14.122 Final Exam

Answer <u>all</u> questions. You have 3 hours in which to complete the exam.

1. (60 Minutes – 40 Points) Answer each of the following subquestions briefly. Please show your calculations and provide rough explanations where you can't give formal statements so I can give you partial credit.

(a) Give a formal definition of what it means for a multistage game with observed actions to be continuous at infinity? Why do we care whether games are continuous at infinity?

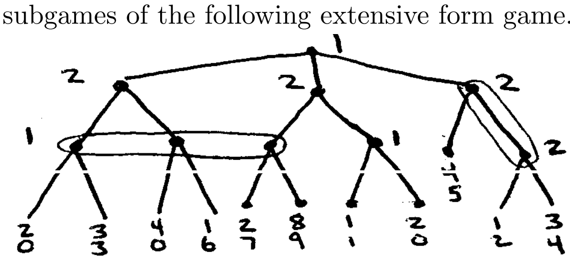
(b) Is the game below solvable by iterated strict dominance? Does it have a unique Nash equilibrium?

	A	B	C
U	1,4	5,0	0,2
D	0,0	4,7	2,2

(c) State Kakutani's theorem. What correspondence is it applied to in the proof that any finite game has a Nash equilibrium? Where does the argument break down if you try to use Kakutani's theorem in the same way to prove the existence of an equilibrium in the "Name the Largest Number" game?

(d) Is the following statement true or false: In generic finite normal form games player 1's equilibrium payoff is positive.

(e) Find all of the subgames of the following extensive form game.



(f) Given an example of a game in which you could argue that the subgame perfect equilibrium concept is too restrictive and rules out a reasonable outcome. Give an example of a game in which you could argue that the subgame perfect equilibrium concept is not restrictive enough and fails to rule out an unreasonable outcome. (Explain briefly what you would argue about each example.)

(g) Find all of the Nash equilibria of the following game.

	A	B	C
U	1,5	4,0	4,1
M	4,1	1,1	2,3
D	3,0	0,2	1,0

(h) Find the Nash equilibrium of the simultaneous move game where player 1 chooses $a_1 \in \Re$, player 2 chooses $a_2 \in \Re$, and the payoffs are

$$u_1(a_1, a_2) = -(a_1 - 1)^2 - \left(a_1 - \frac{a_2 + 2}{2}\right)^2$$

and

$$u_2(a_1, a_2) = -(a_2 - 3)^2 - \left(a_2 - \frac{a_1 + 2}{2}\right)^2.$$

(i) Suppose that in class I presented the following slight variant on Spence's job market signalling model. Nature first chooses the ability $\theta \in \{2, 3\}$ of player 1 (with both choices being equally likely). Player 1 observes θ and chooses $e \in \{0, 1\}$. Player 2 then observes e and chooses $w \in \Re$. The players' utility functions are $u_1(e, w; \theta) = w - ce/\theta^2$ and $u_2(e, w; \theta) = -(w - \theta)^2$. For $c = 4.25$ this model has both a pooling equilibrium:

$s_1^*(\theta = 2) = 0$	$s_2^*(e = 0) = 2.5$	$\mu_2(\theta = 2 \vert e = 0) = 0.5$
$s_1^*(\theta = 3) = 0$	$s_2^*(e = 1) = 2$	$\mu_2(\theta = 2 \vert e = 1) = 1$

and a separating equilibrium:

$s_1^*(\theta = 2) = 0$	$s_2^*(e = 0) = 2$	$\mu_2(\theta = 2 \vert e = 0) = 1$
$s_1^*(\theta = 3) = 1$	$s_2^*(e = 1) = 3$	$\mu_2(\theta = 2 \vert e = 1) = 0$

Suppose that after class two students come up to you in the hallway and ask you to settle an argument they are having about whether the equilibria fail the Cho-Kreps Intuitive Criterion. Assume that Irving argues

> The pooling equilibrium violates the intuitive criterion. The $\theta = 3$ type could make a speech saying 'I am choosing $e = 1$. I know you are supposed to believe that anyone who gets an education is the low type, but this is crazy. The $\theta = 2$ type would be worse off switching to $e = 1$ even if you did choose $w = 3$. I, on the other hand, being the $\theta = 3$ type will be better from having switched to $e = 1$ if you choose $w = 3$. Hence you should believe that I am the high type and give me a high wage.'

Freddy argues

> The separating equilibrium violates the intuitive criterion. It is inefficient. Before he learns his type player 1 could make a speech saying. 'This equilibrium is crazy. Education is of no value, yet with some probability I am going to have to incur substantial education costs. This is entirely due to your arbitrary belief that if I get no education I am the low type. If you instead believed that I was the low type with probability one-half in this case, then the return to education would be sufficiently low so as to allow the inefficiency to be avoided. Moreover, this more reasonable belief would turn out to be correct as I would then choose $e = 0$ regardless of my type.'

What would you tell them?

2. (40 Minutes – 20 Points)

Consider the following multistage game. Player 1 first has to choose how to divide \$2 between himself and player 2 (with only integer divisions being possible). Both players observe the division, and they then play the simultaneous move game with the dollar payoffs shown below.

