Chapter 5

Rationalizability

A player is said to be rational if he maximizes expected value of his utility function, as described in the game. The previous lecture explored the implications of rationality. This was captured by dominance. In natural strategic environments, this often yields weak predictions. Moreover the games in which dominance alone leads to a sharp prediction (e.g. the games with a dominant strategy equilibrium) are not interesting for game theory because in such a game each player's decision can be analyzed separately without requiring a game theoretical analysis.

Nevertheless, in definition of a game, one assumes much more than rationality of the players. One further assumes that it is common knowledge that the players are rational. That is, everybody is rational; everybody knows that everybody is rational; everybody knows that everybody knows that everybody is rational ... up to infinity. If some of these assumptions fail, then one would need to consider a different game, the game that reflects the failure of those assumptions. This lecture explores the implications of the common knowledge of rationality. These implications are precisely captured by a solution concept called *rationalizability*, which is equivalent to iterative elimination of strictly dominated strategies. In this way, rationalizability precisely captures the implications of the assumptions embedded in the definition of the game.

5.1 Definition and Illustration

It is useful to illustrate the solution concept on the leading example of the previous section: (4.1). We have seen there that strategy M is strictly dominated (by a mixture of T and B) and hence it cannot be a best response to any belief. Hence, rationality of player 1 implies that Player 1 does not play M. No other strategy is strictly dominated. For example, for Player 2, her both strategies can be a best reply. If she thinks that Player 1 is not likely to play M, then she must play R, and if she thinks that it is very likely that Player 1 will play M, then she must play L. Hence, rationality of Player 2 does not put any restriction on her behavior. But, what if she thinks that it is very likely that player 1 is rational (and that his payoff are as in (4.1))? In that case, since a rational player 1 does not play M, she must assign very small probability for player 1 playing M. In fact, if she knows that player 1 is rational, then she must be sure that he will not play M. In that case, being rational, she must play R. In summary, *if Player 2 is rational and she knows that player 1 is rational, then she must play R.*

Notice that we first eliminated all of the strategies that are strictly dominated (namely M), then taking the resulting game, we eliminated again all of the strategies that are strictly dominated (namely L). This is called *twice iterated elimination of strictly dominated strategies*. In general, if a player is rational and knows that the other players are also rational (and the payoffs are as given), then he must play a strategy that survives twice iterated elimination of strictly dominated strategies.

Under further rationality assumptions, one can further iteratively eliminate strictly dominated strategies (if there remains any). In example (4.1), recall that rationality of Player 1 requires him to play T or B, and knowledge of the fact that Player 2 is also rational does not put any restriction on his behavior—as rationality itself does not restrict Player 2's behavior. Now, assume that Player 1 also knows that Player 2 is rational and that Player 2 knows that Player 1 is rational (and that the game is as in (4.1)). Then, as the above analysis shows, Player 1 must know that Player 2 will play R. In that case, being rational he must play B.

This analysis yields a mechanical procedure to analyze games, *k-times Iterated Elimination of Strictly Dominated Strategies*: eliminate all the strictly dominated strategies and iterate this *k-times*. In this procedure, one eliminates all the strictly dominated strategies and iterates this k times.

General fact: *If* (1) *every player is rational,* (2) *every player knows that every player is rational,* (3) *every player knows that every player knows that every player is rational, ... and* (k) *every player knows that every player knows that ... every player is rational, then every player must play a strategy that survives k-times iterated elimination of strictly dominated strategies.*

Caution: Two points are crucial for the elimination procedure:

1. One must eliminate only the *strictly* dominated strategies. One cannot eliminate a strategy if it is weakly dominated but not strictly dominated. For example, in the game

	L	R
T	$1, 1$	$0, 0$
B	$0, 0$	$0, 0$

   (T, L) is a dominant strategy equilibrium, but no strategy is eliminated because T does not strictly dominate B and L does not strictly dominate R.

2. One must eliminate the strategies that are strictly dominated by mixed strategies (but not necessarily by pure strategies). For example, in the game in (4.1), M must be eliminated although neither T nor B dominates M.

When there are only finitely many strategies, this elimination process must stop at some k. That is, at some k, there will be no dominated strategy to eliminate. In that case, iterating the elimination further would not have any effect.

