

Brittle Fracture and Sliding Summary

General Resources

M. S. Paterson, Experimental Rock Deformation – The Brittle Field, SpringerVerlag, Berlin, 1978.

C. H. Scholz, The Mechanics of Earthquakes and Faulting, Cambridge University Press, Cambridge, UK, 1990.

Pore Fluid Effects

Physical

○ *Surface Effects in Partially Saturated Soils*

●= *Effective Pressure*

●= *Dilatancy Hardening*

Chemical

○ *Fracture Surface Energy*

●= *Stress Corrosion*

●= *Crack Healing*

○ *Pressure Solution*

○ *Hydrolytic Weakening*

○ *Metamorphic Reactions*

Physical Pore Fluid Effects

Definitions:

- = **Drained** : Pore pressure maintained at a uniform, homogeneous value. Fluid may freely leave and enter the system. Thermodynamically open system.
- = **Undrained**: Thermodynamically closed system. Fluid cannot leave or enter. Fluid pressure determined by thermodynamic equation of state and pore volume.
- **Effectively undrained**: Egress of fluid limited by restricted permeability. For example, local values of pore pressure may vary if dilatancy occurs faster than pore fluid can enter system.

Law of effective stress (Scholz, pp 30-35, Paterson, Chapter 5)

In general, the most physical properties for cracked or porous solids depend on the applied stress state and on the magnitude of the pore fluid pressure. Often it is found that the property can be expressed as depending on the effective stress:

$$\boxed{\text{Property}(\bar{\sigma}) \quad \text{Property}(\sigma - \alpha P_{\text{fluid}})}$$

It is often found empirically that the value of α for inelastic properties is nearly equal to 1. In soils, such an expression was first given by Terzaghi.

Elastic

In equations for elastic strain, values of α are usually found to be less than 1. In fact, the values are different for the elastic properties of the aggregate, α_m , and that for the increase or decrease of the porosity, α_n , are

$$\alpha_m = 1 - \frac{K}{K_s}$$

$$\alpha_n = 1 - \frac{\phi K}{\alpha_m K_s}$$

where K is the modulus of incompressibility for the porous solid and K_s is the modulus for the dense solid, ϕ is the porosity,

Drained Inelastic Properties:

Fracture Tests:

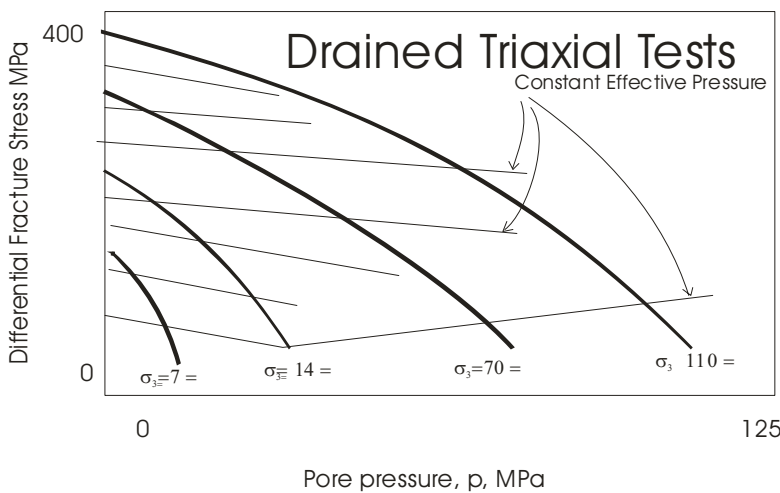
If a uniform pressure applied to all surfaces, internal and external, then an isotropic material will contract uniformly and isotropically. Because the field equations are linear differential equations, the difference of the solution for any triaxial stress state, σ_{ij}' , and that for the isotropic stress, $P\mathbf{I}$, where \mathbf{I} is the unit tensor, will also be a solution.

