Econ 14.04 Fall 2006
Assignment 4: Intertemporal Choice, Shadow Costs, Uncertainty, Exchange.
Due Monday October 23rd.

Reading assignment: Sections 10-12, 28.

1. Consider an agent who values consumption in period 0 and 1 according to the following utility function:
$$u(c_0, c_1) = \ln(c_0) + \delta \ln(c_1)$$
δ is a discount factor ($\delta \leq 1$) which indicates that the agnet prefers to consume today more than he can tomorrow. Suppose that the agent is given a total wealth today of ω and that he may save any portion of this money in order to consume tomorrow. If he saves money he is paid interest r. Thus the agents budget constraint is:
$$c_0 + \frac{c_1}{1+r} = \omega$$
   (a) By renaming variables, notice that this problem is identical to the regular cobb douglas problem.
   (b) Solve for the agents marshallian demand functions.
   (c) At what relation of δ and r will the agent consume the same amount in each period?
   (d) Suppose that instead of being given a fixed endowment, the agent instead has access to a production technology that can make two products x, y that can be sold at prices p_x, p_y. The technology is limited by inputs so that at most:
$$x^2 + y^2 = 2$$
   Solve for the optimal amount of x and y produced that maximize the agents profits.
   (e) Suppose that x is the amount of production that can be made in period 0 and y is the amount of production made in period 1. Suppose that $p_x = 1$, $p_y = 1, r = 0$ Why can we seperate the production decision from the consumption decision in this problem? Give a value for δ such that consumption is held constant in each period and the agent exactly consumes his production.

2. The legrangian multipliers in economics are often called "shadow costs". In this problem we will try to see where this term comes from and build intuition on how to use them to answer economic relevant questions.

   (a) Lets start with a very simple problem:
$$\begin{aligned} M &= \max_x 2x - x^2 \\ ST &: x \leq K \end{aligned}$$

1. Solve this problem using as a constraied optimization problem. For a given x, what is the slope of $2x - x^2$? What is the value of $\lambda(K)$ when $K < 1$? Suppose that the constraint was increased slightly to $K + \varepsilon$ with ε very small - how much should M change by?
2. Suppose that we wanted to determine how much M increases as K is increased. One obvious way to solve this problem would be to resolve the problem with the constraint changed, however we can do something else. Use the envelope theorem to show that the change in M when K goes from K_0 to K_1 is:

$$\int_{K_0}^{K_1} \lambda(K)$$

(b) *Lets take these ideas to a more economic relevant problem. Suppose that we have the following utility function:

$$\begin{aligned} v(p_x, p_y, m) &= \max_{x,y} \alpha \ln(x) + (1-\alpha)\ln(y) \\ ST &: p_x x + p_y y = m \end{aligned}$$

1. Solve for $\lambda(\alpha)$.
2. Find a general formula for how much $v(p_x, p_y, m)$ changes as m changes from m^* to m^{**}
3. Find a general formula for how much $v(p_x, p_y, m)$ changes as p_x changes from p_x^* to p_x^{**}:

3. Suppose that a safety agency is thinking of establishing a criterion under which an area prone to flooding should be evacuated. The probability of flooding is 1%. There are four possible outcomes:

(a) (A) : No evacuation is necessary, and none is performed
(B) : An evacuation is performed that is unnecessary
(C) : An evacuation is performed that is necessary
(D) : No evacuation is performed, and a flood causes a disaster

Suppose that the agency is indifferent between the sure outcome B and the lottery of A with probability p and D with probability $(1-p)$. Further assume that the agency is indifferent between the sure outcome C and the lottery of B with probability q and D with probability $(1-q)$. Suppose also that it prefers A to D and that $p \in (0,1), q \in (0,1)$

(a) Construct a utility function of the expected utility form for the agency.

(b) Consider two different policy criteria:

- Criterion 1: This criterion will result in an evacuation in 90% of the cases in which flooding will occur and an unnecessary evacuation in 10% of the cases in which no flooding occurs.

- Criterion 2: This criterion is more conservative. It results in an evacuation in 95% of the cases in which flooding will occcur and an unnecessary evacuation in 5% of the cases in which no flooding occurs.

Derive the probability distributions over the four outcomes under these two creiteria. Then, by using the utility function in (a), decide which criterion the agency would prefer.

4. Recall that the profit function is given by:

$$\pi(\mathbf{p}) = \max_{y_i(p)\in Y} \sum_{i=1}^{N} y_i(\mathbf{p})p_i$$

(a) Use the envelope theorem to show that if we are given a profit function $\pi(p)$, we can back out the input demand functions as:

$$y_i(\mathbf{p}) = \frac{\partial \pi(\mathbf{p})}{\partial p_i}$$

(b) A common notation is to use $x's$ and $w's$ to denote inputs and their prices and to use $y's$ and $p's$ to denote outputs and output prices. Consider the single input (x) and single output (y) technology described by:

$$y = \begin{cases} 0 & x < 1 \\ \ln(x) & x \geq 1 \end{cases}$$

Find the input demand function $x(p_y, w_x)$ and the profit function $\pi(p_y, w_x)$. At what ratio of prices p_y, w_x will the firm shut down and produce nothing?

5. *Suppose that we are given the production function $f(x_1, x_2) = \min(x_1, x_2)^\alpha$

(a) Draw an isoquant for the case where $\alpha = 1$, and $f(x_1, x_2) = 10$. What are the possible combinations of (x_1, x_2) that will be chosen by a cost minimizing firm?

(b) Find the input demand functions $x_1(p, \omega_1, \omega_2, \alpha), x_2(p, \omega_1, \omega_2, \alpha)$, the supply function $y(p, \omega_1, \omega_2, \alpha)$ and profit funciton $\pi(p, \omega_1, \omega_2, \alpha)$. For what values of α is $\pi < \infty$? What is this requirement called?

(c) Just like in utility maximization, we can start with a cost minimization problem instead of a profit maximization problem.

1. Start by solving a cost minimization problem:

$$\begin{aligned} c(\omega_1, \omega_2, \alpha, y) &= \min \omega_1 x_1 + \omega_2 x_2 \\ ST &: \min(x_1, x_2)^\alpha = \overline{y} \end{aligned}$$

2. From the minimization problem you will have $x_1(\omega_1, \omega_2, \alpha, y), x_2(\omega_1, \omega_2, \alpha, y), c(\omega_1, \omega_2)$. Note that these $x's$ you get here are conditional on y. We call these "conditional factor demand functions".

3. Given your function $c(\omega_1, \omega_2, \alpha, y)$, now solve:

$$\pi(p, \omega_1, \omega_2) = \max_y py - c(\omega_1, \omega_2, \alpha, y)$$

   Notice that this π is identical to the one you found in part (i). Just like in the utility case, we can solve a (typically simpler) cost minimization problem and then use our results to solve for profit maximization as a function of a single variable.

4. Calculate the supply function $y(p, \omega_1, \omega_2, \alpha)$. Plug this into the conditional factor demand functions to get the same results as part (ii).