Definition 5.1 *The elimination process that keeps iteratively eliminating all strictly dominated strategies until there is no strictly dominated strategy is called* Iterated Elimination of Strictly Dominated Strategies*; one eliminates indefinitely if the process does not stop. A strategy is said to be* rationalizable *if and only if it survives iterated elimination of strictly dominated strategies.*

As depicted in Figure 5.1, the procedure is as follows. Eliminate all the strictly dominated strategies. In the resulting smaller game, some of the strategies may become strictly dominated. Check for those strategies. If there is one, apply the procedure one more time to the smaller game. This continues until there is no strictly dominated strategy; the elimination continues indefinitely if the process does not stop. The remaining

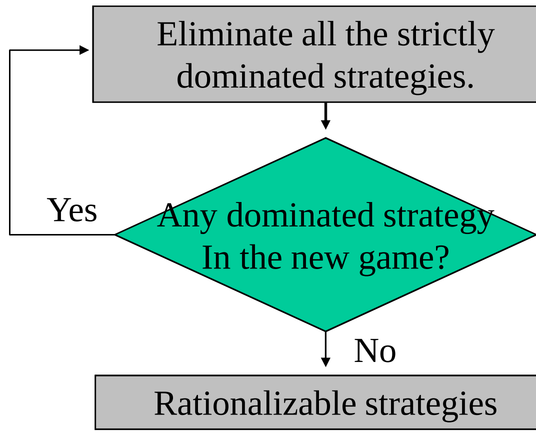

Figure 5.1: Algorithm for rationalizability

strategies are called rationalizable. When the game is finite, the order of eliminations does not matter for the resulting outcome. For example, even if one does not eliminate a strictly dominated strategy at a given round, the eventual outcome is not affected by such an omission. In that case, it is also okay to eliminate a strategy whenever it is deemed to be strictly dominated.

Theorem 5.1 *If it is common knowledge that every player is rational (and the game is as described), then every player must play a rationalizable strategy. Moreover, any rationalizable strategy is consistent with common knowledge of rationality.*

A general problem with rationalizability is that there are usually too many rationalizable strategies; the elimination process usually stops too early. In that case one cannot make much prediction based on such analysis. For example, in the Matching Pennies game

$1\backslash 2$	$Head$	$Tail$
$Head$	$-1,1$	$1,-1$
$Tail$	$1,-1$	$-1,1$

every strategy is rationalizable, and we cannot say what the players will do.

5.2 Example: Beauty Contest

Consider an n-player game in which each player i has strategies $x_i \in [0, 100]$, and payoff

$$u_i (x_1, \ldots, x_n) = -\left(x_i - \frac{2}{3} \frac{x_1 + \cdots x_n}{n} \right)^2 .$$

Notice that, in this game, each player tries to play a strategy that is equal to two thirds of the average strategy, which is also affected by his own strategy. Each person is therefore interested guessing the other players' average strategies, which depends on the other players' estimate of the average strategy.

One iteratively eliminate strictly dominated strategies as follows. First, since each strategy must be less than or equal to 100, the average cannot exceed 100, and hence any strategy $x_i > 200/3$ is strictly dominated by 200/3. Indeed, any strategy $x_i > x^1$ is strictly dominated by x^1 where[1]

$$x^1 = \frac{2(n-1)}{3n-2} 100.$$

To show that $x_i > x^1$ is strictly dominated by x^1, we fix any $(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots x_n)$ and show that

$$u_i (x_1, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots x_n) < u_i \left(x_1, \ldots, x_{i-1}, x^1, x_{i+1}, \ldots x_n \right) . \qquad (5.1)$$

By taking the derivative of u_i with respect to x_i, we obtain

$$\frac{\partial u_i}{\partial x_i} = -2 \left(1 - \frac{2}{3n} \right) \left(x_i - \frac{2}{3} \frac{x_1 + \cdots x_n}{n} \right) .$$

Clearly, $\partial u_i / \partial x_i < 0$ if

$$\left(x_i - \frac{2}{3} \frac{x_1 + \cdots x_n}{n} \right) > 0,$$

which would be the case if

$$x_i > \frac{2}{3n-2} \sum_{j \neq i} x_j \equiv x^*. \qquad (5.2)$$

Hence, u_i is strictly increasing when $x_i < x^*$ and strictly decreasing when $x_i > x^*$. On the other hand, since each $x_j \leq 100$, the sum $\sum_{j \neq i} x_j$ is less than or equal to $(n-1) 100$. Hence, it suffices that

$$x_i > \frac{2}{3n-2} (n-1) 100 = x^1.$$

[1] Here x^1 is just a real number, where superscript 1 indicates that we are in Round 1.