In particular, in the equations for the stress intensity factor around a sharp crack, substituting the effective stress,

$$\bar{\sigma}_{ij} = \sigma_{ij} - p\delta_{ij}$$

yields the same stress intensity factor.

[Paterson, 1978] Fig. 18 After Murrell, 1965



Notice that lines of constant effective pressure are nearly flat and horizontal.

Frictional Strength:

In the case of frictional strength, consider two rough surfaces in contact.

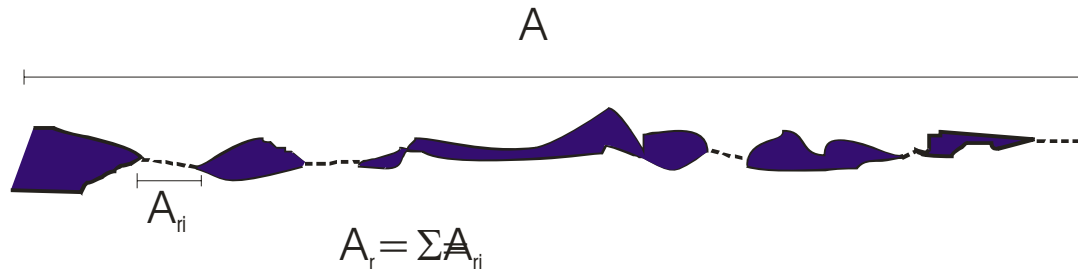
Amontons' law is a relation between the total applied normal load and the shear force necessary to cause sliding.

$$F_{\tau} = \mu F_n$$

$$\tau = \mu \sigma$$

where $\tau = \frac{F_{\tau}}{A_{\tau total}}$ and $\sigma = \frac{F_n}{A_{total}}$

Let the total area (nominal area) of the solid be A and the real area of contact be A_r .



Then, the total load on the nominal area is the applied normal load times the total area $\sigma_n A$

This force is opposed by the sum of the loads at the contacts $F_{n \text{ asperity}}$ plus the pressure in the pores times the area of the pores, $A_t - A_r$.

$$\sigma_n A = F_{n \text{ asperity}} + P_f (A - A_r)$$

The force at the asperity is the hardness, S_n , times A_r .

$$\sigma_n A = S_n A_r + P_f (A - A_r)$$

$$\frac{(\sigma_n - P_f)}{(S_n - P_f)} A = A_r$$

The shear force necessary to cause the asperity to deform is the shear hardness, S_{τ} , times the real contact area.

$$\tau = \frac{S_{\tau}}{(S_n - P_f)} (\sigma_n - P_f) = \mu_{wet} (\sigma_n - P_f)$$

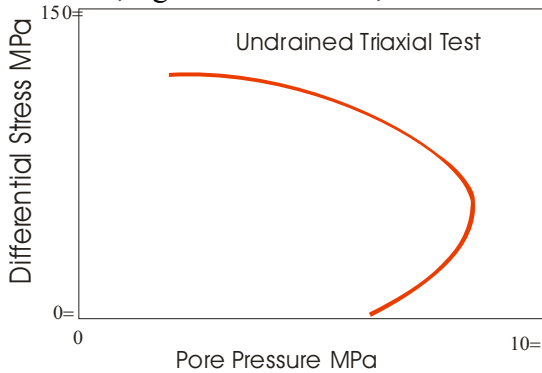
where the equation on the left now appears the same as Amontons' law with the effective normal stress as the independent variable.

Undrained Fracture Strength:

[Brace and Martin, 1968; Brace et al., 1966]

;When the system is undrained, local pore fluid values can deviate from nominal values.

