12.520 Lecture Notes 20

Plates (continued)

Response to a line load (e.g. volcanic chain under water)

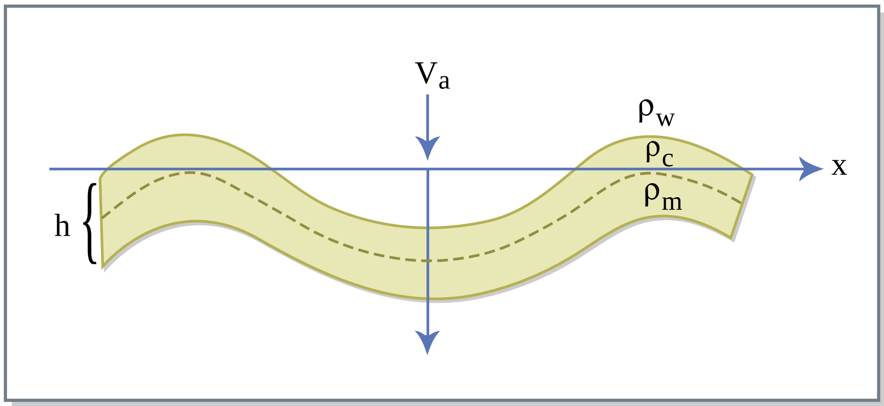

Figure 20.1

Figure by MIT OCW.

$$D\frac{d^4w}{dx^4}+(\rho_m-\rho_w)gw=\begin{cases}0 & \text{at } x\neq 0\\ V_0 & \text{at } x=0\end{cases}$$

Solution to homogeneous equation

$$w=e^{x/\alpha}(C_1\cos\frac{x}{\alpha}+C_2\sin\frac{x}{\alpha})+e^{-x/\alpha}(C_3\cos\frac{x}{\alpha}+C_4\sin\frac{x}{\alpha})$$

with $\alpha=\left[\frac{4D}{(\rho_m-\rho_w)g}\right]^{1/4}$ α is flexural parameter.

Invoke symmetry, boundedness. Determine solution only for $x\geq 0$.

$$\frac{dw}{dx}=0 \quad \text{at } x=0 \qquad C_3=C_4$$

$$w\to 0 \quad x\to\infty \qquad C_1=C_2=0$$

$$w = C_3 e^{-x/\alpha}(\cos\frac{x}{\alpha} + \sin\frac{x}{\alpha})$$

Now to evaluate C_3, go back to the end load problem (or original definition)

$$\frac{dM}{dx} = V + P\frac{dw}{dx}$$

C_3 depends on V_0.

$$\frac{d^3w}{dx^3} = -\frac{1}{2}\frac{V}{D}$$

$$\frac{1}{2}V_0 = D\left.\frac{d^3w}{dx^3}\right|_{x=0} = \frac{4DC_3}{\alpha^3}$$

V_0 is negative load, half supported by each side.

$$w = \frac{V_0\alpha^3}{8D}e^{-x/\alpha}(\cos\frac{x}{\alpha} + \sin\frac{x}{\alpha})$$

$$w_0 = \frac{V_0\alpha^3}{8D}$$

$$w = w_0 e^{-x/\alpha}(\cos\frac{x}{\alpha} + \sin\frac{x}{\alpha})$$

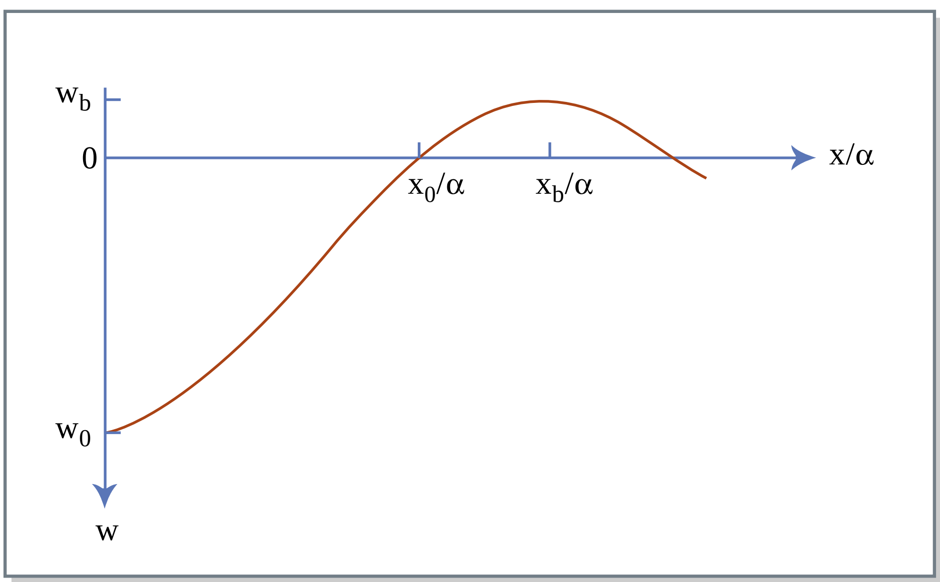

Figure 20.2

Figure by MIT OCW.

$$\frac{x_0}{\alpha} = \tan^{-1}(-1) = \frac{3\pi}{4}$$

$$\frac{x_b}{\alpha} = \sin^{-1}(0) = \pi$$

$$w_b = -w_0 e^{-\pi} = -0.0432 w_0$$

$$x \; ; \; 3^{\mathrm{o}} \; ; \; 300\mathrm{km}$$

$$\alpha = \frac{314}{\pi}\mathrm{km} \; ; \; 100\mathrm{km}$$

Suppose plate is fractured.