Therefore, for any $x_i > x^1$, we have $x^* \leq x^1 < x_i$. Since we have established that u_i is a strictly decreasing function of x_i in this region, this proves that (5.1) is satisfied. This shows that all the strategies $x_i > x^1$ are eliminated in the first round.

On the other hand, each $x_i \leq x^1$ is a best response to some $(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots x_n)$ with

$$x_i = \frac{2}{3n-2} \sum_{j \neq i} x_j.$$

Therefore, at the end of the first round the set of surviving strategies is $[0, x^1]$.

Now, suppose that at the end of round k, the set of surviving strategies is $\left[0, x^k\right]$ for some number x^k. By repeating the same analysis above with x^k instead of 100, we can conclude that at the end of round $k+1$, the set of surviving strategies is $\left[0, x^{k+1}\right]$ where

$$x^{k+1} = \frac{2(n-1)}{3n-2} x^k.$$

The solution to this equation with $x^0 = 100$ is

$$x^k = \left[\frac{2(n-1)}{3n-2}\right]^k 100.$$

Therefore, for each k, at the end of round k, a strategy x_i survives if and only if

$$0 \leq x_i \leq \left[\frac{2(n-1)}{3n-2}\right]^k 100.$$

Since

$$\lim_{k \to \infty} \left[\frac{2(n-1)}{3n-2}\right]^k 100 = 0,$$

the only rationalizable strategy is $x_i = 0$.

Notice that the speed at which x^k goes to zero determines how fast we eliminate the strategies. If the elimination is slow (e.g. when $2(n-1)/(3n-2)$ is large), then many strategies are eliminated at very high iterations. In that case, predictions based on rationalizability will heavily rely on strong assumptions about rationality, i.e., everybody knows that everybody knows that ... everybody is rational. For example, if the n is large or the ratio $2/3$ is replaced by a number close to 1, the elimination is slow and the predictions of rationalizability are less reliable. On the other hand, if n is small or the ratio $2/3$ is replaced by a small number, the elimination is fast and the predictions of rationalizability are more reliable. In particular, the predictions of rationalizability for this game is more robust in a small group than a larger group.

It is important that one analyzes the game that describes the actual situation. For example, when the above game is played in classroom, there are often some students who would rather move the mean in an unexpected direction and upset the other students than get the prize of being closest to the two thirds of the average. Those students bid 100 instead. In such experiments, the resulting outcome is often different from the rationalizable solution of 0 for the above game, which does not take into account the existence of such students. In fact, some students bid 0 in the first time they play the game and switch to relatively higher bids in the follow up games. To analyze that situation, consider the following variation.

For example, in the beauty contest game suppose that there are m mischievous students with utility function

$$u_i(x_1, \ldots, x_n) = \left(x_i - \frac{x_1 + \cdots x_n}{n}\right)^2.$$

The remaining $n - m$ students are as before. The best response of a mischievous student is 0 if the expected value of $\sum_{j \neq i} x_j/(n-1)$ is greater than 50, and it is 100 otherwise. Hence at the first round all strategies other than 0 and 100 are eliminated for the mischievous students.

