Massachusetts Institute of Technology
Department of Earth, Atmospheric, and Planetary Sciences

12.409 Observing Stars and Planets, Spring 2002

Useful Optics Information

What you know so far (or at least may know so far) by experience with the LX200:

- Views through the telescope and finder scope are inverted
- Diagonal mirror view is right-side-up but left/right reversed
- You see fainter objects in the finder than you can with the unaided eye, and objects which are fainter still with the telescope itself
- Of the eyepieces in your kit,
    - 6.4 mm has the smallest field of view, highest magnification, and dimmest images
    - 40 mm has the widest field of view, least magnification, and brightest images
- Rings of color (particularly red and blue) around bright objects, especially bright stars and planets near the horizon
- A very defocused image of a point source has a "doughnut" shape

The SECRETS behind these phenomena will be revealed herein!

Contents

1 How Do Telescopes Work?

1.1 Light-Gathering and Image Formation

Optical telescopes make use of 2 phenomena:

- **Reflection** of light, by mirrors (Figure 1), and
- **Refraction** of light, by lenses (Figure 2)
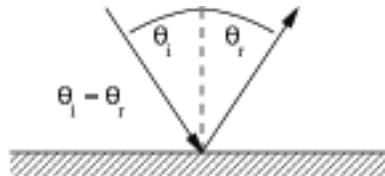
Figure 1: Light ray reflected from flat surface

Refraction is the bending of light ray as it passes from one medium to another.
Application of Snell's Law:

$$n_1 \sin\theta_i = n_2 \sin\theta_r \qquad (1)$$

where the value of n, the refractive index, is characteristic of the material the ray is
passing through:

$n = 1.0000$ for a perfect vacuum
$n = 1.0002$ for air
$n \approx 1.5$ for glass

n is really $\frac{speed\ of\ light\ in\ vacuum}{speed\ of\ light\ in\ medium}$. Figure 2 illustrates the case where $n_2 > n_1$.
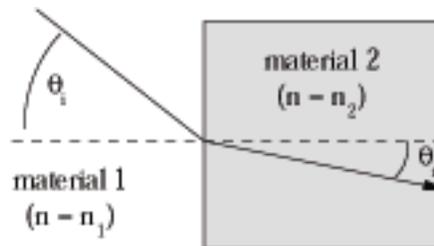
Figure 2: Light ray refracted at a boundary between 2 media

That's how you make a lens, as shown in Figure 3. The distance labeled *f* is the **focal
length** of the lens. The image of an object at "infinity" will be formed at the distance *f*
behind the lens. As we'll see later in Section 2, telescopes come in a variety of optical
arrangements. Many designs contain both refractive and reflective optics, but for the
sake of simplifying the following presentation, only lenses are used for the sample
telescopes. In fact, for our purposes reflection and refraction are equivalent, in the sense
that one can in principle construct a system using only lenses which is optically
indistinguishable from a system using mirrors. By using our lens as an **objective** to
collect light from some far-away object to create an image, we find we have constructed
the basic **astronomical refractor telescope**.
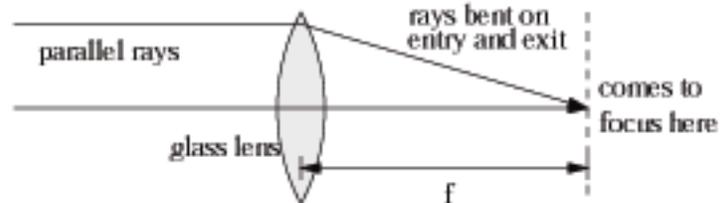
Figure 3: Refraction by lens

1.2 Image Size Depends On Focal Length

Note that our refractor as described so far has no eyepiece lens and thus will not allow someone to directly view the image it has created, since the human visual system isn't designed to use already-focused light rays. Even so, our simple instrument *is* in fact a telescope, and to see how and where the image is formed you could hold a white card or piece of photographic film at the focus, as in Figure 4 in which two stars are separated in the sky by angle θ and are being observed.
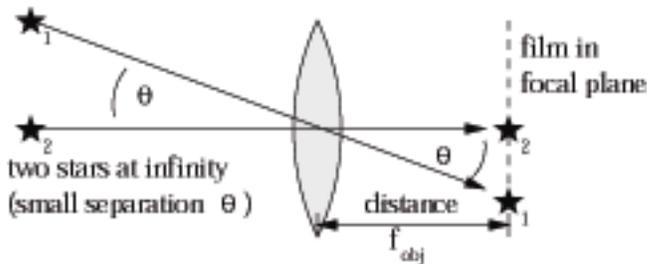
Figure 4: Film at focus