	A	B	C
U	X,X	0,0	-2,-2
D	0,0	1,1	-2,-2

Assume that each player is risk neutral and has utility equal to the sum of the number of dollars he or she receives in the divide the dollar game and the dollar payoff he receives in the second stage game

(a) Draw a tree diagram to represent the extensive form of this game. How many pure strategies does each player have in the normal form representation of this game?

(b) Show that for any x the game has a Nash equilibrium in which player chooses to give both dollars to player 2 in the initial divide-the-two-dollars game.

(c) For what values of x will the game have an unique subgame perfect equilibrium?

(d) For what values of x is there a subgame perfect equilibrium in which player 1 gives both dollars to player 2 in the initial divide-the-two-dollars game.

(e) Can the game have a subgame perfect equilibrium in which player 1's total payoff is less than 2?

3. (40 Minutes – 20 Points)

Harvard and MIT are both considering whether to admit a particular student to their economics Ph.D. programs. Assume that MIT has read the student's application carefully and knows the quality q of the student. Assume that Harvard faculty members are too busy to read applications carefully. Instead they must base their decisions on their prior about the student's ability. Harvard's prior is that q may be 1, 2 or 3 and that each of these values is equally likely.

Assume that each school must make one of two decisions on the student: admit with financial aid or reject (the student has no source of support and could not attend graduate school without financial aid). The schools make these decisions simultaneously.

Assume that each school's payoff in the game is 0 if they do not offer the student admission, -1 if the student is offered admission and turns them down (this is costly both because the school loses prestige and because the slot could have been given to another student), and $q - 1.5$ if the student is offered admission and decides to come.

Assume that if the student is admitted to both schools she chooses to come to MIT with probability 0.65 and to go to Harvard with probability 0.35.

In the following questions treat this as a two player game between Harvard (player 1) and MIT (player 2).

(a) What type spaces Θ_1 and Θ_2 would you use to represent this situation as a static game of incomplete information? How many elements are in each set? Write down the values of the utility functions $u_i(a_1, a_2; \theta_1, \theta_2)$ for a couple values of $i, a_1, a_2, \theta_1, \theta_2$ to illustrate how to compute them. How many pure Bayesian strategies does each player have?

(b) What actions are strictly (conditionally) dominated for each possible type of each player?

(c) Find the Bayesian Nash equilibrium of this game.

(d) Would Harvard be any better off if it could observe MIT's admission decision before making its decision?

4. (40 Minutes – 20 Points)

Consider the following model in which a worker's choice of health play may signal his health to his insurance company. Suppose nature first chooses the health of the worker choosing $\theta \in \{\text{healthy}, \text{not healthy}\}$. Assume that the probability that the worker is healthy is q. Player 1 observes whether he is healthy and then chooses $a_1 \in \{\text{HMO}, \text{full insurance}\}$. Player 2, the competitive insurance market, observes a_1 and then chooses a price p for the chosen plan.

Assume that the HMO plan will always pay half of the worker's health care expenses, while the full insurance plan will pay all of the worker's health care expenses. Assume that the worker's expected health care expenses are \$1000 if the worker is healthy and \$2000 if the worker is not healthy. Assume that the worker is risk neutral and that his expected utility from each health plan is equal to 2000 minus the sum of p and his unreimbursed health care expenditures, i.e.

$$\begin{aligned}
u_1(\text{HMO}, p; \text{healthy}) &= 1500 - p \\
u_1(\text{full insurance}, p; \text{healthy}) &= 2000 - p \\
u_1(\text{HMO}, p; \text{not healthy}) &= 1000 - p \\
u_1(\text{full insurance}, p; \text{not healthy}) &= 2000 - p
\end{aligned}$$

To model a competitive insurance market assume that player 2's has a quadratic utility function that makes it want to set p equal the expected payments that will be made under a plan, i.e.

$$\begin{aligned}
u_2(\text{HMO}, p; \text{healthy}) &= -(p - 500)^2 \\
u_2(\text{full insurance}, p; \text{healthy}) &= -(p - 1000)^2 \\
u_2(\text{HMO}, p; \text{not healthy}) &= -(p - 1000)^2 \\
u_2(\text{full insurance}, p; \text{not healthy}) &= -(p - 2000)^2
\end{aligned}$$

(a) Does the model have a separating PBE where only the unhealthy workers buy full insurance?

(b) For what values of q does this model have a pooling PBE where all types of player 1 buy full insurance?

(c) Suppose that rather than there being just two types of player 1 there are a continuum of possible types. In particular, assume that player 1 observes his expected health care expenditures for the year before making his health plan choice and that player 2's prior is that these are uniformly distributed on $[0, 2000]$. What kinds of equilibria seem like they might be possible in this model? Show that there is no PBE in which player 1 buys full insurance with positive probability.

(d) What does this model suggest about the dangers of a free market in health insurance? What modifications to the model would be necessary if you wanted to think about the inefficiency of health insurance more seriously?