12.864 Inference from Data and Models 8 March 2004
Problem Set No. 3 Due: 15 March 2004

1. Show that the weighted/tapered least-squares objective function with Lagrange multipliers (Eq. 2.169 of the notes), produces the same solution as objective function Eq. (2.122) of the notes. (Eq. 2.173 of the notes should read $\tilde{\mu} = \mathbf{W}^{-1}\mathbf{n}$.)

2. (a) You have three equations in two unknown x_i,

$$\begin{aligned} x_1 + x_2 + n_1 &= 1, \\ x_1 - x_2 + n_2 &= 2, \\ x_1 + x_2 + n_3 &= 1.5. \end{aligned}$$

There is reason to believe that $\langle n_i \rangle = 0$, $\langle n_1^2 \rangle = \langle n_2^2 \rangle = 1, \langle n_3^2 \rangle = 16$. Find a defensible estimate of x_i, n_i, explaining what you are doing (don't just give me numbers).

(b) Suppose now, in addition, that it is thought that $\langle x_1^2 \rangle = 10 \langle x_2^2 \rangle, \langle x_1 x_2 \rangle = 3$. What is a new solution using this information?

3. Using an eigenvector/eigenvalue analysis, solve (a)

$$\left\{ \begin{array}{ccc} 1 & -1 & -2 \\ -1 & 2 & -1 \\ -2 & -1 & 6 \end{array} \right\} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \tag{1}$$

and (b)

$$\left\{ \begin{array}{ccc} 1 & -1 & -2 \\ -1 & 2 & -1 \\ 1.5 & 2 & -2.5 \end{array} \right\} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \tag{2}$$