14.03/14.003 Problem Set 4
Fall 2012

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1 Short Questions (20 points)

As always, explain your answer in one or two sentences or equations. Answers without explanations will not receive credit.

1. **(5 points)** According to the Coase theorem, agents will always reach a Pareto efficient bargain to internalize an externality if property rights over that externality are complete and bargaining is costless. One policy implication is that all agents should be indifferent among all allocations of property rights over externalities since the final outcome will always be Pareto efficient.

   False. There are distributional consequences of property rights allocations: who pays what depends on who owns what. Agents should not be indifferent.

2. **(5 points)** A second policy implication of the Coase Theorem is that the allocation of property rights has no effect on the amount of the externality produced. [Hint: Consider the ownership of "mineral" rights on private property. Mineral rights allow a person or company to drill for oil and gas on a given property. In many U.S. states, homeowners own the "surface" rights to their properties while the state owns the mineral rights. In other states, homeowners own both surface and mineral rights. If oil or gas is discovered in the area, extraction firms may offer substantial payments to mineral rights-holders for permission to drill on all or part of their land.]

   False. The division of property rights may have income effects that affect the amount of externality that one party will accept. This idea is conveyed by the smoky Edgeworth box in the lecture note where two parties bargain over beans and smoking. If the party who doesn't like smoking is allocated ownership of clean air, this produces less smoking than if the smoker is allocated ownership of the right to smoke. Even though both outcomes are Pareto efficient, the division of property rights affects the wealth of each party and hence the final allocation. Allocating property rights is like moving the endowment in the Edgeworth box.

3. **(5 points)** The Massachusetts Secretary of Transportation wants to correct the externality caused by traffic congestion on the Massachusetts Turnpike during rush hour. She is considering two alternative policies. One policy is to cap the number of cars using the Pike at peak hours by auctioning a fixed number Q^* of rush hour permits each day. Here, Q^* is chosen to maximize total vehicle throughput, measured in vehicle miles traveled per hour.[1] The alternative policy is to charge drivers a peak travel time fee of P^*, which they pay using EZ Pass (an electronic toll system). The fee of P^* would yield Q^* rush hour drivers on the Pike on an *average*

[1] When the number of cars traveling simultaneously on the Pike exceeds Q^*, total vehicle throughput falls.

day. Assume that both the toll and the auction have identical transaction costs for drivers and the state of Massachusetts. An advisor to the Secretary makes the following argument in favor of tolls: "The demand for commuting fluctuates daily. Most Mondays, many people need to get work downtown. On Thursdays and Fridays, more people tend to work from home. By setting a price of P^* rather than a cap of Q^*, we will allow the number of Pike users to increase flexibly when demand is especially high." Explain whether or not you agree with the advisor's argument and why.

*If the efficient carrying capacity is Q^*, then we don't want more drivers on the road when demand is higher. We want the price of permits to rise instead. So, this is actually an argument **against** tolls and in favor of permits.*

4. **(5 points)** The Second Welfare Theorem says that any Pareto efficient allocation can be attained as a market equilibrium by making appropriate transfers, assuming of course that the four necessary conditions for efficient markets are satisfied. Starting from the initial endowment E, the policymaker would like the economy to reach the point X^*, which lies on the contract curve. Absent transfers, however, the economy would reach the point X', which is also on the contract curve but not the policymaker's preferred point. Having taken 14.03/003, the policymaker decides to implement a *proportional* tax t^* (10 percent, for example) on all goods purchased. Note that if the initial price ratio is $P = p_1/p_2$, then the taxed initial price ratio will be $P_t = p_1(1+t^*)/p_2(1+t^*) = P$. The tax revenue will be used to make the lump-sum transfers needed to reach X^*. Will the policymaker's scheme satisfy the Second Welfare Theorem? Explain.

   No. This is a transaction tax. Agents will reduce the amount they trade so as not to lose part of their endowment along the way. For example, if an agent sold and then repurchased her entire initial endowment, she would lose $t^* \times (p_1E_1 + p_2E_2)$. Yes, agents may get back some of their wealth back in transfers under this scheme, but this does not change the cost of transacting at the margin (unless agents understand exactly how the money will be recycled—even then, the agent that is not expected to gain in net from the lumps transfers will have an incentive not to transact).