For each round k there are such that x_i survives k rounds of iterated elimination for a regular student iff $\underline{x}^k \leq x_i \leq \bar{x}^k$. Note that for $k = 0$, $\underline{x}^k = 0$ and $\bar{x}^k = 100$. In the earlier rounds, both 0 and 100 are available for mischievous students, and in that case the lower bound remains $\underline{x}^k = 0$ because 0 is a best response to 0 for regular students. To compute the upper bound, fix a regular student i. The expected value of $\sum_{j \neq i} x_j$ can take any value in $[0, 100m + (n - m - 1)\bar{x}^{k-1}]$, where $100m + (n - m - 1)\bar{x}^{k-1}$ is obtained by taking the highest possible bid for each remaining students, m mischievous students playing 100 and $(n - m - 1)$ regular students playing $\bar{x}^{k-1}$. The best reply to this value give us the upper bound:

$$\bar{x}^k = \frac{2}{3n-2}[100m + (n - m - 1)\bar{x}^{k-1}] \tag{5.3}$$

which is obtained by substituting $100m + (n - m - 1)\bar{x}^{k-1}$ for $\sum_{j \neq i} x_j$ in 5.2. As above, all $x_i > \bar{x}^k$ is eliminated. Note that as $k \to \infty$, $\bar{x}^k$ converges to

$$\bar{x}^\infty = \frac{\frac{2}{3n-2} \cdot 100m}{1 - \frac{2}{3n-2}(n - m - 1)} = \frac{200m}{n + 2m} \tag{5.4}$$

(One can obtain $\bar{x}^\infty$ by substituting $\bar{x}^\infty$ for $\bar{x}^k$ and $\bar{x}^{k-1}$ in 5.3.)

The lower bound $\underline{x}^k$ depends on whether 0 remains a best response to a mischievous student. This is the case when

$$\frac{\bar{x}^k(n-m)+100(m-1)}{n-1} \geq 50$$

If $m \geq n/4$, then $\bar{x}^\infty$ satisfies the above inequality. In that case, all $\bar{x}^k$ satisfy the inequality, and neither 0 nor 100 is eliminated for the mischievous students. In that case, the rationalizable strategies are $\{0, 100\}$ for mischievous students and $[0, 200m/(n+2m)]$ for the regular students. If $m \geq n/4$, then $\bar{x}^\infty$ fails the above inequality. Then, there exists k^* such that $\bar{x}^k$ fails the inequality for every $k \geq k^*$, and $\bar{x}^k$ satisfies the inequality for all $k < k^*$. In that case at round $k^* + 1$, 0 is eliminated for mischievous students. Consequently, at round $k = k^* + 2$ and after, for any regular student i, the lowest value for $\sum_{j \neq i} x_j$ is $100m + (n-m-1)\underline{x}^{k-1}$. As in the above analysis, the best response to this yields the lower bound at k:

$$\underline{x}^k = \frac{2}{3n-2}[100m + (n-m-1)\underline{x}^{k-1}] \tag{5.5}$$

Of course, as $k \to \infty$, $\underline{x}^k$ converges to

$$\underline{x}^\infty = \bar{x}^\infty = \frac{200m}{n+2m}.$$

In that case, the unique rationalizable strategy is $200m/(n+2m)$ for regular students and 100 for the mischievous students. The rationalizable strategy is plotted in Figure 2. Note that the mischievous students have a large impact. For example, when 10% of the students are mischievous, the rationalizable strategy for regular students is $20/1.2 \cong 16.667$, and the average rationalizable bid is 25.

5.3 Exercises with Solution

1. [Homework 2, 2011] Compute the set of rationalizable strategies in the following game.

	a	b	c	d
w	$3,1$	$1,0$	$0,2$	$1,1$
x	$1,0$	$0,10$	$1,0$	$0,10$
y	$2,1$	$1,0$	$0,0$	$0,0$
z	$0,0$	$1/2,0$	$3,1$	$0,0$

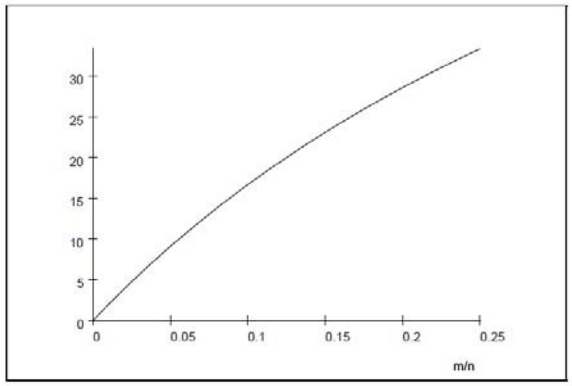

Figure 5.2: Rationalizable strategy as a function of the fraction of the mischievous students