By similar triangles θ is unchanged, so the separation of the stars in the image is proportional to their angular separation in the sky.
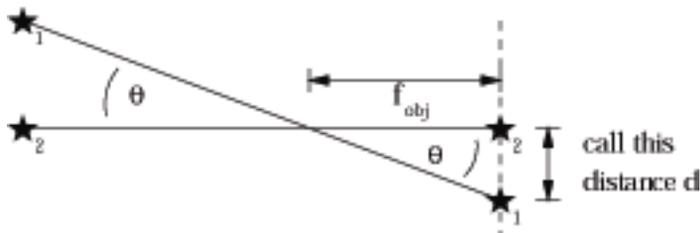
Figure 5: Angular separation transformed to linear distance

Also, from Figure 5 see that:

$$\tan\theta = {d}/{f_{Obj}} \qquad (2)$$

where d is the linear distance between the stars in the image, and f_{Obj} is the focal length of the lens. Now, (physicists love to pull sneaky tricks like this one) since θ is so small, what with the stars being at infinity and all, $\tan\theta \approx \theta$. This gets us

$$\theta = \frac{d}{f_{Obj}} \Rightarrow \frac{1}{f_{Obj}} = \frac{\theta}{d} \tag{3}$$

Thus, $1/f_{Obj}$ is a constant (with units of $\frac{radians}{length}$) directly relating angular separation in the sky to linear distance on the image!

Let's fix up the units to be something convenient: 1 radian ≈ 206265 arcseconds
So the image scale of the objective lens (also called the **plate scale**) is

$$plate\ scale \approx \frac{206265}{f_{Obj}(in\ mm)}\ arc\sec\ per\ mm \tag{4}$$

EXAMPLE: What is the linear diameter of the Moon on an image taken at the straight cassegrain focus of an LX200?

- First, we need the image scale. For the LX200, f_{Obj} = 2000 mm, so the image scale at "cassegrain focus", by Equation (4), is

    206265 arcsec/2000 mm = 103 arcsec/mm

- To use the image scale to determine image size, we now need to know the angular size of the target object. The disk of the Moon subtends a diameter of about $1/2^o$, which is 1800".

$$\frac{1800\ arcsec}{103\ \frac{arc\sec}{mm}} \approx 17\ mm$$

- Thus, if we try to image the Moon onto a CCD detector (the SBIG ST-7E you'll be using in class are approximately 7 mm × 5 mm), the whole thing won't fit. In this case we would need to either use different optics, or resign ourselves to splicing together an array of images.

1.3 Image Brightness Depends On Focal Ratio

The **brightness** of the image you get depends upon *two* things (the symbol ∝ is used here to indicate proportionality):

1. How much light you collect from the object in the first place, which depends *only* on the area of your objective lens, or mirror (like raindrops into a bucket).

The following is how "how much does a telescope help us increase apparent brightness" was computed in the "Can we observe X tonight" handout.

$$A_{Obj} = \pi r_{Obj}^2 = \pi \left(\frac{d_{Obj}}{2}\right)^2 = \frac{\pi}{4} d_{Obj}^2 \Rightarrow \mathrm{Im}age\ Brightness \propto d^2$$

So our 8-inch-objective telescope collects 64 times as much light as does a 1-inch-objective finder scope.

2. How big an image size you're spreading the light over.

If you keep the amount of light constant, IMAGE BRIGHTNESS $\propto$ 1/AREA, and since AREA $\propto$ f^2 as shown in Figure 6, the IMAGE BRIGHTNESS $\propto$ 1/f^2
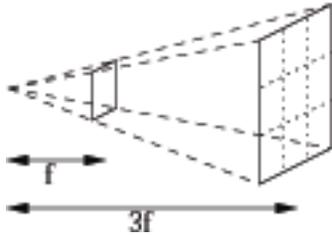
Figure 6: Exposed area is proportional to distance squared

