

Lecture 6

Nash Equilibrium

14.12 Game Theory

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Road Map

1. Definition
2. Examples
3. Mixed-strategy Nash Equilibrium
4. Relation to other solution concepts
5. Population Dynamics





Nash Equilibrium

Definition: A strategy-profile $s^* = (s_1^*, \dots, s_n^*)$ is a **Nash Equilibrium** iff, for each player i , and for each strategy s_i , we have

$$\begin{aligned} u_i(s_1^*, \dots, s_{i-1}^*, s_i^*, s_{i+1}^*, \dots, s_n^*) \\ \geq u_i(s_1^*, \dots, s_{i-1}^*, s_i, s_{i+1}^*, \dots, s_n^*), \end{aligned}$$

i.e., no player has any incentive to deviate if he knows what the others play.

Hawk-Dove game

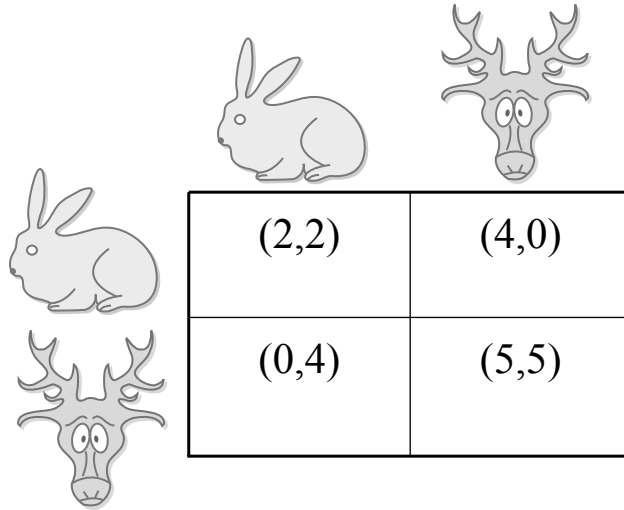
	 
	

$\left(\frac{V-c}{2}, \frac{V-c}{2}\right)$ $(V, 0)$ $V < c$

$(0, V)$ $(V/2, V/2)$

Image by MIT OpenCourseWare.

Stag Hunt



A 2x2 payoff matrix for the Stag Hunt game. The rows represent the choices of Player 1 (Rabbit or Stag) and the columns represent the choices of Player 2 (Rabbit or Stag). The payoffs are listed in the cells of the matrix.





		
	(2,2)	(4,0)
	(0,4)	(5,5)

Image by MIT OpenCourseWare.

Equilibrium in Mixed Strategies

What is a strategy?

- A complete contingent-plan of a player.
- What the others think the player might do under various contingencies.
- A social convention





What do we mean by a mixed strategy?

- The player is randomly choosing his pure strategies.
- The other players are not certain about what he will do.
- The distribution of the behavior in a society.

Mixed Strategy Nash Equilibrium

- A mixed strategy profile $\sigma^* = (\sigma_1^*, \dots, \sigma_n^*)$ is a **Nash Equilibrium** iff, for each player i , σ_i^* is a “best response” when all the other players play according to σ^* .
- i.e. if $\sigma_i^*(s_i) > 0$, s_i is a best response to σ_{-i}^* .

Stag Hunt

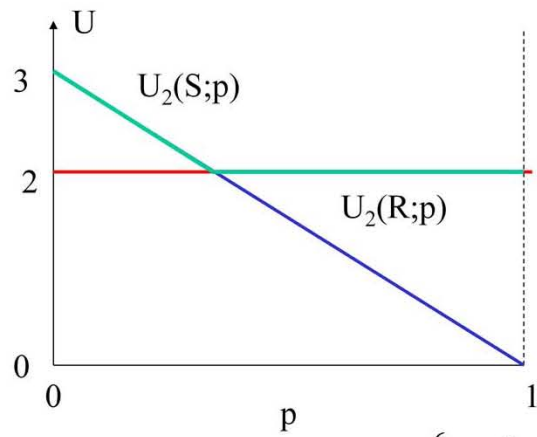
			
	(2,2)	(2,0)	p
	(0,2)	(3,3)	1-p

Assume: Player 2 thinks that, with probability p , Player 1 targets for Rabbit.