2 International trade (22 points)

Suppose Cheese (C) and Watches (W) in Switzerland are produced using only labor and the production functions are:

$$C = 10\sqrt{L_c}$$

$$W = \alpha L_w$$

L_c and L_w represent labor devoted to the production of cheese and watches, respectively, and $\alpha > 0$ is a constant. Suppose labor supply in the country is fixed at $L = 400$, and the utility function of the representative Swiss consumer is $U(C,W) = \sqrt{CW}$. Note that this utility function implies that the marginal utility of a Swiss consumer from one extra unit of watch consumption is higher if she has more cheese to consume, which is a natural assumption about Swiss preferences for Cheese and Watches.

1. **(5 points)** Suppose $\alpha = 1$. That is, one unit of labor can produce one watch. Derive and draw the Swiss production possibility frontier for watch and cheese production.

   From the production functions, we get: $L_C = \frac{C^2}{100}$ and $L_W = \frac{W}{\alpha}$. Substituting this into $L_C + L_W = 400$ we get:

$$\frac{C^2}{100} + \frac{W}{\alpha} = 400$$

or, equivalently,

$$C^2 + 100W = 40000$$

This is the production possibility frontier (PPF) with $\alpha = 1$.

2. **(5 points)** *Now make no assumptions on the value of α.* Suppose Switzerland didn't sell or buy any Cheese or Watches to other countries. What would be the equilibrium price ratio and equilibrium quantities of Cheese and Watches in the Swiss domestic market? [Note: Your answers will depend on α, which you should treat as an unknown for now. Of course, the solution will require finding the point of tangency between the PPF and the highest feasible indifference curve.]

   The marginal rate of substitution is

   $$MRS = -\frac{dW}{dC} = \frac{W}{C}$$

   In equilibrium, the marginal rate of substitution is equal to the price ratio and to the slope of the production possibility frontier. To find the slope of the production possibility frontier, we can totally differentiate it to get:

   $$2CdC + \frac{100}{\alpha}dW = 0$$

   which gives us $-\frac{dW}{dC} = \frac{\alpha}{50}C$. Combining all these, we get that in equilibrium:

   $$\frac{W}{C} = \frac{\alpha}{50}C = \frac{p_C}{p_W}$$

   We can substitute the first equality into the production possibility frontier to get:

   $$\frac{50}{\alpha}W + \frac{100}{\alpha}W = 40000$$

   which gives us $W^* = \frac{800\alpha}{3}$. Then, $C^* = \frac{200}{\sqrt{3}}$ and the price ratio is $\frac{p_C}{p_W} = \frac{4}{\sqrt{3}}\alpha$

3. **(3 points)** How do the equilibrium quantities change when α increases? How does the price ratio change? Notice that you can interpret α as a productivity parameter: when α rises, watchmakers get more watches with the same amount of L_w. In light of this interpretation of α, explain the intuition behind your mathematical results.

   When α increases, the optimal quantity of Cheese is unchanged and the production of Watches increases. The price ratio also increases. We can certainly interpret α as a productivity parameter, since it determines how many Watches can be produced with a given amount of labor. When labor becomes more productive in Watches, logically the optimal quantity of watches produced increases and the relative price of watches falls.

4. **(5 points)** Assume now that Switzerland can trade with other countries – specifically, France. Suppose the French price ratio is $\frac{p_c}{p_w} = 1$. At the French price ratio, how much would Swiss consumers want to consume and how much would Swiss producers want to produce of each good? Find consumption bundle (C_c, W_c) and production bundle (C_p, W_p). In your solution assume $\alpha > 1/4$. For what values of α will Switzerland want to export Cheese to France? How about Watches?