Solution: For player 1, strategy x is dominated by a mixed strategy that puts probability 1/2 on w and probability 1/2 on z. No other strategy is dominated. After elimination of x, strategies b and d become dominated; both b and d are dominated by any strategy that puts positive probabilities on a and c and zero probability on b and d. Strategies b and d are eliminated in the second round. In the next round, y is eliminated because it becomes dominated by a mixed strategy that puts probability 1/2 on w and probability 1/2 on z. The eliminations so far leaves the following strategies:

	a	c
w	$3,1$	$0,2$
z	$0,0$	$3,1$

One can easily see that the strategy a and then w are eliminated next, yielding (z, c) as the only rationalizable strategies. The games with unique rationalizable strategy are called *dominance-solvable*. We got one of them here.

2. [Midterm 1, 2011] Compute the set of all rationalizable strategies in the following

game.

	w	x	y	z
a	0,3	0,1	3,0	0,1
b	3,0	0,2	2,4	1,1
c	2,4	3,2	1,2	10,1
d	0,5	5,3	1,2	0,10

(a) **Solution**: Strategy x is strictly dominated by the mixed strategy σ_2 with $\sigma_2(w) \in (1/3, 1/2)$ and $\sigma_2(y) = 1 - \sigma_2(w)$. In the first round, x is therefore eliminated. (No other strategy is eliminated in that round.) In the second round, d is strictly dominated by b and eliminated. In the third round, z is strictly dominated by σ_2 above and eliminated. In the fourth round, c is strictly dominated by b and eliminated. There are no other elimination, and the set of rationalizable strategies is $\{a, b\} \times \{w, y\}$.

3. [Midterm 1, 2001] Find all the pure strategies that are consistent with the common knowledge of rationality in the following game. (State the rationality/knowledge assumptions corresponding to each operation.)

1\2	L	M	R
T	$1,1$	$0,4$	$2,2$
M	$2,4$	$2,1$	$1,2$
B	$1,0$	$0,1$	$0,2$

Solution: Clearly, one needs to compute rationalizable strategies and state the underlying rationalizability assumptions along the way.

Round 1 For player 1, M strictly dominates B. Since **Player 1 is rational**, he will not play B, and we eliminate this strategy:

1\2	L	M	R
T	$1,1$	$0,4$	$2,2$
M	$2,4$	$2,1$	$1,2$

Round 2 Since **Player 2 knows that Player 1 is rational**, he knows that Player 1 will not play B. Given this, the mixed strategy that assigns probability 1/2 to each of the strategies L and M strictly dominates R. Since

Player 2 is rational, in that case, he will not play R. We eliminate this strategy:

$1\backslash 2$	L	M
T	$1,1$	$0,4$
M	$2,4$	$2,1$

Round 3 Since **Player 1 knows that Player 2 is rational and that Player 2 knows that Player 1 is rational**, he knows that Player 2 will not play R. Given this, M strictly dominates T. Since **Player 1 is rational**, he will not play T, either. We are left with

$1\backslash 2$	L	M
M	$2,4$	$2,1$

.

Round 4 Since **Player 2 knows that Player 1 is rational, and that Player 1 knows that Player 2 is rational, and that Player 1 knows that Player 2 knows that Player 1 is rational**, he knows that Player 1 will not play T or B. Given this, L strictly dominates M. Since **Player 2 is rational**, he will not play M, either. He will play L.

$1\backslash 2$	L
M	$2,4$

Thus, the only strategies that are consistent with the common knowledge of rationality are M for Player 1 and L for Player 2.

4. [Midterm 1, 2011] Compute the set of all rationalizable strategies in the following game. Simultaneously, Alice and Bob select arrival times t_A and t_B, respectively, for their meeting, where $t_A, t_B \in \{0, 1, 2, \ldots, 100\}$. The payoffs of Alice and Bob are

$$u_A(t_A, t_B) = \begin{cases} 2 - (t_A - t_B)^2 & \text{if } t_A < t_B \\ -(t_A - t_B)^2 & \text{otherwise} \end{cases}$$

$$u_B(t_A, t_B) = \begin{cases} 2 - (t_A - t_B)^2 & \text{if } t_B < t_A \\ -(t_A - t_B)^2 & \text{otherwise,} \end{cases}$$

respectively. [Note that t_A and t_B are integers between 0 and 100.]

