Critiques & Alternatives to Expected Utility Theory

14.123 Microeconomic Theory III
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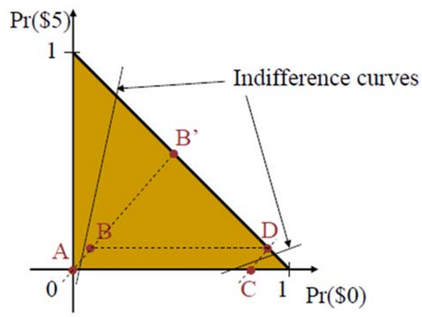
Calibration Paradox

- Expected utility theory implies that DM is approximately risk neutral against small risks for “almost all” initial wealth levels.
- Actual decision makers may reject small gambles with positive expected gain for a range of initial wealth levels.
- Implied risk aversion leads to extreme risk-aversion against large risks:
- CARA DM who rejects a gamble ($1,-$1;0.6,0.4), rejects any gamble that has $2 loss with probability ½.
- DM who rejects ($1,-$1;0.6,0.4) for wealth levels in [w-$100, w+$100] requires nearly ∞ gain to compensate a $100,000 loss with probability 1/2 at wealth w.

Allais Paradox

- Choose A or B, then C or D.
  - (A) Win $1 million for sure.
  - (B) Win $5M with 10% chance, $1M with 89%, nothing with 1%.
  - (C) Win $1M with 11% chance, nothing with 89%.
  - (D) Win $5M with 10% chance, nothing with 90%.
- Choice of A and D violates expected utility

Allais Paradox, Graphically



"Common consequence" paradox: A ≻ B but D ≻ C.

"Common ratio" paradox: A ≻ B' but D ≻ C.

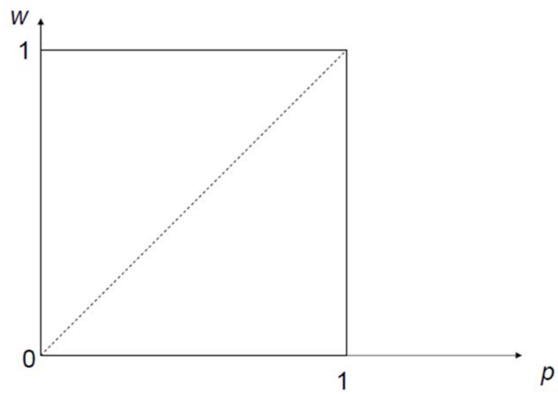
Resolutions

- Indifference curves *fan out.*
- Betweenness without Independence
- Weighted Expected Utility:

  $W(p) = \Sigma_{x\in X}\, \gamma(x)p(x)u(x)/[\Sigma_{x\in X}\, \gamma(x)p(x)]$.
- Rank-Dependent Expected Utility

  $R(p) = \int \mathrm{u}(x)\ dw(p(x))$.
- And many others

Probability Weighting Function



Ellsberg Paradox

- An urn contains 99 balls, colored, Red, Black and Green
- There are 33 Red balls;
- The combination of the other colors is not known.
- You choose a color and we draw a ball.
- If the ball is of the color chosen, you win $1. What color would you choose?
- If the ball is **not** of the color chosen, you win $1.
- What color would you choose?

Resolution: Ambiguity Aversion

- Compounded lotteries are not reduced to simple lotteries
- Ambiguity aversion:

  $\max_a \min_p E_p[u(a)]$
- Smooth ambiguity aversion:

  $\max_a E[v(E_p[u(a)])]$

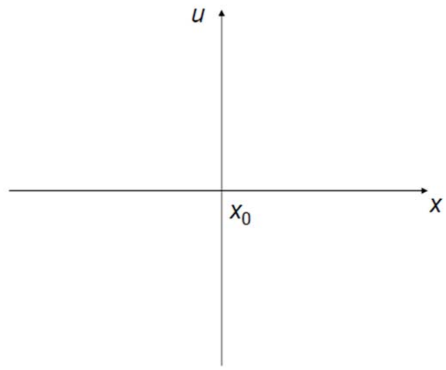
Framing

- "Outbreak of disease is about to kill 600 people. Choose treatment program A or B; then C or D."
  - (A) 400 people die.
  - (B) Nobody dies with 1/3 chance, 600 people die with 2/3 chance.
  - (C) 200 people saved.
  - (D) All saved with 1/3 chance, nobody saved with 2/3 chance.
- 78% of subjects pick B, 28% of subjects (in different group) pick D. But A is equivalent to C, B is equivalent to D (apart from wording).

Prospect Theory

- "Edit the decision problem"
- Distort the probabilities using inverted S shape
- Apply a reference-dependent S shaped utility function
  - Risk aversion towards gains
  - Risk taking towards losses
  - "Loss aversion"

Prospect Theory
Reference-dependent Utility Function



Prospect Theory Formula

- $U(x|w,x_0) = \int u(x|x_0)dw(F(x))$
- Properties & Problems:
  - What is reference point?
  - Framing
  - Dynamic Programming