Putting points 1 and 2 together we get:

$$\text{IMAGE BRIGHTNESS} \propto d^2 \times 1/f^2 = d^2/f^2 = (d/f)^2$$

The square root of the reciprocal of this quantity is called the **focal ratio** or **f-number**, familiar to those of you who've used SLR cameras.

$$f - number = focal\ ratio = \frac{f}{d} = \frac{focal\ length}{objective\ diameter} \qquad (5)$$

For our telescopes, the f-number is fixed at 2000mm/200mm = "f/10"

On a camera lens, the f-numbver is adjustable *by virtue of an iris diaphragm which is used to effectively reduce d; f stayes fixed unless you're using a zoom lens.*

LOW F-NUMBERS: brighter image, wide field (many arcsec per mm) so individual objects appear *smaller*. Better for galaxies, faint nebulae, and the Milky Way, or for allowing shorter exposure times for bright objects (Moon, planets).

HIGH F-NUMBERS: dimmer image, narrow field, so individual objects appear *larger*. Better for limiting the accumulation of sky brightness fogging during long-exposures, or for larger images of bright objects.

"narrow field" here does not *mean that as you close up the aperture on your camera lens you narrow your field; rather, it means that for two optical systems with the same diamter objective but with different f-numbers, the system with the higher f-number has a smaller field.*

Finally, even with the advantage afforded us by the "bigger eye" of the telescope, there are some rather anti-social effects which will cause us difficulties with our attempted viewing of faint objects:

- Extinction—light is scattered and absorbed as it passes through the atmosphere. This effect is minimum when the object you're observing is directly overhead, and is maximum when it's down near the horizon.

- Contrast (or lack thereof)—In Cambridge, there's *lots* of ambient light scattered in a sky with lots of dust and haze for it to reflect back at us from (though less, there's some in Westford as well, particularly if you look low in the east toward the nearby city of Lowell or to the southeast toward Boston).

1.4 Resolving Power (and limit thereof)

So far, our treatment of telescope operation has used refraction and reflection only. This is a subset of what's called **geometric optics**, and describes a sort of an "ideal world" case. Geometrically, point-source stars in the sky would appear in the image as perfect Euclidean points as in the left side of Figure 7, but here in the real world we get blobs instead as shown on the right. Why isn't the real world geometric?
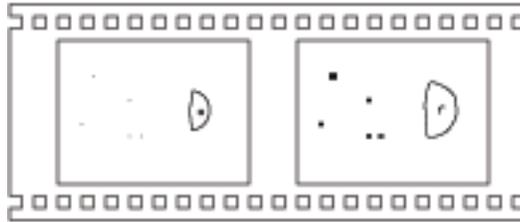
Figure 7: Ideal vs. Real-World resolution

- First, because of the physics of light being partly wave-like, a point source imaged through a circular aperture such as a telescope will *not* produce a point image but will in fact yield a small circular blot called a **diffraction disk** (also known as an Airy disk) on the image instead.

  The physics of this are included in subject 8.03

- The disk diameter is well-defined and is inversely proportional to the diameter of the original aperture, so by successively increasing the aperture size from that of the unaided eye, to finder scope, to binoculars, to telescope, we can decrease the blot size by virtue of successively larger objectives, and thus increase the resolution available in the image…

- … until we hit the limit of observing conditions, that is. Somewhere around the small-binocular-sized aperture range, another player kicks in to hold image blurs back to the 1"-5" range, in the guise of atmospheric **seeing**—you're looking up through an atmosphere complete with turbulence and density variations due primarily to temperature variations (e.g. when viewing from campus, the Bldg. 42 steam plant smokestack). Seeing at the best ground-based sites (such as Las Campanas, Chile) sometimes gets down in the range 0.4" to 0.8" . Good seeing at Wallace is typically 3-5". (Even with the seeing limit, larger apertures still win on the image brightness front; at least you can get *brighter* seeing blotches…)

Breaking the seeing-limited barrier was one of the major motivations behind orbiting the Hubble Space Telescope, which could in fact be diffaction-limited to some truly amazing resolution if the primary mirror had been figured properly…

- ▪ Finally, the resolution of the image you actually see or record with some detector can be further limited by the structure of the detector used, if it's coarser than that intrinsically available in the image. For example, the coarseness of the structure of light detectors on the retina limits resolution perceived by the eye to about 1'.

