14.126 GAME THEORY

PROBLEM SET 1

MIHAI MANEA

Question 1

Provide an example of a 2-player game with strategy set $[0, \infty)$ for either player and payoffs continuous in the strategy profile, such that no strategy survives iterated deletion of strictly dominated strategies ($S^\infty = \emptyset$), but the set of strategies remaining at every stage is nonempty ($S^k \neq \emptyset$ for $k = 1, 2, \ldots$).

Question 2

In the normal form game below player 1 chooses rows, player 2 chooses columns, and

A

	L	R
U	9	0
D	0	0

B

	L	R
U	0	9
D	9	0

C

	L	R
U	0	0
D	0	9

D

	L	R
U	6	0
D	0	6

player 3 chooses matrices. We only indicate player 3's payoff. Show that action D is not a best response for player 3 to any independent belief about opponents' play (mixed strategy for players 1 and 2), but that D is not strictly dominated. Comment.

Question 3

Each of two players $i = 1, 2$ receives a ticket with a number drawn from a finite set Θ_i. The number written on a player's ticket represents the size of a prize he may receive. The two prizes are drawn independently, with the value on i's ticket distributed according to F_i. Each player is asked simultaneously (and independently) whether he wants to exchange

Date: February 17, 2016.

his ticket for the other player's ticket. If both players agree then the prizes are exchanged; otherwise each player receives his own prize. Find all Bayesian Nash equilibria (in pure or mixed strategies).

Question 4

A game $G = (N, S, u)$ is said to be symmetric if $S_1 = S_2 = \cdots = S_n$ and there is some function $f : S_1 \times S_1^{n-1} \to \mathbb{R}$ such that $f(s_i, s_{-i})$ is symmetric with respect to the entries in s_{-i}, and $u_i(s) = f(s_i, s_{-i})$ for every player i.

(1) Consider a symmetric game $G = (N, S, u)$ in which S_1 is a compact and convex subset of a Euclidean space and u_i is continuous and quasiconcave in s_i. Show that there exists a symmetric pure-strategy Nash equilibrium (i.e. a pure-strategy Nash equilibrium where every player uses the same strategy).

(2) Suggest a definition for symmetric Bayesian games, $G = (N, A, \Theta, u, T, p)$, and find broad conditions on such a game G that ensure that G has a symmetric Bayesian Nash equilibrium.

(3) Consider a Cournot oligopoly with inverse-demand function P and a cost function γ that is common to all firms. Each firm's cost depends on its production level and its idiosyncratic cost parameter, which is drawn from a finite set C. Assume the vector of cost parameters $(c_1, \ldots, c_n)$ is symmetrically distributed. Each firm i privately knows its own cost c_i, but not the others' costs, and independently chooses a quantity q_i to produce. Find conditions on P and γ that guarantee existence of a symmetric Bayesian Nash equilibrium in this game. (Note that the profit of each firm i is $q_i P(q_1 + \cdots + q_n) - \gamma(q_i, c_i)$.)

Question 5

Let $N = \{0, 1, \ldots, n\}^2$ be a two dimensional grid. Say that two points (x, y) and (x', y') in N are *neighbors* if $|x - x'| + |y - y'| = 1$. At each point $i \in N$, there is a firm, also denoted by i. As in a Cournot oligopoly, simultaneously, each firm i chooses a quantity $q_i \in [0, 1]$ to produce at zero marginal cost, and sells at price

$$P_i(\theta, q, \alpha) = \theta - q_i - \sum_{k=1}^{\infty} \alpha^{k-1} \left(\sum_{j \in N_i^k} q_j / \left| N_i^k \right| \right)^k .$$

Here, $\theta \in [1,2]$ is a common demand parameter, and $\alpha \in [0,1)$ is an interaction parameter with respect to distant neighbors. N_i^k is the k-th iterated set of neighbors of i: thus N_i^1 is the immediate neighbors of i (e.g., $N_{(0,0)}^1 = \{(1,0),(0,1)\}$), N_i^2 is the neighbors of neighbors of i (e.g., $N_{(0,0)}^2 = \{(0,0),(0,2),(2,0),(1,1)\}$), and so on. The payoff of firm i is its profit: $q_i P_i$.

The value of α is common knowledge, but θ is unknown, drawn from some finite set $\Theta \subseteq [1,2]$. The players' information about θ is represented by a finite type space T, with some joint prior $p \in \Delta(\Theta \times T)$.

(1) For any choice of a Bayesian Nash equilibrium $q_\alpha^* : T \to [0,1]^N$ of the above Bayesian game (for each α), and for any $t_i \in T_i$, find $\lim_{\alpha \to 0} q_\alpha^*(t_i)$.

[It suffices to find a formula that consists of iterated expectations of the form $E_{ij_1 \cdots j_k}[\theta | t_i] \equiv E[E[\cdots E[\theta | t_{j_k}] \cdots | t_{j_1}] | t_i]$, where $i, j_1, \ldots, j_k \in N$. Your formula does not need to be in closed form, but it should not refer to q^*.]

(2) Simplify your result in part (a) under the assumption that $E_{ij}[\theta | t_i] = E[\theta | t_i]$ for all i, j, and t_i.