

# Fall 2018 14.01 Problem Set 1-*Solutions*

## 1 Positive vs. Normative Statements (16 points)

Identify whether each of the following statements is positive or normative. Briefly justify your answer.

1. (4 points) The government has a duty to provide basic healthcare and education to every citizen.
2. (4 points) The cost of health insurance is too high.
3. (4 points) The median earnings of a full-time worker with a college degree are almost twice as high as those of a high-school graduate with no college education.
4. (4 points) The current unemployment rate is 3.9%, the lowest it has been since December 2000.

*Solution: The first statement is normative as it provides an opinion about what the role of the government should be. The second statement is also normative since saying that it is “too high” is implicitly expressing that it should be lower. The last two statements are positive as they are just describing facts about the US economy.*

## 2 True or False (20 points)

For each of the following statements, indicate if they are True or False. Justify your answer.

1. Bill is a football coach. He evaluates his players based on three criteria: height, strength, and speed. Bill prefers one player over another if he is better in at least two of these criteria. Assume that there are no players with the exact same height, nor the exact same strength, nor the exact same speed.
  - (a) (4 points) Bill's preferences are complete.

- (b) (4 points) Bill's preferences are transitive.
2. (4 points) Ann and Bob are utility maximizing consumers. Given their income and market prices, Ann chooses a bundle that gives her a utility  $U_{Ann} = 100$ , while Bob chooses a bundle that gives him a utility  $U_{Bob} = 110$ . Therefore, we know that Bob is happier than Ann.
3. (4 points) John's utility function for food ( $f$ ) and clothes ( $c$ ) is given by  $U(f, c) = (f^\alpha + c^\alpha)^{\frac{1}{\alpha}}$ , with  $\alpha > 0$ . John's preferences satisfy the principle of diminishing marginal rate of substitution for any value of  $\alpha$ .
4. (4 points) Ava has preferences over two goods that satisfy completeness, transitivity, non-satiation and the indifference curves have a strictly diminishing marginal rate of substitution. Suppose that the price of one of these goods increases (and the price of the other one remains the same). Claim: Ava's utility must be strictly lower after the price increase. How does your answer depend on the composition of the initial consumption bundle?

*Solution:*

- 1.a. True, if we are comparing two players based on three criteria, it must be that one is better than the other in at least two of them, so we can always compare any two players.
- 1.b. False, consider the following counterexample. We have three players: A, B and C.

	A	B	C
Height	2	1	3
Strength	3	2	1
Speed	1	3	2

so A is preferred to B, B is preferred to C, and C is preferred to A, thus violating transitivity.

2. False, utility is ordinal so we can't tell who is happier.
3. False, if  $\alpha > 1$  the indifference curves are concave, so they don't satisfy the principle of diminishing MRS.
4. False, the utility can't be higher after the price increase, but it could stay the same if in the initial bundle she was only consuming the good that didn't change price. If the initial bundle had strictly positive quantities of both goods, then we can tell that the utility must be strictly lower after the price increase.

### 3 Demand for Video Games (16 points)

We have the following weekly demand data for the video game *Grand Theft Auto* in a US town. We also have the price data for a Playstation at the same time.

Price of Playstation (in \$)	Price of Grand Theft Auto (in \$)	Quantity of games demanded
300	10	100
310	10	95
320	10	90
300	11	99
310	9	96

1. (6 points) Write down the equation for the demand for *Grand Theft Auto* in the following form:

$$Q_g^D = \alpha + \beta_g P_g + \beta_p P_p$$

where  $Q_g^D$  and  $P_g$  are quantity demanded and price of *Grand Theft Auto*,  $P_p$  is the price of Playstation, and  $\alpha, \beta_g, \beta_p$  are constants.

*Solution:* Using the first two observations, we have that  $-5 = \Delta Q_g^D = \beta_p \Delta P_p = \beta_p 10$ , so  $\beta_p = -\frac{1}{2}$ . Using the first and fourth observation, we have that  $-1 = \Delta Q_g^D = \beta_g \Delta P_g = \beta_g$ , so  $\beta_g = -1$ . We can then use any observation to find the constant,  $\alpha = 260$ , so the demand curve is

$$Q_g^D = 260 - P_g - \frac{1}{2}P_p$$

2. (4 points) Does the demand function that you found in part (1) satisfy the law of demand? Explain.

*Solution:* Yes, we have that  $\beta_g < 0$  so it satisfies the law of demand.

3. (6 points) The supply curve for *Grand Theft Auto* in this town is

$$Q_g^S = 2P_g$$

Solve for the equilibrium price and quantity, as a function of  $P_p$ . How does the equilibrium price and quantity depend on  $P_p$ ? Briefly explain the intuition.