His payoff from targeting Rabbit: $U_2(R;p) = .$ His payoff from targeting Stag: $U_2(S;p) =$
 She is indifferent iff

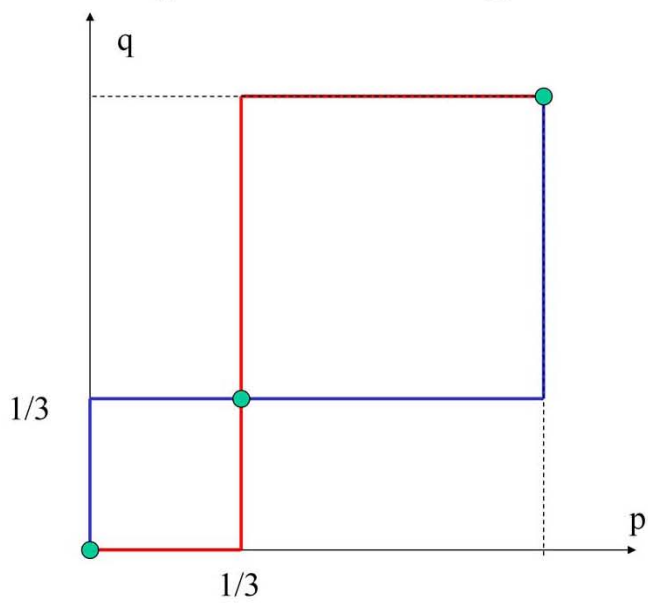
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Mixed-strategy equilibrium in Stag-Hunt game



$$q^{BR}(p) = \begin{cases} 0 & \text{if } p < 1/3 \\ q \in [0,1] & \text{if } p = 1/3 \\ 1 & \text{if } p > 1/3 \end{cases}$$

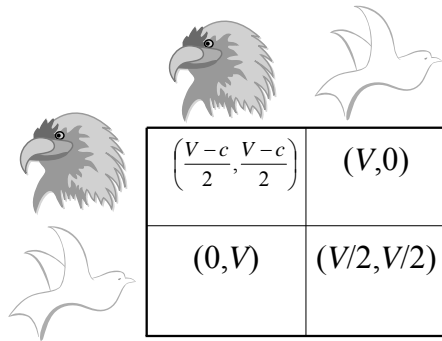
Best responses in Stag-Hunt game







Relation to Other Solution Concepts

- Dominant Strategy \Rightarrow Nash Equilibrium
- Nash Equilibrium \Rightarrow Rationalizability

Hawk-Dove game



The table shows the payoff matrix for the Hawk-Dove game. The rows represent Player 1's strategy (Hawk or Dove) and the columns represent Player 2's strategy (Hawk or Dove). The payoffs are given as (Player 1, Player 2).

		
		
	$\left(\frac{V-c}{2}, \frac{V-c}{2}\right)$	$(V, 0)$
	$(0, V)$	$(V/2, V/2)$

- $h = \text{Pr}(2 \text{ plays Hawk})$
- $d = \text{Pr}(2 \text{ plays Dove})$
- Indifference of 1:
 $(V-c)h/2 + Vd = Vd/2$
- $h = V/c$
- Nash Equilibria =
 $(h = V/c, d = (c-V)/c)$
 $(h=0, d=1)$
 $(h=1, d=0)$

Image by MIT OpenCourseWare.

Evolution of Hawks and Doves

- There are H hawks and D doves; H and D large.
- Animals are randomly matched and get “payoffs” as in left.
- The “payoff” of an animal is the number of its offsprings.
- What is the ratio of Hawks 1M years later?

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