Paterson, fig. 20. After Heck, 1972

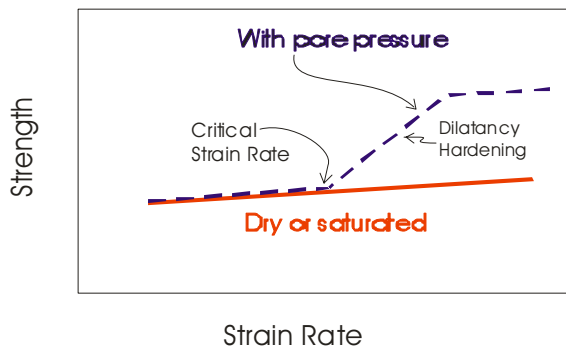


Deviation of pore pressure measured in an undrained triaxial test

Undrained tests may develop reduced pore pressures which in turn can cause the aggregate strength to increase.

Transiently Undrained and Dilatancy Hardening

[Brace et al., 1966; Brace et al., 1968; Olgaard et al., 1995]



When deformation occurs below a critical strain rate the material is able to drain itself and the pore pressure can be maintained at the nominal value. The characteristic length of distribution of the fluid is greater than the sample length.

As the strain rate is increased the amount of dilation exceeds the ability of

the fluid to enter the sample and the local pore pressure is reduced below the nominal value. The net result is that the sample is stronger than before.

Example:

For a sample with permeability k , storage capacity β_s , filled with a pore fluid of viscosity, η , the flux of fluid caused by local pressure gradients is given by Darcy's Law:

$$j = -\frac{k}{\mu} \nabla P_f \quad \text{where } j = \frac{1}{A} \frac{dV}{dt}$$

$$\frac{1}{LA} \frac{dV}{dt} \dot{\epsilon} \quad \text{and} \quad \beta_s = \frac{1}{V_{\text{sample}}} \frac{dV_{\text{pore}}}{dP_{\text{fluid}}} \quad \text{which implies} \quad \frac{\Delta P_{\text{fluid}}}{L} \approx \frac{1}{L \cdot \beta_s} \frac{\Delta V_{\text{pore}}}{\beta_c} = \frac{\Delta \epsilon = k}{L \eta \beta_c}$$

so

$\dot{\epsilon}_{\text{crit}} = \frac{\Delta \epsilon = k}{L^2 \eta \beta}$	Critical Strain Rate	Pore fluid
	10E-4	Water
	10E-7	Silicone Oil (1000 times water viscosity)

Chemical Pore Fluid Effects:

Stress Corrosion/Static Fatigue

[*Lockner, 1993; Lockner, 1998*]

Crack Healing

[*Hickman and Evans, 1987*]

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Permeability

Darcy's Law:

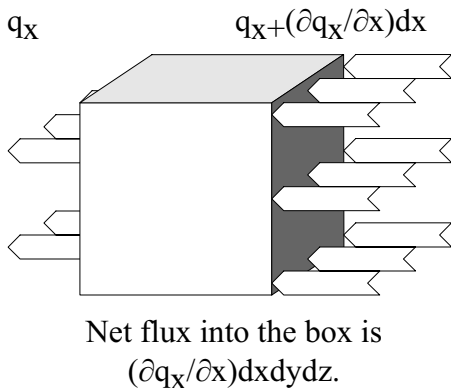
In experiments in sand packed beds Darcy (1856) discovered that the volume of water flowing through a bed was linearly proportional to the pressure gradient driving the flow, dp/dx , to the cross sectional area of the bed, A , where the constant of proportionality is K , i.e.

$$Q_x = -K \frac{dp}{dx} A.$$

If $K = k/\mu$, and $q = Q/A$, then

$$q = -\frac{k}{\mu} \frac{dp}{dx} \quad \left(= -\frac{k}{\mu} \nabla p \right). \quad \text{Darcy's Law (Eq. 1)}$$

This equation is similar in form to those for diffusive flow, heat flow, and flow of electric current (see the table below). The response of the material is linearly proportional to the gradient of the driving potential. In the case of pressure, the gradient dp/dx is the net force on a small parcel of fluid with unit area and length dx . All these equations may be loosely cast as constitutive equations, that is, equations which govern the response of the material to a driving force.