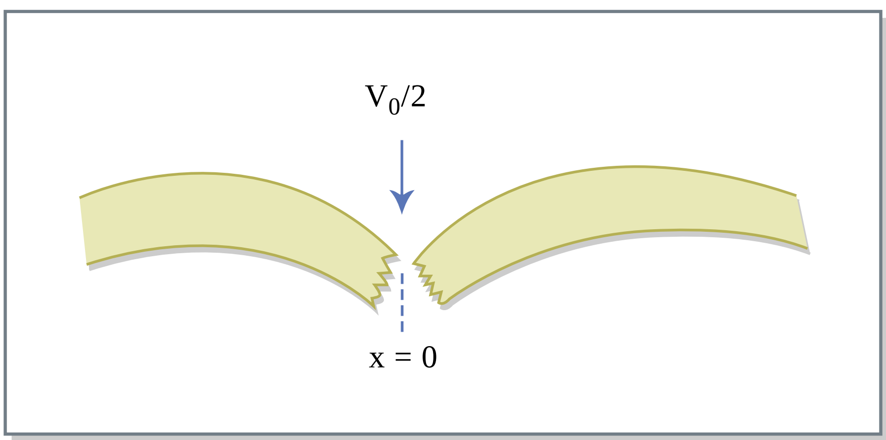

Figure 20.3
Figure by MIT OCW.

Same equation. Different boundary conditions $\Rightarrow$ different profile.

$$M = 0 \ \text{ at } x = 0 \qquad \frac{d^2 w}{dx^2} = 0$$

$$w = \frac{V_0 \alpha^3}{4D} e^{-x/\alpha} \cos\frac{x}{\alpha}$$

$$w_0 = \frac{V_0 \alpha^3}{4D}$$

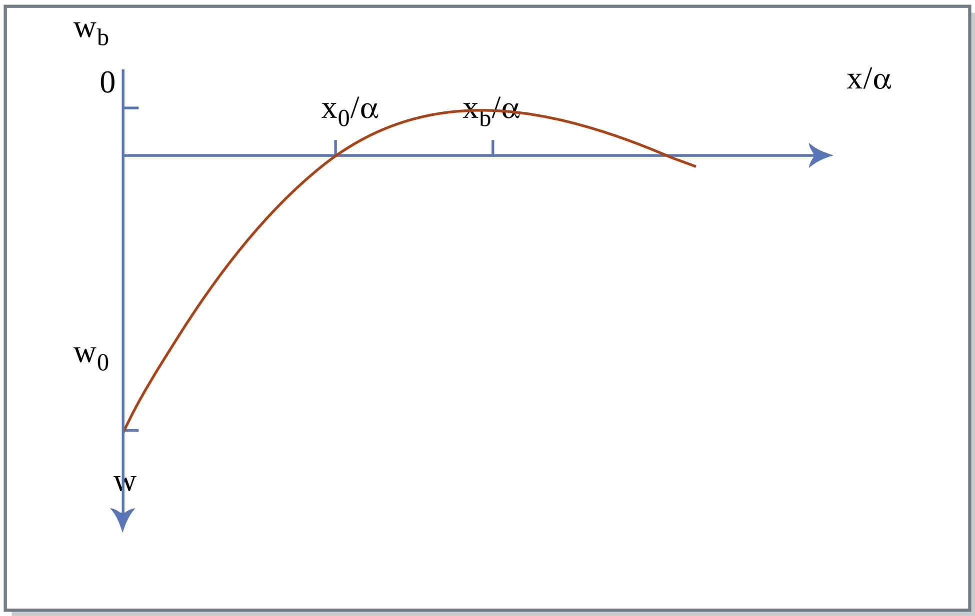

Figure by MIT OCW.
Figure 20.4

$$x_0 = \frac{\pi}{2}\alpha, \qquad x_b = \frac{3\pi}{4}\alpha, \qquad w_b = -0.067 w_0$$

h ; 35km unbroken

h ; 50km broken

Thickness of broken lithosphere is about 1.5 times thickness of the unbroken lithosphere.

Bending at a trench.

