Lecture 6
Nash Equilibrium

14.12 Game Theory

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Road Map

1. Definition
2. Examples
3. Mixed-strategy Nash Equilibrium
4. Relation to other solution concepts
5. Population Dynamics

Nash Equilibrium

Definition: A strategy-profile $s^* = (s_1^*, \ldots, s_n^*)$ is a **Nash Equilibrium** iff, for each player i, and for each strategy s_i, we have

$$u_i(s_1^*, \ldots, s_{i-1}^*, s_i^*, s_{i+1}^*, \ldots, s_n^*)$$
$$\geq u_i(s_1^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*),$$

i.e., no player has any incentive to deviate if he knows what the others play.

Hawk-Dove game

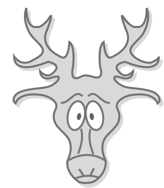
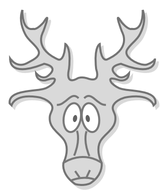
 

	$\left(\frac{V-c}{2},\frac{V-c}{2}\right)$	$(V,0)$
	$(0,V)$	$(V/2,V/2)$

V < c

Image by MIT OpenCourseWare.

Stag Hunt

		
	(2,2)	(4,0)
	(0,4)	(5,5)

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Equilibrium in Mixed Strategies

What is a strategy?

- A complete contingent-plan of a player.
- What the others think the player might do under various contingencies.
- A social convention

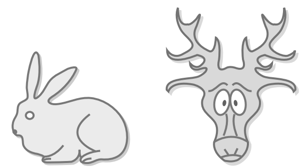
What do we mean by a mixed strategy?

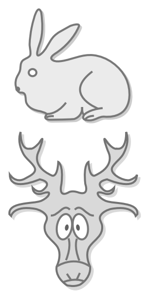
- The player is randomly choosing his pure strategies.
- The other players are not certain about what he will do.
- The distribution of the behavior in a society.

Mixed Strategy Nash Equilibrium

- A mixed strategy profile $\sigma^* = (\sigma_1^*, \ldots, \sigma_n^*)$ is a **Nash Equilibrium** iff, for each player *i*, σ_i^* is a "best response" when all the other players play according to σ^*.
- i.e. if $\sigma_i^*(s_i)>0$, s_i is a best response to σ_{-i}^*.

Stag Hunt





(2,2)	(2,0)	p
(0,2)	(3,3)	1-p

Assume: Player 2 thinks that, with probability p, Player 1 targets for Rabbit.

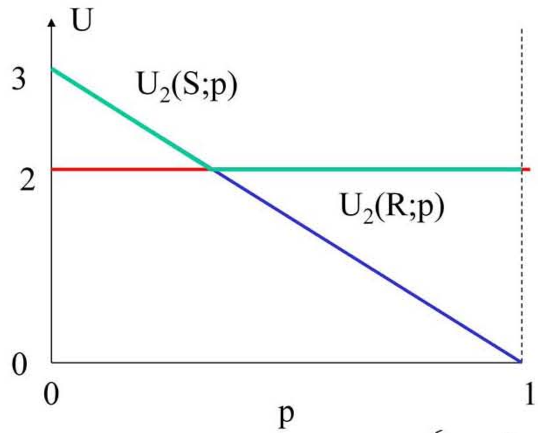
His payoff from targeting Rabbit:
$U_2(R;p) = .$

His payoff from targeting Stag:
$U_2(S;p) =$

She is indifferent iff

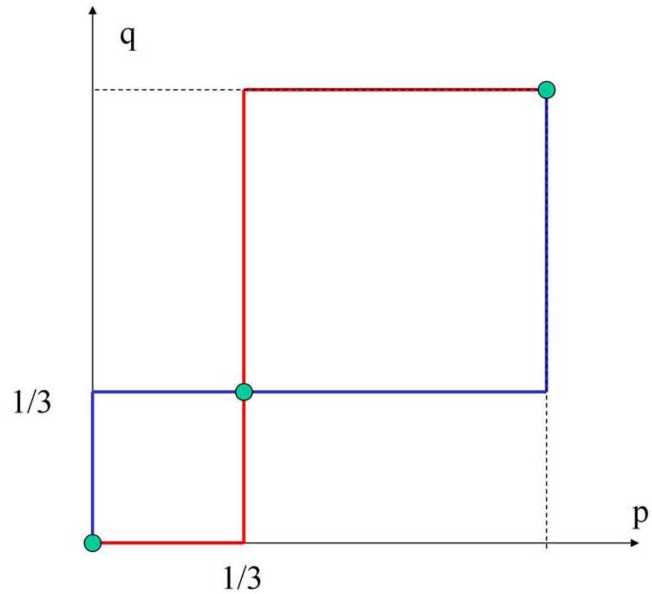
Image by MIT OpenCourseWare.

Mixed-strategy equilibrium in Stag-Hunt game



$$q^{BR}(p) = \begin{cases} 0 & \text{if } p < 1/3 \\ q \in [0,1] & \text{if } p = 1/3 \\ 1 & \text{if } p > 1/3 \end{cases}$$

Best responses in Stag-Hunt game



Relation to Other Solution Concepts

- Dominant Strategy => Nash Equilibrium
- Nash Equilibrium => Rationalizability

Hawk-Dove game

	$\left(\frac{V-c}{2}, \frac{V-c}{2}\right)$	$(V,0)$
	$(0,V)$	$(V/2,V/2)$

- h = Pr(2 plays Hawk)
- d = Pr(2 plays Dove)
- Indifference of 1:
  $(V\text{-}c)h/2 + Vd = Vd/2$
- $h = V/c$
- Nash Equilibria =
  $(h = V/c,\ d = (c\text{-}V)/c)$
  $(h{=}0,\ d = 1)$
  $(h = 1,\ d = 0)$

Image by MIT OpenCourseWare.

Evolution of Hawks and Doves

- There are *H* hawks and *D* doves; *H* and *D* large.
- Animals are randomly matched and get "payoffs" as in left.
- The "payoff" of an animal is the number of its offsprings.
- What is the ratio of Hawks 1M years later?

14.12 Economic Applications of Game Theory
Fall 2012