There's a subtle distinction here between increasing the **resolution** available in the image itself by using a larger objective (at least up to the limit imposed by the seeing), and increasing the **magnification** of the image as you've done when observing visually by changing to a shorter focal-length eyepiece, which is dealt with in the following section. In short: *magnifying a non-resolved blotch will yield only a bigger non-resolved blotch.*

1.5 Magnification and Field of View

Our demonstration telescope objective so far has been fine by itself for imaging, but for visual observing we need to add an **eyepiece** which will let us see the image. An eyepiece is just another lens, but instead of using it to collect light (as we do for an objective lens) we'll use it as if it were a magnifying lens like those you've played with before—what you were doing was putting an *object* at the focal point and observing it from the other side (Figure 8) .
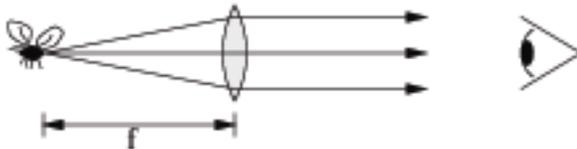
Figure 8: Magnifying lens

To *you*, it looks like the image of the fly is at infinity—this is the most comfortable viewing arrangement for your eye (i.e. your eye focuses as if the fly were at infinity). So, if we put these two lenses together, the eyepiece and the objective, we finally have a refractor telescope for visual observing (Figure 9).
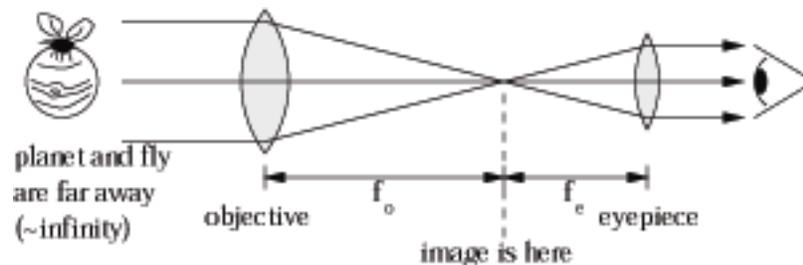
Figure 9: Refractor telescope

The finderscope on the LX200 is this type of telescope.

Notice that the image you see is *inverted* (Figure 10).
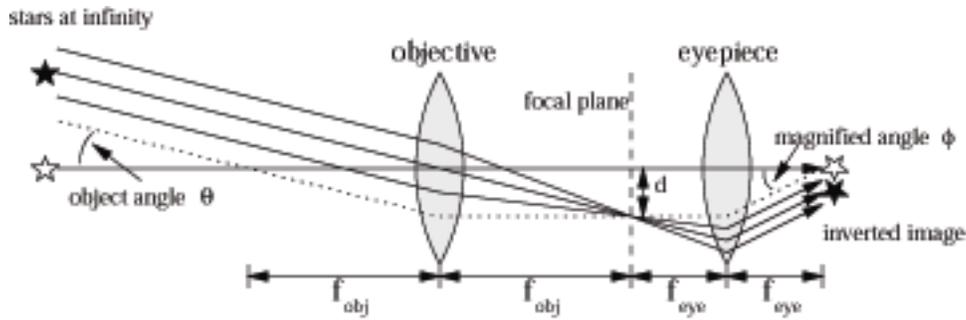
Figure 10: Refractor with eyepiece

$$\theta \approx \tan\theta = \frac{d}{f_{Obj}} \; ; \; \phi \approx \tan\phi = \frac{d}{f_e}$$

$$\frac{\phi}{\theta} = \frac{apparent\ angle}{real\ angle} = \frac{f_{Obj}}{f_e} \to \text{this is the magnification}$$

$$M = \frac{f_{Obj}}{f_e} \qquad\qquad (6)$$

You see that the magnification of a telescope is variable, in the sense that exchanging among eyepieces of different focal length changes the magnification you get. Using equation (6), you can calculate the magnifications that are obtained with the eyepieces in your observing kit and the 8" LX200 (you can check your math with handout 3). As power increases, the image brightness decreases rapidly as does the perceived image sharpness.

Field of view (FOV):
It varies with the focal length of the eyepiece and also with the particular optical design of the eyepiece (i.e. a 25 mm "orthoscopic" probably has a different field of view than a 25 mm "Kellner").