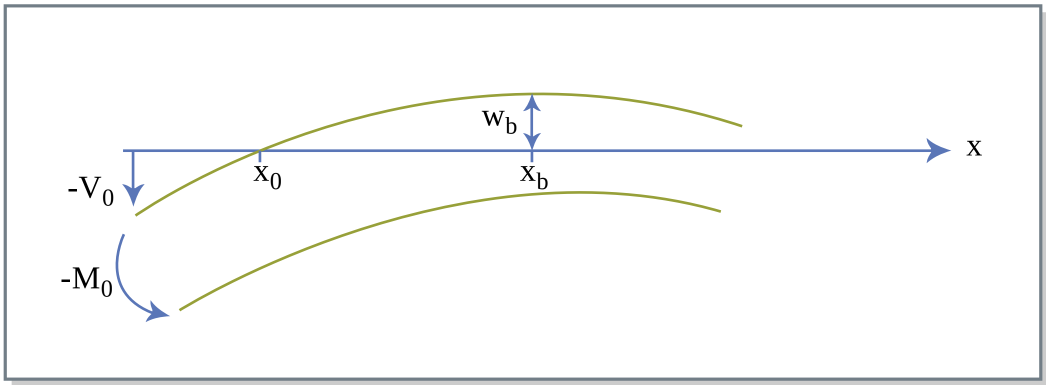

Figure 20.5
Figure by MIT OCW.

$$w = e^{-x/\alpha}\left(C_3 \cos\frac{x}{\alpha} + C_4 \sin\frac{x}{\alpha}\right)$$

$$C_4 = -\frac{M_0 \alpha^2}{2D}$$

$$C_3 = (V_0 \alpha + M_0)\frac{\alpha^2}{2D} \qquad \text{(entire } V_0 \text{ supported)}$$

$$w = \frac{\alpha^2 e^{-x/\alpha}}{2D}\left\{-M_0 \sin\frac{x}{\alpha} + (V_0 \alpha + M_0)\cos\frac{x}{\alpha}\right\}$$

Observables -- x_0, x_b, w_b

$$\tan\frac{x_0}{\alpha} = 1 + \alpha V_0 / M_0$$

$$\tan\frac{x_b}{\alpha} = -1 - 2M_0 / \alpha V_0$$

$$x_b - x_0 = \frac{\pi}{4}\alpha$$

$$w_b = \frac{\alpha^2 e^{-x_b/\alpha}}{2D}\left\{-M_0 \sin\frac{x_b}{\alpha} + (V_0\alpha + M_0)\cos\frac{x_b}{\alpha}\right\}$$

$$w_b = -\frac{\alpha^2 M_0}{2D} e^{-[(x_b - x_0)/\alpha]} e^{-x_0/\alpha} \frac{\sin\left((x_b - x_0)/\alpha\right)}{\cos\left(x_0/\alpha\right)}$$

Plates Summary:

Successes – “moats” and “rises” ubiquitous. Elastic flexure seems to apply.

Problems:

- Geophysical measurements of *w* are difficult.
- Reference level?
- Other sources of topography.

Alternative approach – use gravity

Consider same observed topography

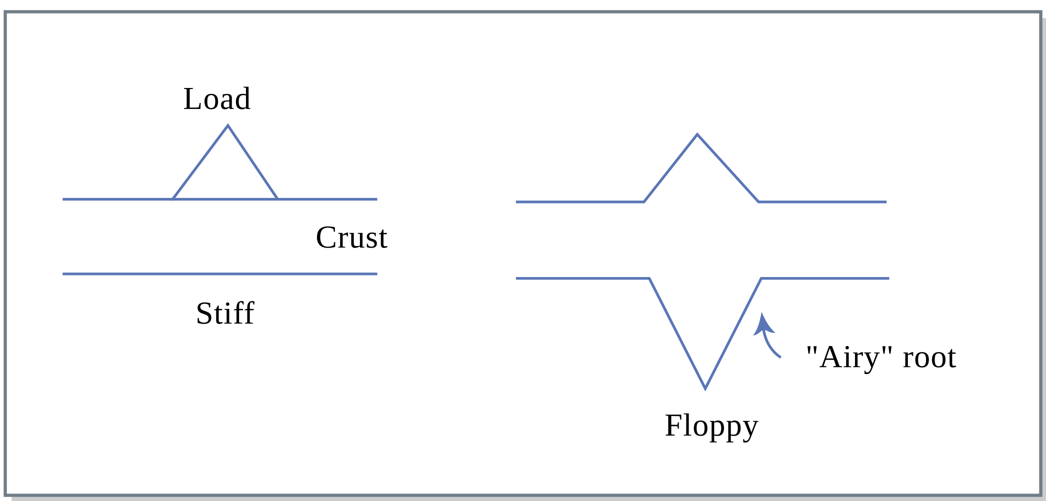

Figure 20.6

Figure by MIT OCW.

Fourier Series $t(x) \to t(k)$

Apply load $t_c = t_0 \cos kx$

Get displacement $D \to w = w_0 \cos kx$

$$w_0 = \frac{t_0}{\frac{\rho_m}{\rho_c} - 1 + \frac{D}{\rho_c g} k^4}$$

$$\delta g = g_0 \cos kx$$

$$g_0 : \rho_c (t_c - e^{-kH} w_0)$$

where H is crustal thickness and w_0 depends on k and D.

$$\delta g(k) \to \delta g(x)$$

Compare $\delta g(x)$ with data.