   At the French price ratio, the producers will want to equate the slope of the production possibility frontier to the new price ratio:

   $$\frac{\alpha}{50}C = 1$$

Thus, the new quantity of Cheese produced is $C_p = \frac{50}{\alpha}$. *Substituting this into the production possibility frontier, we get that the quantity of Watches produced is* $W_p = 400\alpha - \frac{25}{\alpha}$. *Now let's consider the consumers. They will equalize the marginal rate of substitution with the French price ratio:*

$$\frac{W_c}{C_c} = 1$$

Consumers will want to consume an equal amount of Cheese and Watches. Since the consumers cannot consume more than the value of their production, they are limited by a budget set:

$$W_c + C_c = 400\alpha + \frac{25}{\alpha}$$

Since $W_c = C_c$ *at the optimum as we just established, we will get*

$$W_c = C_c = 200\alpha + \frac{25}{2\alpha}$$

Switzerland will want to export Watches, if its domestic demand for Watches is less than its production of Watches. That is if α *is such that:*

$$W_p = 400\alpha - \frac{25}{\alpha} > W_c = 200\alpha + \frac{25}{2\alpha}$$

$$\alpha > 0.43$$

Thus, Switzerland will export Watches and import Cheese if $\alpha > 0.43$ *and import Watches and export Cheese if* $0.24 < \alpha < 0.43$.

5. **(4 points)** Consider the expressions for the optimal consumption and production bundles under trade with France that you found in part (4). How are Swiss exports affected by a rise α? Provide an intuitive explanation. How are Swiss consumption of both goods affected by a rise in α? Provide an intuitive explanation.

   We can check that production of Cheese is decreasing in α, *while production of Watches and consumption of both Cheese and Watches are increasing in* α. *In words, this means that, as the production possibility frontier is expanded in the direction of watches, Switzerland will tend to specialize more in the production of Watches, and will be able to afford more of both Watches and Cheese under trade.*

3 Externalities (28 points)

There are 150 hunters in a community. They each choose whether to hunt in the forest or on the plains. The plains are so large that each hunter can catch 0.05 tons of game no matter how many other hunters are there. The forest can get crowded, however. If there are x hunters in the forest, each of them catches $\frac{1}{2}x^{-\frac{1}{2}}$ tons of game (so, in total, $x * \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2}x^{\frac{1}{2}}$ tons of game are hunted in the forest). Forest game and plains game are perfect substitutes in consumption and there are no other costs associated with hunting in the forest or the plains. The demand for game is perfectly elastic at price $5 per ton. *(Note: You can ignore discreteness throughout the question.)*

1. **(5 points)** If each hunter is free to choose whether to hunt in the forest or the plains, how many hunters will go to the forest, how many to the plains, and what will be the average yield for the 150 hunters?
   Hunters will hunt in the forest up to the point where they are indifferent between hunting in the forest and hunting on the plains, i.e. the point at which a hunter obtains the same yield from hunting in the forest and

on the plains. In equilibrium:

$$\begin{aligned} \frac{1}{2}N^{-\frac{1}{2}} &= 0.05 \\ N &= 100 \end{aligned}$$

There will be 100 hunters in the forest, and 50 hunters on the plains. The average yield is the same for everyone, and is equal to 0.05.

2. **(3 points)** What is the nature of the externality in this example? Using the Coase Theorem, analyze why the market does not internalize this externality.
Dissipative externality. The extra hunters do not take into account the congestion they are causing in the forest when deciding whether or not to "enter". There are no property rights assigned. Given the high number of parties involved, bargaining is likely to be costly, hence by the Coase theorem the market will not internalize this externality and the market equilibrium will be inefficient.

3. **(5 points)** If the government restricts access to the forest, how many hunters should it allow in the forest to maximize the total yield in the community? What is the average yield from the forest and what are total profits? Is this a Pareto improvement over the allocation in part (1)?
The government want to maximize:

$$\max_N 0.05(150 - N) + N \cdot (\frac{1}{2}N^{-\frac{1}{2}})$$

$$\begin{aligned} [N] : -0.05 + \frac{1}{4}N^{-\frac{1}{2}} &= 0 \\ N &= 25 \end{aligned}$$