The only definitive way to determine your field of view with a given eyepiece is to put a star near the celestial equator at the east edge of your field, turn off your clock drive, and time it drifting across the field. If it takes t seconds of time for a star at declination δ to cross the width of the field, the field diameter d in arcseconds will be:

$$d(arc\sec) = t(\sec) \cdot \cos\delta \cdot 15(^{arc\,sec}\!/_{sec})$$

An approximate method, taking the FOV of the eye to be $40°$: Using an eyepiece, your eye still sees a FOV of $40°$ but now you know that the telescope magnifies angles by a factor of M, so

$$\text{FOV} \approx 40°/M$$

A few angular sizes, to help give you a feel of numbers vs. real-life:

Object	Angular size
Sun, Moon	$0.5° = 30'$
M57 (Ring Nebula)	$1' = 60''$
Saturn ring extent	$40''$
Mars, at opposition 1990	$18''$
Mercury, Nov 1987	$6.5''$

2 Telescope Designs

2.1 Refractors
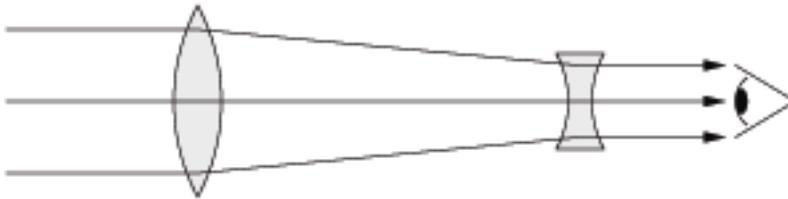
Figure 11: Galileo's refractors
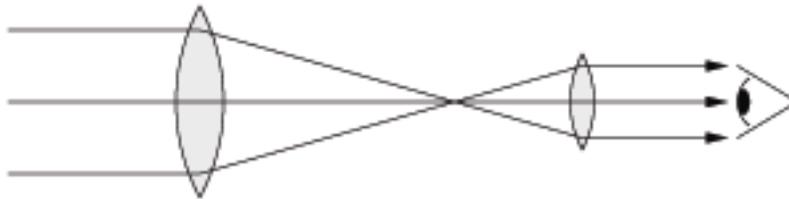
Figure 12: Kepler's refractors

…and here we have our friend the refractor from Page 7.

Advantages:
- They're tough—sealed tubes; little or no maintenance
- No obstruction (as found in Newtonian or Cassegrain reflectors in the next section) in light path to foul up the contrast
- Unsurpassed image quality for Moon, planets, double stars

Disadvantages:
- very expensive per inch of aperture
- (lenses have to be perfect; larger → more difficult to make → more expensive)
- big and heavy
- require tall mountings—shaky, expensive (again)
- **Chromatic aberration** (Figure 13): Refracted light splits up into a spectrum, and because different wavelengths focus at different distances, objects suddenly appear to grow red and blue rings around themselves that really don't belong there at all.

Even though our LX200s are generally referred to as reflectors (as described in an upcoming section) the "corrector plate" is (as you may have suspected) in fact a lens, as are the eyepieces. These elements could in principle cause chromatic aberration, but the corrector doesn't bend light enough to cause any appreciable amount, and the combination of lenses built into a particular eyepiece design are specifically chosen to mostly cancel out each other's aberration and thus yield "good color correction".

The fringe colors you may see around bright objects when observed at very low altitudes are due to differential refraction by the atmosphere itself: same physics, different culprit.
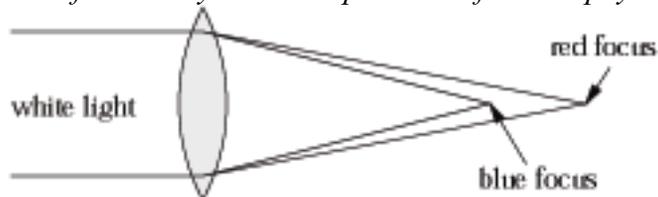
Figure 13: Chromatic aberration

Big refractors are a *pain.*

2.2 Reflectors

All large modern instruments are **reflectors** of some kind. A **parabolic** mirror will focus parallel rays to a single point, as shown in Figure 14.
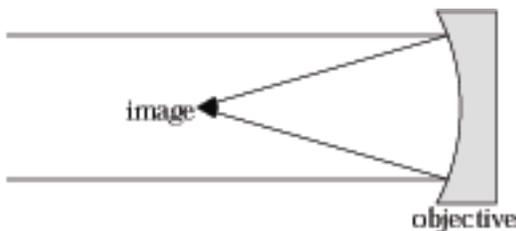
Figure 14: Focus of parabolic mirror