Solution: If the set of remaining strategies from the earlier rounds is $\{0, \ldots t_{\max}\}$ for some $t_{\max} > 0$, then the $t_{\max}$ is strictly dominated by $t_{\max} - 1$ and is eliminated. (Proof: For $t_B = t_{\max}$,

$$u_A (t_{\max} - 1, t_{\max}) = 1 > 0 = u_A (t_{\max}, t_{\max}),$$

and for any $t_B < t_{\max}$,

$$u_A (t_{\max} - 1, t_B) = -(t_{\max} - 1 - t_B)^2 > -(t_{\max} - t_B)^2 = u_A (t_{\max}, t_B),$$

showing that $t_{\max} - 1$ strictly dominates $t_{\max}$ for Alice. The same argument applies for Bob.)

Therefore, we eliminate 100 in round 1, 99 in round 2, ..., and 1 in round 100. The set of rationalizable strategies is $\{0\}$ for both players.

5. [Midterm 1 make up, 2007] Consider the following game:

1\2	L	R
T	$1,1$	$1,0$
B	$0,1$	$0,10000$

   (a) Compute the rationalizable strategies.

   Solution: First B and then R are eliminated. The rationalizable strategies are T for Player 1 and L for Player 2.

   (b) Now assume that players can tremble: when a player intends to play a strategy s, with probability $\epsilon = 0.001$, Nature switches it to the other strategy s'. For instance, if player 2 plays L (or intends to play L), with probability ϵ, R is played, with probability $1 - \epsilon$, L is played. Assume that the trembling probabilities are independent. Compute the rationalizable strategies for this new game.

   Solution: Taking into the Nature's move, the new game is as follows in normal form:

1\2	L	R
T	$1-\varepsilon, 1-\varepsilon+10000\varepsilon^2$	$1-\varepsilon, \varepsilon+10000(1-\varepsilon)\varepsilon$
B	$\varepsilon, 1-\varepsilon+10000\varepsilon(1-\varepsilon)$	$\varepsilon, 10000(1-\varepsilon)^2+1-\varepsilon$

To see how the payoffs are computed consider (T, L). If this strategy profile is intended, the outcome is (T, L) with probability $(1-\varepsilon)^2$ [nobody trembles], (T, R) with probability $(1-\varepsilon)\varepsilon$ [only Player 2 trembles], (B, L) with probability $(1-\varepsilon)\varepsilon$ [only Player 1 trembles], and (B, R) with probability ε^2 [everybody trembles]. We mix the payoff vectors with the above probabilities to obtain the table. One can use the structure of payoffs to shorten the calculations. For example, Player 1 gets 1 if he does not tremble and gets 0 otherwise, yielding $1-\varepsilon$.

To compute the rationalizable strategies, note that B is still dominated by T and is eliminated in the first round. In the second round, we cannot eliminate R, however. Indeed, the payoffs from L and R are approximately 1 and 10, respectively. Hence, L is eliminated in the second round, yielding (T, R) as the only rationalizable strategy profile.

This example shows that rationalizability may be sensitive to the possibility of trembling, depending on the relative magnitude of trembling probabilities and the payoff differences.

5.4 Exercises

1. [Homework 1, 2004] Consider the following game in normal form.

	A	B	C	D
a	$0,-1$	$4,4$	$0,0$	$2,0$
b	$0,3$	$0,0$	$4,4$	$1,0$
c	$5,2$	$2,0$	$1,3$	$1,3$
d	$4,4$	$1,0$	$0,1$	$0,5$

(a) Iteratively eliminate all strictly dominated strategies; state the assumptions necessary for each elimination.

(b) What are the rationalizable strategies?

2. Compute the set of rationalizable strategies in the following game:

	A	B	C	D
a	$2,0$	$2,4$	$0,0$	$0,-1$
b	$1,-2$	$-2,-2$	$4,2$	$0,1$
c	$1,3$	$0,0$	$1,3$	$5,2$
d	$0,5$	$-1,0$	$0,1$	$4,4$

3. [Midterm 1, 2000] Consider the following game.