The government should allow 25 hunters in the forest in order to maximize the total yield. The average yield from the forest is now:

$$\frac{1}{2}N^{-\frac{1}{2}} = \frac{1}{2}(25)^{-\frac{1}{2}} = 0.1$$

which is higher than it was under no government intervention. Profits are now:

$$5 * (0.05 * (150 - 25) + \frac{1}{2} * 25^{\frac{1}{2}}) = 75$$

whereas profits in the first part of the question were:

$$5 * (0.05 * 150) = 37.5$$

We see that this is indeed a Pareto improvement over the allocation in part (1). No hunter is worse off, i.e.

all of the hunters are obtaining at least as great a yield as they were previously, but the hunters in the forest are now obtaining larger yields and earner higher profits.

4. **(5 points)** The government decides to sell permits for hunting in the forest. It chooses a price p and sells as many permits as hunters wish to buy at this price. What price p should the government charge per hunting permit to achieve the optimal allocation determined in part (3)?

   We want to set the price for permits p such that exactly 25 hunters want to hunt in the forest. The indifference condition is profits from hunting in the forest equal profits from hunting on the plains.

$$\begin{aligned} 5 * (\frac{1}{2} * 25^{-\frac{1}{2}}) - p &= 5 * 0.05 \\ p &= 0.25 \end{aligned}$$

   A permit to hunt in the forest which costs 0.25 will equate the profits associated with forest and plains hunting.

   Some of you may have interpreted the problem as requiring hunters to obtain a permit per unit hunted. Under this interpretation, the indifference condition is the following:

$$\begin{aligned} (5 - p) * (\frac{1}{2} * 25^{-\frac{1}{2}}) &= 5 * 0.05 \\ p &= 2.5 \end{aligned}$$

5. **(5 points)** Assume the revenues from the sale of permits in part (4) are given back in equal amount to the 150 hunters as a lump sum. How does the welfare of the hunters under the permit scheme in part (4) compare to their welfare with no government intervention? Explain.

   *Everyone is better off compared to the equilibrium with no government intervention. Note that this is not automatic (i.e. just because we internalized the externality, does not mean that everyone is better off). Total welfare is higher because we are at an efficient outcome where the externality has been internalized. Without the redistribution of the revenues from the sale, everyone is the same as with no government intervention (think about the indifference condition above). The total revenues from the sale of permits are 0.25*25 = 6.25 and are distributed equally across all hunters.*

6. **(5 points)** Wildlife ecologists observe that following the reduction in the number of forest hunters, some animals have been migrating from the plains to the forests. This increases the productivity of hunting in the forest but reduces it in the plains. Taking into account this migratory response, should the government be permitting a smaller or larger number of hunters to hunt in the forest than what you calculated in part (3)?

   In general, if an activity generates a negative externality and the market doesn't correct it (e.g., through Coasean mechanisms), the equilibrium will have an inefficiently high level of the negative externality-generating activity (and vice versa for a positive externality). In parts (1) through (5) above, each hunter caused negative externalities for other hunters and, accounting for these social costs, there was too much hunting in equilibrium. Hence, you calculated that reducing the number of hunters would improve social efficiency. In part (6), we learn that not *hunting also creates negative externalities for plains hunters. Accounting for this negative externality, it's likely that the efficient level of forest hunting is* higher *than what we calculated above. Accordingly, efficiency demands a greater number of forest hunters than parts (1) through (5) suggested.*

4 Causal Inference using Instrumental Variables (IV) (30+5 points)

In this problem, we seek to evaluate the effectiveness of charter schools in Boston. Charter schools are schools that are publicly funded but privately run; they function with considerably more independence than traditional public schools and are often characterized by longer instructional hours and different pedagogical techniques, such as the 'No Excuses' philosophy embraced by many charters. There has been rapid growth of charter schools in recent years, and educators, government officials and, of course, economists are interested in the causal effect of charter school attendance on student outcomes such as test scores.

Massachusetts state law caps the number and total enrollment of charter schools in a way that results in excess demand for charter schools in Boston. Thus, Boston uses a lottery to allocate spots in its charter schools to students who request to attend a charter. If a student wins the lottery, she is offered a spot in a charter school. If a student does not win the lottery, she attends a traditional Boston public school. For simplicity, assume that students who wish to attend a charter enter a centralized lottery, and are offered admission to at most one charter school.