In addition to a constitutive relation, the amount of fluid and solid must be conserved. The equation for conservation is often called a continuity equation. Consider fluid flow into and out of a small cube of porous material, with sides, dx , dy , dz , the amount of fluid that flows into the box is

$$\left(q_x - \frac{\partial q_x}{\partial x} \frac{dx}{2} \right) dy dz \quad (\text{Rate of flow in}).$$

$$\left(q_x + \frac{\partial q_x}{\partial x} \frac{dx}{2} \right) dy dz \quad (\text{Rate of flow out}).$$

The net total flow into the element in the x direction

$$\frac{\partial q_x}{\partial x} dx dy dz.$$

Or for flow in three dimensions, the net flow into or out of the box is

$$\nabla_{\mathbf{q}} dx dy dz = \left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} \right) dx dy dz.$$

Some fluid may remain in the solid because the porous material has a storage capacity for the fluid. Defining the storage capacity, β_s , as the volume of fluid stored in the material for an incremental increase in pressure, the net rate of gain of fluid in the porous solid is

$$\beta_s \frac{\partial p}{\partial t} dx dy dz.$$

The continuity equation equates the storage of fluid with the net flow in or out, i.e. forces the fluid to be conserved:

$$\frac{\partial q_x}{\partial x} - \beta_s \frac{\partial p}{\partial t} = 0 \quad \text{or in three dimensions, } \nabla_{\mathbf{q}} - \beta_s \frac{\partial p}{\partial t} = 0$$

Finally combining the continuity equation with Darcy's law, gives an equation for fluid flow, which is analogous to Fourier's equation for heat flow:

$\nabla^2 p - \frac{\beta_c \mu}{k} \frac{dp}{dt} = 0$	Fourier's Law (Eq. 2)
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Dynamic Viscosity	$\tau = \eta \frac{du}{dy}$	Pa s	10 poise = 1 Pa s
Thermal conductivity	$q = -k \nabla T$	W/mK	
Permeability	$q = \frac{k}{\eta} \nabla P$	m ² (10 ¹² darcy)	q, volumetric flow rate, m/s k, permeability, m ² η, viscosity, Pa s del P, or dp/dx, pressure gradient
Resistivity	$I = r \nabla \psi$	(ohm m) ⁻¹	
Diffusion coefficient	$Q = -D \frac{\partial C}{\partial x}$ $q = -D \nabla C$ $j_i = -\frac{D_i C_i}{RT} \nabla \mu_i$	m ² /s	

General Solutions for Fourier's Equation:

Source solution	$u = t^{-1/2} \exp\left(-\frac{x^2}{4kt}\right)$	Source of heat at $x=0$, $2\rho c(\pi k)$
Error function	$A \cdot \operatorname{erf}\left[\frac{x}{2(kt)^{1/2}}\right] = A \cdot \int_0^x t^{-1/2} \cdot \exp\left[-\frac{\xi^2}{2kt}\right] \cdot d\xi$	Distributed source
$f(x/4kt)$	$t^m f\left[x / (4kt)^{1/2}\right]$ where $\frac{\partial^2}{\partial z^2} f(z) + 2\frac{\partial}{\partial z} f(z) - 4mf(z) = 0$	
Exponentials	$P \cdot \exp[kA^2 t + Ax]$	
Steady State	$Ax + B$	conduction
Double Power Series	$v = \sum_{k=2}^{\infty} \frac{x^2}{k!} \phi^{(k)} + \sum_{k=4}^{\infty} \frac{x^4}{k!} \ddot{\phi}^{(k)} + \dots + x \sum_{k=3}^{\infty} \frac{x^3}{k!} \psi^{(k)} + \dots$ where ψ, ϕ are functions of time.	
Power series	$v = a_0 + a_1 x + a_2 (x^2 + 2kt) + a_3 (x^3 + 6ktx) + a_4 (x^4 + 12kx^2 t + 12kt^2) + \dots$	

Equivalent Channel Model

M. S. Paterson, The equivalent channel model for permeability and resistivity in fluid-saturated rock - A re-appraisal, *Mech. Mater.*, 2, 345-352, 1983.

