14.12 Game Theory – Midterm I
10/18/2007

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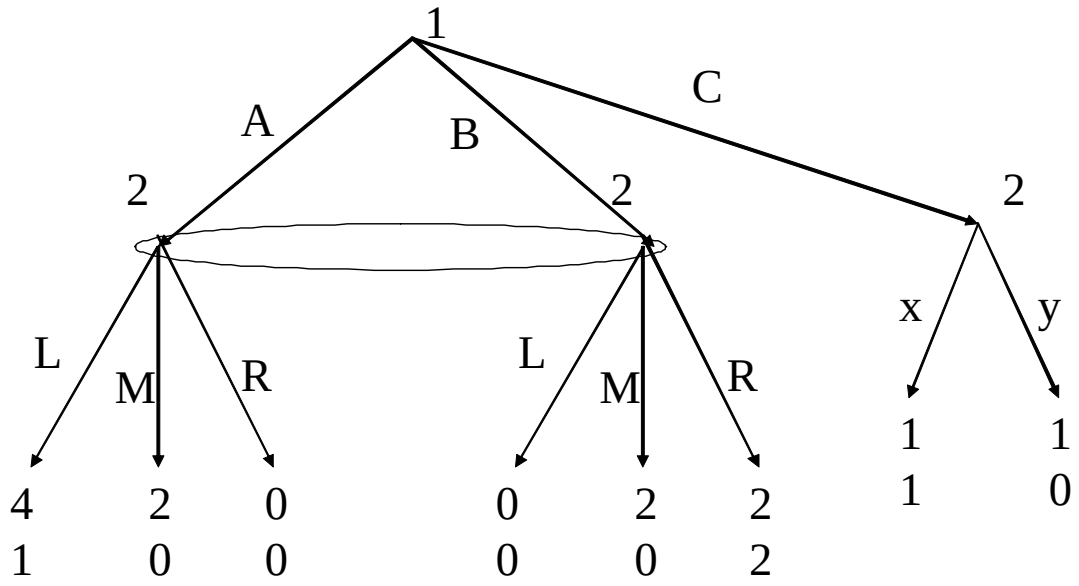
Instructions. This is an open book exam; you can use any written material. You have one hour and 20 minutes. You need to show your work when it is needed. Good luck!

1. Consider the following game.



   (a) (10 pts) Write this game in normal form.

   (b) (10 pts) Compute the set of all rationalizable strategies.

   (c) (10 pts) Find all pure strategy Nash Equilibria.

   (d) (10 pts) Compute a mixed strategy Nash equilibrium.

2. Consider the Cournot duopoly with linear demand function $P = 1 - Q$, where P is the price and $Q = q_1 + q_2$ is the total supply.[1] Firm 1 has zero marginal cost. Firm 2 has marginal cost $c(q_2) = q_2$, so that the total cost of producing q_2 is $q_2^2/2$.

   (a) (10 points) Compute all the Nash equilibria.

   (b) (15 points) Compute the set of all rationalizable strategies. Explain your steps.

3. (35 points) [**Read the bonus note at the end before you answer the question**.] This question is about arbitration, a common dispute resolution method in the US. We have a Worker, an Employer, and an Arbitrator. They want to set the wage w. If they determine the wage w at date t, the payoffs of the Worker, the Employer and the Arbitrator will be $\delta^t w$, $\delta^t (1 - w)$ and $w(1 - w)$, respectively, where $\delta \in (0, 1)$. The timeline is as follows:

[1] Recall that in Cournot duopoly Firms 1 and 2 simultaneously produce q_1 and q_2, and they sell at price P.

- At $t = 0$,
  - the Worker offers a wage w_0;
  - the Employer accepts or rejects the offer;
  - if she accepts the offer, then the wage is set at w_0 and the game ends; otherwise we proceed to the next date;
- at $t = 1$,
  - the Employer offers a wage w_1;
  - the Worker accepts or rejects the offer;
  - if he accepts the offer, then the wage is set at w_1 and the game ends; otherwise we proceed to the next date;
- at $t = 2$, the Arbitrator sets a wage $w_2 \in [0, 1]$ and the game ends.

Compute an equilibrium of this game using backward induction.

Bonus: If you solve the following variation instead, then you will get extra 10 points (45 points instead of 35 points). *Final Offer Arbitration:* At $t = 2$, the Arbitrator sets a wage $w_2 \in \{w_0, w_1\}$, i.e., the Arbitrator has to choose one of the offers made by the parties.

14.12 Economic Applications of Game Theory

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