Let Y_i be the test score of student i and

$$Z_i = \begin{cases} 1 & \text{if } i \text{enters and wins the charter school lottery} \\ 0 & \text{if } i \text{enters and does not win the charter school lottery} \end{cases}$$

$$D_i = \begin{cases} 1 & \text{if } i \text{attends a charter school} \\ 0 & \text{if } i \text{does not attend a charter school} \end{cases}$$

1. **(2 points)** We are interested in estimating T^*, the causal effect of attending a charter school on student test scores. Write the definition of T^* using formal (causal) notation.
   We are interested in estimating the causal effect of attending a charter school on student test scores.

$$T^* = E[Y_1 - Y_0 | D = 1]$$

2. **(5 points)** Suppose we decided to estimate T^* by comparing students who attend charter schools to students who do not attend charter schools. We will call this estimator $\hat{T}$.

   (a) Write out $\hat{T}$ using formal notation.

$$\hat{T} = E[Y_1 | D = 1] - E[Y_0 | D = 0]$$

   (b) Do you think that $\hat{T}$ will provide an unbiased estimate of T^*? Why or why not? If not, will $\hat{T}$ be upward biased, downward biased or can the bias not be signed? Provide an explanation for your view.
   $\hat{T}$ is unlikely to provide an unbiased estimate of T^*. Students who attend charter schools entered the lottery in order to get there, whereas students who do not attend charter schools are comprised of the students who lost the lottery and the students who never entered the lottery at all. We have no reason to believe that the students who enter the lottery are comparable to those who did not enter the lottery. The students who attend charter schools may have parents who are more invested in their education and would have had better test scores had they not attended a charter school ($E[Y_0|D = 1] > E[Y_0|D = 0]$). In this case, the bias would be positive, i.e. using $\hat{T}$ we would overestimate the effect attending a charter school on student test scores. [But it would be fine to argue that the bias cannot be signed. All we can

say with confidence is that we ***cannot*** *be confident that $\hat{T}$ is unbiased.]*

$$\begin{aligned} Bias &= \hat{T} - T^* \\ &= (E[Y_1|D=1] - E[Y_0|D=0]) - (E[Y_1|D=1] - E[Y_0|D=1]) \\ &= E[Y_0|D=1] - E[Y_0|D=0] > 0 \end{aligned}$$

3. **(5 points)** Suppose that every student who wins the charter school lottery chooses to accept the offer and attends a charter school. Explain how you would estimate T^*. Call this estimator $\bar{T}$. Does $\bar{T}$ provide an unbiased estimate of T^*? Why or why not?
   If every student who wins the charter school lottery chooses to go to a charter school, then we can estimate T^* simply by comparing the test scores of students who won the lottery to the test scores of students who lost the lottery.

   $$\overline{T} = E[Y_1|D=1, Z=1] - E[Y_0|D=0, Z=0]$$

   Of those who enter the lottery, the students who attend a charter school are randomly chosen.

   $$E[Y_0|D=1, Z=1] = E[Y_0|D=0, Z=0]$$

   So, we are able to rewrite $\bar{T}$:

   $$\begin{aligned} \bar{T} &= E[Y_1|D=1, Z=1] - E[Y_0|D=1, Z=1] \\ &= E[Y_1|D=1] - E[Y_0|D=1] \\ &= T^* \end{aligned}$$

4. **(5 points)** When the lottery results come out, you learn that some students who won the lottery ended up declining the offer to attend a charter school and instead chose to attend a Boston public school. Does $\bar{T}$ provide an unbiased estimate of T^*? Why or why not? If it is likely to be biased, can you sign the bias (explain)?
   $\bar{T}$ is unlikely to be an unbiased estimate of T^*. Instead of charter school attendance being randomly assigned, there are some students who win the lottery but decline the offer—thus, attendance is endogenous among lottery winners. We can no longer assume that the counterfactuals are balanced among the students who conditional on entering the lottery, attend and do not attend charters. [Note that the counterfactuals of students who win and lose the lottery still remain balanced, but we are not trying to estimate the causal effect of the lottery on test scores, we are trying to estimate the causal effect of attending the charter.]