*Solution:* We find the intersection between the two curves and obtain

$$\begin{aligned} P_g &= \frac{260}{3} - \frac{1}{6}P_p \\ Q_g &= \frac{520}{3} - \frac{1}{3}P_p \end{aligned}$$

Both quantity and price depend negatively on  $P_p$ . The intuition is that the increase in a complementary good (Playstation) shifts the demand for videogames curve inwards, leading to a drop in the equilibrium price and quantities.

## 4 Indifference Curves (18 points)

In each of the following examples, the consumer consumes only two goods,  $x$  and  $y$ . Based on the information given in each statement, sketch a plausible set of indifference curves (draw at least two curves on a set of labeled axes and indicate the direction of higher utility). Then, write down a possible form of the utility function  $u(x, y)$  that is consistent with your graph.

1. (6 points) Alan likes wearing both right shoes ( $x$ ) and left shoes ( $y$ ). He always needs to wear them as a pair, having a right shoe is useless without the left one and viceversa.

*Solution: right shoes and left shoes are perfect complements, so a possible utility function is  $u(x, y) = \min\{x, y\}$ .*

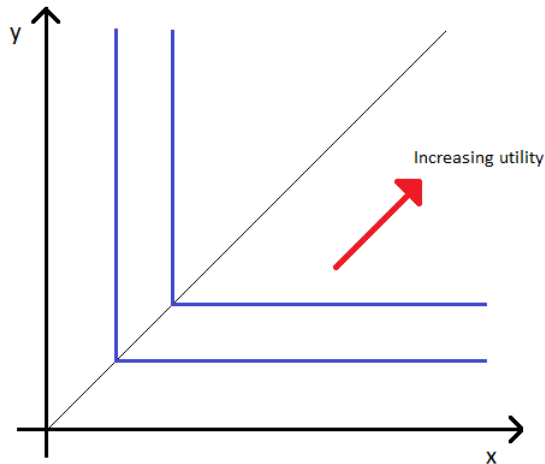


Figure 1:

2. (6 points) Emma likes pizza ( $x$ ) but hates vegetables ( $y$ ). She is only willing to eat an extra unit of vegetables if she gets to eat an extra unit of pizza.

*Solution: note that in this case the utility must be decreasing in  $y$  and increasing in  $x$ . It must also satisfy that when we increase both pizza and vegetables by one unit the utility stays the same. A possible utility function is  $u(x, y) = x - y$ .*

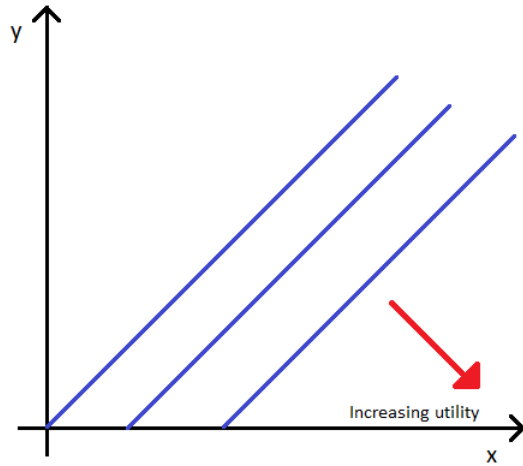


Figure 2:

3. (6 points) Mary likes Coke ( $x$ ) and Pepsi ( $y$ ). She is indifferent between them as she is unable to tell the difference between Coke and Pepsi.

*Solution: Coke and Pepsi are perfect substitutes, so a possible utility function is  $u(x, y) = x + y$ .*

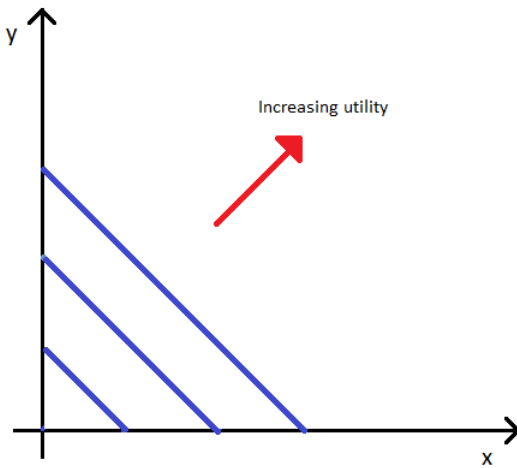


Figure 3:

## 5 Utility Maximization (30 points)

Chloe consumes only books ( $x$ ) and video games ( $y$ ). Her preferences can be represented by the following utility function:  $U(x, y) = xy^2$ . The price of books is  $p_x$ , the price of video games is  $p_y$ , and Chloe has an income of  $m$  dollars.

