12.802 **Problem Set 5** **Spring 2008**

1. A Poincaré wave with $x-$wave number $k_1 < 0$ and wave number l_1 approaches a wall at $x = 0$ from the right.

   a) What angle does the group velocity make with the $x-$axis?

   b) What is the frequency of the wave?

   c) If the amplitude of the incident wave is a_i, find the complex amplitude a_r of the reflected wave and discuss what a_r complex means.
   (Hint: Write the boundary condition for the velocity at the wall.)

2. Consider a kelvin wave in a channel of depth D. The free surface elevation is:

$$\eta = \eta_0 \cos(kx - \omega t) e^{\frac{-f}{c_0} y}$$

   a) Find the relative vorticity and the potential vorticity. Discuss why the potential vorticity is zero.

   b) Calculate the kinetic and potential energies (the kinetic energy must be integrated over the water column). Check for equipartition.

   c) Discuss the trajectories of the fluid elements as the wave passes.

3. A topographic Rossby wave obeys the dispersion relation:

$$\omega = -\frac{\beta k}{k^2 + l^2}$$

   where the convention used is $\omega > 0, k < 0$

$$\beta = -\frac{f_0}{H_0}\frac{\partial H}{\partial y}$$, where f_0 is the Coriolis parameter (constant)

   and H_0 the undisturbed depth, is the equivalent $\beta-$effect due to a depth $H(y)$ that decreases monotonically northward over a distance L large compared to the wavelength of the wave.

   A wave packet with initial wavenumbers (k_0, l_0) and initial frequency ω_0 begins propagating northward $(y > 0)$ from the initial latitude y_0.

   a) Find the x and y components of the group velocity.

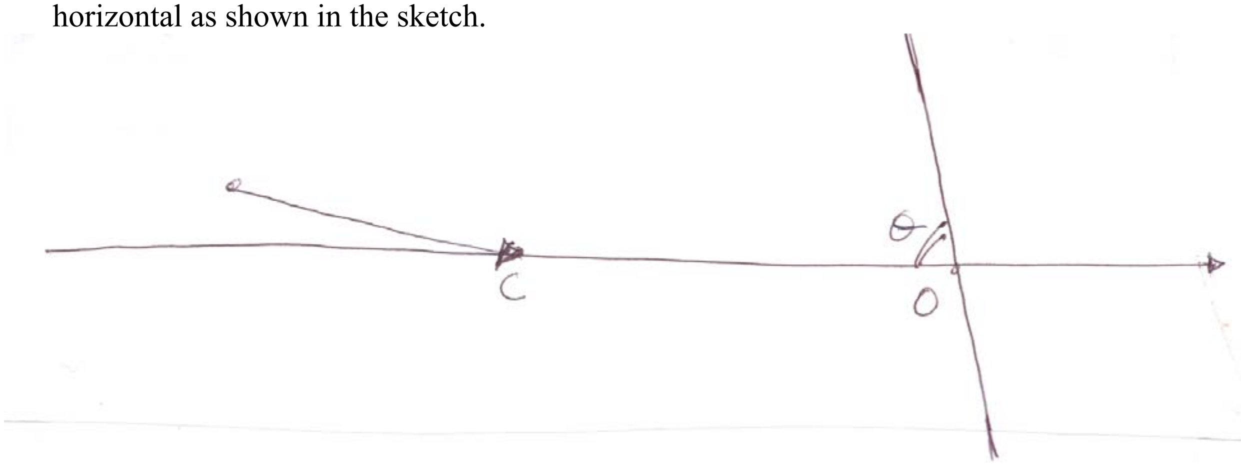
b) Derive the ray equations for (ω, k, l) and discuss their implications.

c) Derive the expression for l along the path of the wave packet (the ray path).

What happens for $l = 0$? Discuss why this condition defines a turning latitude y_T where the wave is totally reflected. At $y_T, \beta = \beta_T$. What is β_T in terms of the initial conditions?

d) Find the shape of the ray $\frac{dy}{dx}$ near the turning latitude y_T. (Hint: Expand $\beta_T = \beta(y - y_T)$ around y_T.)

4. A Rossby wave group velocity it impingent on an eastern wall inclined at an angle θ to the horizontal as shown in the sketch.



Find graphically the incident and reflected wave numbers, the reflected group velocity. Find also the analytic values of the incident and reflected wave numbers and their difference.

5. Consider a plane Rossby wave with a free surface $\eta = \eta_0 \cos(kx + ly - \omega t)$

   a. Calculate the kinetic and potential energy in the quasi-geostrophic limit. Is there a particular wavelength for which equipartition holds even though it is not true in general?

   b. Calculate c_{gx} as a function of *k*. Evaluate its maximum positive and negative value and the corresponding value of *k*. Where is it zero? Sketch c_{gx} as a function of *k*.