14.12 Game Theory

Muhamet Yildiz

Fall 2012

Homework 2

1. Consider the following game.

	a	b	c	d
w	2,0	0,5	1,0	0,4
x	4,1	2,1	0,2	1,0
y	2,1	5,0	0,0	0,3
z	0,0	1,0	4,1	0,0

   (a) Compute the set of rationalizable strategies.

   We find the rationalizable strategies by iterated strict dominance.

   w is dominated by a mixed strategy putting 2/3 on x and 1/3 on z.
   a is dominated by a mixed strategy putting 3/5 on c and 2/5 on d.
   b is dominated by a mixed strategy putting 3/5 on c and 2/5 on d.
   y is dominated by a mixed strategy putting 1/2 on x and 1/2 on z.
   d is dominated by c.
   x is dominated by z.
   The set of rationalizable strategies is $\{z\} \times \{c\}$

   (b) Compute the set of all Nash equilibria.

   The only nash equilbrium is (z, c) because there is only 1 rationalizable strategy for each player

2. Consider the following game.

   (a) Find all Nash equilibria in pure strategies.

   To find the nash equilibrium of the extensive form game, we must write it as a normal form game. The cells in bold are pure strategy nash equilibria.

   (b) Find a Nash equilibrium in which Player 1 plays a mixed strategy (without putting probability 1 on any of his strategies).

   To find a mixed strategy, we look at which strategies allow player 1 to make player 2 indifferent between any of his strategies. Since we found pure NE on A and B , we look for some mixing between those two. By putting probability 3/4 on A and 1/4 on B, we make player 2 indifferent between all of his strategies. Then he can mix with a total

Table 1: Table

	Ll	Lr	Rl	Rr
A	**3,1**	**3,1**	0,0	0,0
B	0,0	0,0	**1,3**	**1,3**
C	1.5	0.5	1	0

probability of 1/4 on Ll and Lr and 3/4 probability on some combination of Rl and Rr. This makes player 1 indifferent. In addition, we must check that C is not the best response to player 2's strategy. To do that, we need $1.5\sigma(Ll)+0.5\sigma(Lr)+\sigma(Rl) < 0.75$. For this to be true, player 2 must put positive probability on Rr.

There is another set of mixed equilibria: Player 2 plays Rl and Player 1 mixes, putting probability $p \in [1/3, 1]$ on B and $1-p$ on C.

3. Use backwards induction to compute a Nash equilibrium of the following game.

   After L, player 2 plays B and player 1 plays A. Player 1's equilibrium utility from L is 3. After Rr, player 1 plays y, so after R player 2 will choose to play l. Player 1's equilibrium utility from R is 2, so player 1 will play L. The nash equilibrium from backwards induction is LAy, Bl.

4. (a) For $p > q + c(1-2x)$, all kids go to firm 2. Thus, the revenue for firm 1 and 2 are zero and q , respectively. For $p = q + c(1-2x)$, kids from $x_0 \leq x$ are indifferent and kids from $x_0 > x$ prefer firm 2. The revenue for firm 1 and 2 are $\frac{1}{2}px$ and $\frac{1}{2}px + q(1-x)$, respectively. Similarly, for $q > p+c(1-2x)$, the revenue for firm 1 and 2 are p and zero. For $q = p + c(1-2x)$, the revenue for firm 1 and 2 are $px + \frac{1}{2}p(1-x)$ and $\frac{1}{2}q(1-x)$. For $|p-q| < c(1-2x)$, we have an interior solution: there is a "mid-point" x^* such that $x < x^* < 1-x$ and kid at x^* is indifferent. In other words,

$$c|x^* - x| + p = c|x^* - (1-x)| + q$$

   Solving, we get

$$x^* = \frac{1}{2} + \frac{q-p}{2c}$$

   Note that $|p-q| < c(1-2x)$ implies $x < x^* < 1-x$. For $|p-q| < c(1-2x)$, the revenue for firm 1 (located at x) is

$$x^*p = \left\{\frac{1}{2} + \frac{q-p}{2c}\right\} \cdot p$$

   For firm 2 (located at $1-x$), the revenue is

$$(1-x^*)\,q = \left\{\frac{1}{2} + \frac{p-q}{2c}\right\} \cdot q$$

   (b) Strategy of firm 1 is to choose $p \in [0,\ \infty]$ and Strategy of firm 2 is to choose $q \in [0,\ \infty]$. Utility (payoff) of firm 1 is zero if $p > q + c(1-2x)$, p if $q > p + c(1-2x)$, $\frac{1}{2}px$ if $p =$

$q+c\,(1-2x)\,,\, px+\frac{1}{2}p(1-x)$ if $q=p+c\,(1-2x)$, and $\left\{\frac{1}{2}+\frac{q-p}{2c}\right\}\cdot p$ if $|p-q|<c\,(1-2x)$. Utility of firm 2 is 0 if $q>p+c(1-2x)$, q if $p>q+c(1-2x)$, $\frac{1}{2}qx$ if $q=p+c(1-2x)$, $qx+\frac{1}{2}q(1-x)$ if $q=p-c(1-2x)$, and $\left\{\frac{1}{2}+\frac{p-q}{2c}\right\}\cdot q$ if $|p-q|<c\,(1-2x)$.

(c) If $q \geq p+c(1-2x)$, firm 2 would deviate to $q=p+c(1-2x)-\epsilon$, where $\epsilon>0$ and ϵ is small, as

$$\left[\frac{1}{2}+\left\{\frac{p-(p+c(1-2x)-\epsilon)}{2c}\right\}\right]\{p+c(1-2x)-\epsilon\}-\frac{1}{2}\{p+c(1-2x)\}\,x>0$$

similarly, if $p \geq q+c(1-2x)$, firm 1 would deviate to $p=q+c(1-2x)-\epsilon$, where $\epsilon>0$ and ϵ is small. Thus, to search Nash equilibrium, we only need to consider the case $|p-q|<c\,(1-2x)$. Best response functions are given by the first order conditions (FOC): $q^{BR}(p)=\frac{p+c}{2}$, $p^{BR}(q)=\frac{q+c}{2}$. Solving, we get $p=q=c$. This is the unique Nash equilibrium.

14.12 Economic Applications of Game Theory
Fall 2012