- (4 points) Write down Chloe's budget constraint.  
*Solution:*  $p_x x + p_y y \leq m$  (it is also correct to right it as an equality).
- (4 points) Calculate the Marginal Rate of Substitution (at an arbitrary bundle  $(x, y)$ ).  
*Solution:*  $MRS = -\frac{MU_x}{MU_y} = \frac{y}{2x}$ .
- (6 points) Find the equation that describes Chloe's demand for books and videogames for any possible value of  $p_x, p_y$  and  $m$ .

*Solution:* Setting  $MRS = MRT$  and using the budget constraint with equality we get

$$\begin{aligned}x &= \frac{m}{3p_x} \\y &= \frac{2m}{3p_y}\end{aligned}$$

- (6 points) Repeat part 3 when the utility function is  $U(x, y) = \min\{x, y\}$ .  
*Solution:* With these preferences, the consumer will always choose bundles such that  $x = y$ . Replacing this condition in the budget constraint, we get that

$$\begin{aligned}p_x x + p_y y &= (p_x + p_y) x = m \\ \Rightarrow x = y &= \frac{m}{p_x + p_y}\end{aligned}$$

- (4 points) Let us now go back to the original utility function  $U(x, y) = xy^2$ . Suppose that the government imposes a tax on videogames, such that if the price of a videogame is  $p_y$ , the consumer must pay  $(1 + \tau)p_y$ . What is the new demand function for videogames? Plot the demand before the tax and after the tax in the same graph (you don't need to assume any particular values for  $m, \tau$ , it is enough to provide a qualitative graph). Briefly explain the intuition.  
*Solution:* the new demand function is  $y = \frac{2m}{3(1+\tau)p_y}$ , so the new demand function is down and to the left of the original one. The intuition is that for any given value of  $p_y$ , Chloe now has to pay a higher price, so she will demand a lower quantity.
- (6 points) Let's go back to the case without taxes. Suppose that  $p_x = 2, p_y = 2, m = 30$ . Suppose that Chloe has one "buy-10-get-10-free" coupon for books (that is, she will get 10 free books if she buys at least 10 books). How many

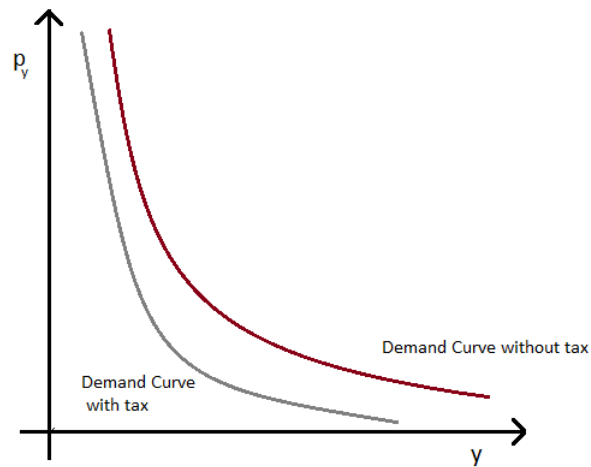


Figure 4:

books and how many videogames will Chloe consume? Carefully draw the budget set and the highest attainable indifference curve on the same graph.

*Solution:* The highest indifference curve will be either tangent to the budget constraint or will pass through the kink at the bundle  $(20, 5)$ . The only tangency point is at  $(5, 10)$ , and evaluating the utility function at these two points we find that  $U(20, 5) = U(5, 10) = 500$ , so Chloe is indifferent between buying  $(x, y) = (5, 10)$  or using the coupon and getting  $(x, y) = (20, 5)$ .

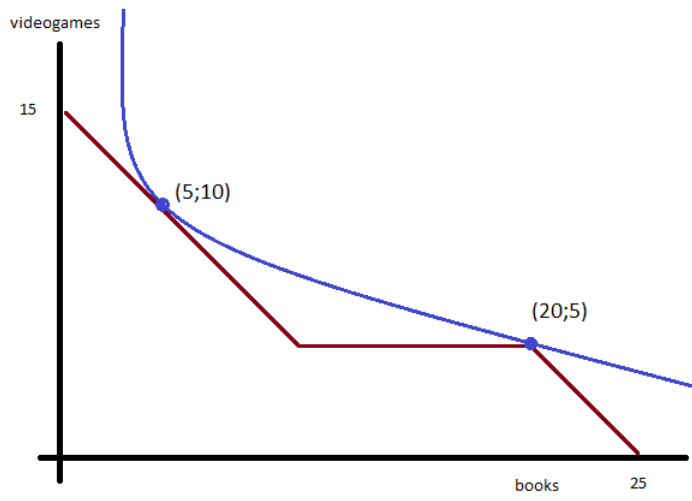


Figure 5:



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