$1\backslash 2$	L	M	R
T	$3,2$	$4,0$	$1,1$
M	$2,0$	$3,3$	$0,0$
B	$1,1$	$0,2$	$2,3$

   (a) Iteratively eliminate all the strictly dominated strategies.

   (b) State the rationality/knowledge assumptions corresponding to each elimination.

   (c) What are the rationalizable strategies?

4. [Homework 1, 2004] Consider the game depicted in Figure 5.3 in extensive form (where the payoff of player 1 is written on top, and the payoff of 2 is on the bottom).

   (a) Write this game in strategic form.

   (b) What are the strategies that survive the *iterative elimination of weakly-dominated strategies* in the following order: first eliminate all weakly-dominated strategies of player 1; then, eliminate all the strategies of player 2 that are weakly dominated in the remaining game; then, eliminate all the strategies of player 1 that are weakly dominated in the remaining game, and so on?

5. [Homework 1, 2001] Compute the set of rationalizable strategies in the following game that is played in a class of n students where $n \geq 2$: Without discussing with anyone, each student i is to write down a real number $x_i \in [0, 100]$ on a paper and submit it to the TA. The TA will then compute the average

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

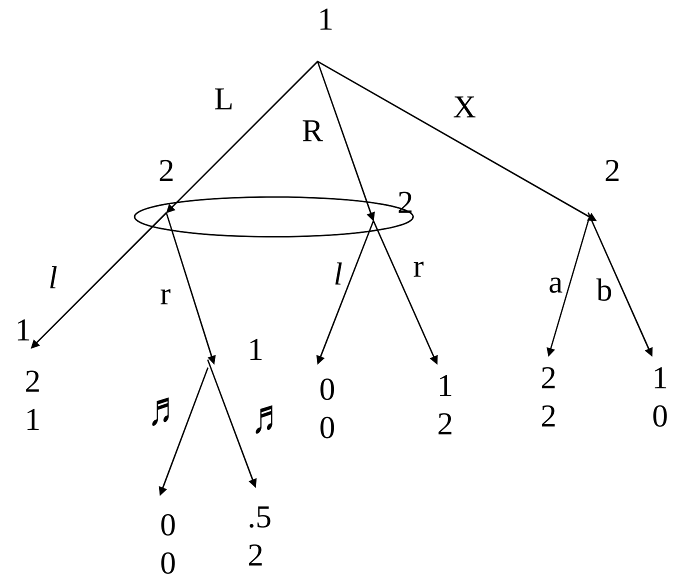

Figure 5.3:

of these numbers. The students who submit the number that is closest to $\bar{x}/3$ will share the total payoff of 100, while the other students get 0. Everything described above is common knowledge. (Bonus: would the answer change if the students did not know n, but it were common knowledge that $n \geq 2$?)

6. [Homework 2, 2011] There are n students. Simultaneously, each student i submits a real number $x_i \in [0, 100]$ and each student receives the payoff of

$$u_i(x_1, \ldots, x_n) = 100 - \left(x_i - \frac{2}{3} med(x_1, \ldots, x_n)\right)^2,$$

where med finds the median.

   (a) Write this game formally in normal form.

   (b) Compute the sets of rationalizable strategies and Nash equilibria.

   (c) Answer part (b) assuming that there are $m \in (0, (n-1)/2)$ mischievous students with payoff $(x_i - med(x_1, \ldots, x_n))^2$.

   (d) Bonus: Answer part (c) for $m \in (n/2, n)$.

7. [Midterm 1, 2007] Compute the set of all rationalizable strategies in Exercise 4 in Section 3.5.)

8. [Midterm 1, 2005] Compute the set of all rationalizable strategies in the game in Figure 3.14. (See Exercise 2 in Section 3.5.)

9. [Homework 1, 2001] Consider the game in Figure 5.4.

