 2 observes player 1's decision and decides whether to burn a util herself, which would reduce her payoffs by 1 for each action profile in BoS. After both players observe each other's burning decisions, they play BoS.

The iterative procedure is as follows. Let S_i be player i's pure strategy space.

- For step $t = 0$, set $S_i^0 = S_i$.
- At any step $t \geq 1$, for each player i and information set h of i, let $\Delta_i^t(h)$ be the set of all beliefs $\mu_i(h) \in \Delta(S_{-i}^t)$ such that $\mu_i(s_{-i}|h) > 0$ only if h can be reached by some strategy in $S_i \times S_{-i}^t$. For each $s_i \in S_i^t$, eliminate s_i if there exists an information set h for player i such that s_i is not sequentially rational at h with respect to any belief $\mu_i(h) \in \Delta_i^t(h)$. Let S_i^{t+1} denote the set of remaining strategies.
- Iterate until no further elimination is possible.

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Question 2

(a) Consider the repeated game $RG(\delta)$, where the stage game is matching pennies:

	H	T
H	1,-1	-1,1
T	-1,1	1,-1

For any discount factor $\delta \in (0, 1)$, find all the subgame-perfect equilibria of the repeated game.

(b) A game $G = (N, A, u)$ is said to be a *zero-sum game* if $\sum_{i\in N} u_i(a) = \sum_{i\in N} u_i(a')$ for all $a, a' \in A$. For any discount factor $\delta \in (0, 1)$ and any two-player zero-sum game, compute the set of all payoff vectors that can occur in an SPE of the repeated game $RG(\delta)$.

Question 3

Consider the three-player coordination game shown below.

	A	B
A	1,1,1	0,0,0
B	0,0,0	0,0,0

A

	A	B
A	0,0,0	0,0,0
B	0,0,0	1,1,1

B

Show that each player's minmax payoff is 0, but that there is $\varepsilon > 0$ such that in every SPE of the repeated game $RG(\delta)$, regardless of the discount factor δ, every player's payoff is at least ε. Why does this example not violate the Fudenberg-Maskin folk theorem?

Question 4

Consider a repeated game with imperfect public monitoring. Assume that the action space and signal space are finite. Let $E(\delta)$ be the set of expected payoff vectors that can be achieved in perfect public equilibrium, where public randomization is available each period. Show that if $\delta < \delta'$, then $E(\delta) \subseteq E(\delta')$.

Question 5

Consider a two-player, infinitely repeated game in which players maximize average discounted value of stage payoffs with discount factor $\delta \in (0, 1)$. At each date t, simultaneously each player i invests $x_{i,t} \in \{0,1\}$ in a public good, $y_t \in \{0,1\}$, where

$$\mathbb{P}\left(y_t = 1 | x_{1,t}, x_{2,t}\right) = \begin{cases} 2/3 & \text{if } x_{1,t} + x_{2,t} = 2 \\ 1/2 & \text{if } x_{1,t} + x_{2,t} = 1 \\ r & \text{if } x_{1,t} + x_{2,t} = 0 \end{cases}$$

where $r \in (1/3, 5/12)$ is a parameter. The stage payoff of player i is $4y_t - x_{i,t}$.

(1) Assuming that all the previous moves are publicly observable, compute the most efficient symmetric subgame-perfect equilibrium (for each $\delta \in (0,1)$).

(2) Assume the previous levels of public goods (i.e., y_s with $s < t$) are publicly observable but individual investments are not. Find the range of δ under which the grim trigger strategy profile is a public perfect equilibrium (Grim trigger: $x_{1,t} = x_{2,t} = 0$ if y has ever been 0 and $x_{1,t} = x_{2,t} = 1$ otherwise).

(3) In part (b), find the range of δ under which the following is a public perfect equilibrium: start with $x_{1,t} = x_{2,t} = 1$, and for any $t > 0$, select $x_{1,t} = x_{2,t} = y_{t-1}$.

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