   We cannot confidently sign the bias in this case. We might hypothesize that the students who decline to attend the charter school despite winning the lottery are generally satisfied with their education at traditional Boston public schools and likely would have had higher test scores, absent any intervention. If this is the case ***and*** charter schools raise scores, *then $\bar{T}$ would be biased downward ($E[Y_0|D=0, Z=1] > E[Y_0|D=1, Z=1]$). A sufficient answer to this question is that we can no longer be confident that $\bar{T}$ is unbiased and we cannot confidently say what the bias will be.*

5. **(5 points)** In order to estimate the causal effect of attending a charter school on student test scores, we wish to employ the charter school lottery as an instrumental variable (IV) for attending a charter school.

   (a) **(3 points)** State the formal conditions that are necessary for the charter school lottery to serve as a valid instrument.
   First stage: the charter school lottery must have a casual effect on the probability of going to a charter school

$$E[D|Z=1] > E[D|Z=0]$$

   Exclusion restriction: the only way the charter school lottery can affect test scores is through raising the probability of attending a charter school.

$$E[Y|D=1, Z=1] = E[Y|D=1, Z=0]$$

   (b) **(2 points)** Do you think these conditions are likely to be satisfied? Which of the conditions can be directly tested?
   The charter school lottery is likely to affect charter school attendance, so the first stage will hold (and we can test this!). The exclusion restriction is also likely to hold here. It seems plausible that the only way the lottery can affect test scores is through raising the probability of attending a charter school. (An alternative story is that winning the lottery also has a direct effect on student test scores through, for example, the positive motivational effect of winning the lottery. This does not seem particularly credible.) The first stage assumption can be tested, but the exclusion restriction cannot.

6. **(3 points)** Consider the following data collected for each group:

	Won Lottery	Lost Lottery
E[Test Score]	76	72
E[Attended Charter]	0.8	0

   (a) What is the causal effect of winning the charter school lottery on the chance of going to a charter school?
   The causal effect of winning the charter school lottery on the chance of going to a charter is:

$$E[D|Z=1] - E[D|Z=0] = 0.8 - 0 = 0.8$$

   (b) What is the causal effect of winning the charter school lottery on student test scores?
   The causal effect of winning the charter school lottery on student test scores is:

$$E[Y|Z=1] - E[Y|Z=0] = 76 - 72 = 4$$

7. **(2 points)** Explain how you would construct an instrumental variables (IV) estimate of the causal effect of attending a charter school on student test scores. What is your instrumental variables causal effects estimate, $\hat{T}_{IV}$?
We construct an IV estimate of the causal effect of attending a charter school on student test scores by dividing the reduced form by the first stage:

$$\hat{T}_{IV} = \frac{E[Y|Z=1] - E[Y|Z=0]}{E[D|Z=1] - E[D|Z=0]} = \frac{4}{0.8} = 5$$

8. **(3 points)** Does $\hat{T}_{IV}$ correspond to the Average Treatment Effect (ATE) or the Average Effect of Treatment on the Treated (ATT)? Explain.

   $\hat{T}_{IV}$ corresponds to the ATT, as it measures the effect of attending a charter school on student test scores for the students who accepted the offer to attend (i.e. won the lottery and attended).

9. [EXTRA CREDIT **(5 points)**] Now suppose that a student can influence her probability of winning the lottery if her parents lobby Boston school district administrators on her behalf. Would our IV estimate of the causal effect of attending a charter school still be valid (i.e., unbiased)? Discuss which of the two IV conditions would be violated and why.

   If a student can influence her probability of winning, it's likely that the exclusion restriction will be violated. A student whose parents are willing to go great lengths in order to get her into charter school may be more invested in her educational attainment. In this case, winning the lottery not only has an indirect effect on test scores through raising the probability of attending a charter school, but also a direct effect on test scores through the parental investment channel.

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