For laminar fluid flow in a pipe with no slip along the boundary between the pipe and the fluid, the expression for Poiseuille flow is

$$q' = -\frac{1}{2} R^2 \frac{1}{\mu} \frac{dp}{d\xi}$$

where q' is the flow rate perpendicular to the cross-section of the pipe. R is 1/4 the diameter of the pipe and ξ is the linear coordinate along the pipe.

For channels of different dimension, assume the flow can be expressed by

$$q' = -CR_h^2 \frac{1}{\mu} \frac{dp}{d\xi} =$$

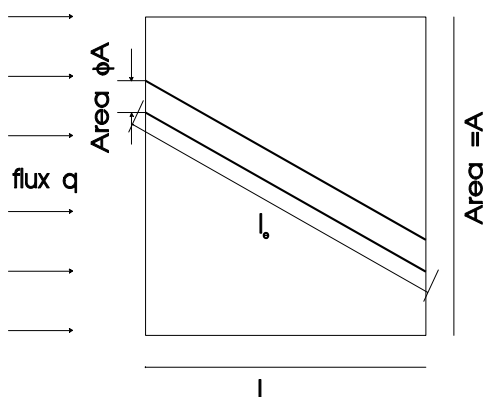
For a channel with a tortuous path the equivalent path length, l_e , the differential in the x direction is $dx=dv$. Then the Poiseuille equation is

$$q' = -CR_h^2 \frac{1}{\mu} \frac{1}{l_e} \frac{dp}{dx}$$

The flow rate, q' , is a vector parallel with the pipe. The component of the flow in the x direction is $(1/l_e)q'$. The total volume of the flow is the area of the pipe, ϕA , times the flow rate, q' [assuming that the porosity is randomly oriented (?)]. Then the bulk flow rate, i.e. the flow rate for the total area A , is the total flow rate, $q'\phi A$ divided by the area A , which is $\phi q'$. And substituting with Poiseuille's equation:

$$q = -C\phi R_h^2 \left(\frac{1}{l_e}\right)^2 \frac{1}{\mu} \frac{dp}{dx}$$

Comparing this equation with Darcy's Law gives the expression for permeability



$$k = C \phi R_h^2 \left(\frac{1}{l_e} \right)^2 \frac{CR_h^2}{F'} \text{ where } F = \frac{\left(\frac{l_e}{l} \right)^2}{\phi}$$

where F' is a factor which characterizes the extent to which the porous body acts as a channel. Paterson calls this factor the relative tortuosity, i.e. the tortuosity per unit porosity.

Resistivity

For electrical conductivity, ohm's law predicts that the current density, j , is proportional to the voltage gradient.

$$j = -k \frac{dV}{dx} = -\frac{k_f}{F} \frac{dV}{dx}, \quad \text{Equation 3}$$

The factor ϕ and ϕ_f are the conductivity of the rock and the electrolyte fluid; F is the formation factor. By exact analogy with the fluid flow case, the total current passing through the rock is

$$J = jA = \left(\kappa_f \frac{\phi A l}{l_e} \frac{1}{l_e} \right) \left(-l \frac{dV}{dx} \right)$$

The first term is the conductance of the element. The second term is the total voltage drop from one end to the other. The equation assumes that all the current passes through the liquid, and that the equivalent channel model accurately represents the porosity. Then the current density is

$$j = -\frac{\phi}{(l/l_e)^2} \kappa_f \frac{dV}{dx}$$

By comparison with (3) the formation factor is seen to be $F = (l_e/l)^2 / \phi$.

And, finally, the resistivity is

$k = CR_h / F$	Equation 4
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