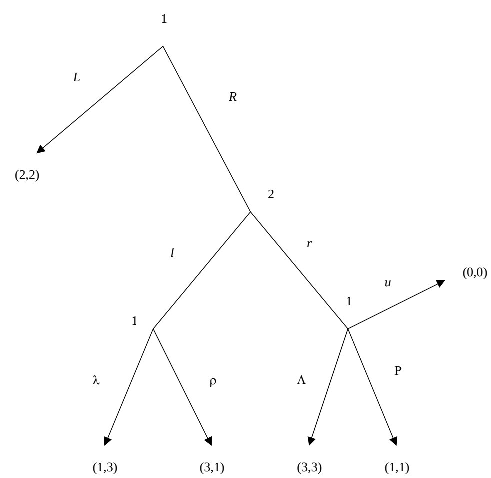

Figure 5.4:

   (a) Write this game in the strategic form.

   (b) What are the strategies that survive the *iterative elimination of weakly-dominated strategies* in the following order: first eliminate all weakly-dominated strategies of player 1; then, eliminate all the strategies of player 2 that are weakly dominated in the remaining game; then, eliminate all the strategies of player 1 that are weakly dominated in the remaining game, and so on?

10. [Homework 1, 2002] Consider the game depicted in Figure 5.5 in extensive form.

   (a) Write this game in strategic form.

   (b) What are the strategies that survive the *iterative elimination of weakly-dominated strategies* in the following order: first eliminate all weakly-dominated strategies of player 1; then, eliminate all the strategies of player 2 that are

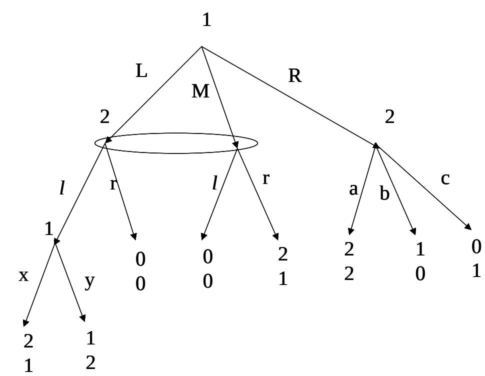

Figure 5.5:

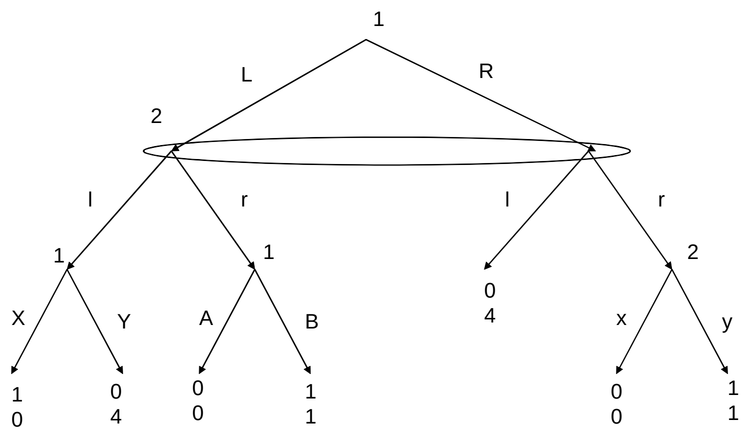

Figure 5.6:

weakly dominated in the remaining game; then, eliminate all the strategies of player 1 that are weakly dominated in the remaining game, and so on?

11. [Homework 1, 2006] Consider the game depicted in Figure 5.6 in extensive form.

    (a) Write this game in strategic form.

    (b) Iteratively eliminate all weakly dominated strategies.

    (c) What are the rationalizable strategies?

12. Consider any collection of sets $Z_1 \subseteq S_1, \ldots, Z_n \subseteq S_n$ such that there exists no $z_i \in Z_i$ that is strictly dominated when the others' strategies are restricted to be

in Z_{-i}. That is, for every $z_i \in Z_i$ and every mixed strategy σ_i of player i, there exists a strategy profile z_{-i} of other players such that $z_j \in Z_j$ for every $j \neq i$ and

$$u_i(z_i, z_{-i}) \geq \sum_{s_i \in S_i} \sigma_i(s_i)\, u_i(s_i, z_{-i}).$$

Show that each $z_i \in Z_i$ is rationalizable.

13. Show that the set of rationalizable strategies satisfy the above property that no rationalizable strategy is dominated when others' strategies are resticted to be rationalizable.

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14.12 Economic Applications of Game Theory
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