Advantages:
- *No* chromatic aberration!
- Most light-collection capability for the money (you don't need perfect glass)
- Fewer optical surfaces
- Open tube less likely to dew up on you

Disadvantages:
- The focus is *in* the incoming light path. This means that whatever you use to either collect the light or redirect it and then collect will form a **central obstruction**.
- Open tube permits image-fuzzing air currents to wander around inside the tube as it cools to outside temperature
- Open-tube telescopes also require more careful treatment and maintenance, what with trying to keep dust (and worse) out and also keep the mirrors aligned.
- Also… parabolic mirrors work really well for images which are right on the central optical axis, but off-axis stuff is increasingly distorted into football-shaped blobs (i.e. only the very center of the field is undistorted) This effect ("**coma**") is particularly bad with short f-ratios (A **spherical** mirror would fix this problem, but won't focus all incoming light to a focal point like a paraboloid does).
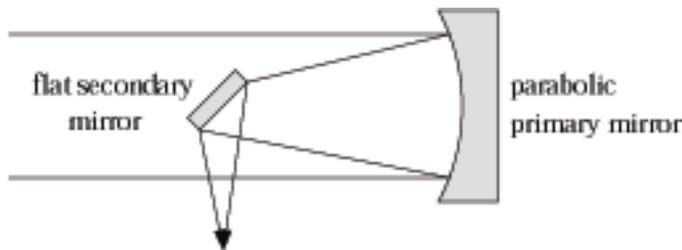
Figure 15: Newtonian reflector

Newtonian reflectors (Figure 15) are ideally suited to short, stable mounts, ,and are of simple construction (many amateurs make their own!)
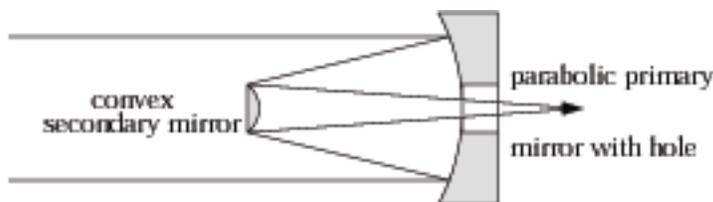
Figure 16: Cassegrain reflector

A **Cassegrain** reflector (Figure 16) involves a rearrangement of the light path more convenient for larger telescopes. The telescopes one finds at observatories typically are Cassegrains (including the 16-inch and 24-inch 'scopes at Wallace).

So why doesn't the image have a whole in it due to the central obstruction?

First, lets think about a slightly different question. What happens to the image of the box in Figure 17 as the aperture A gets smaller? Will the size of the image shrink or will a part of it be chopped off? No, because *each tiny portion of a lens forms a complete image* and therefore the final observed image is simply the sum of all of the independent contributions!
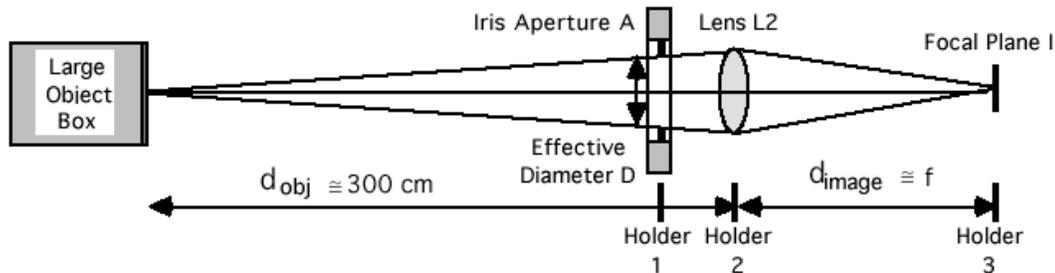
Figure 17[*]: Aperture experiment

Applying this to our refractor means that each part of the mirror also creates a complete image. If you were to block another part of the mirror (as shown in Figure 18), we can see that and image is still created but with somewhat less intensity. How much less intensity (hint: see handout 4)?
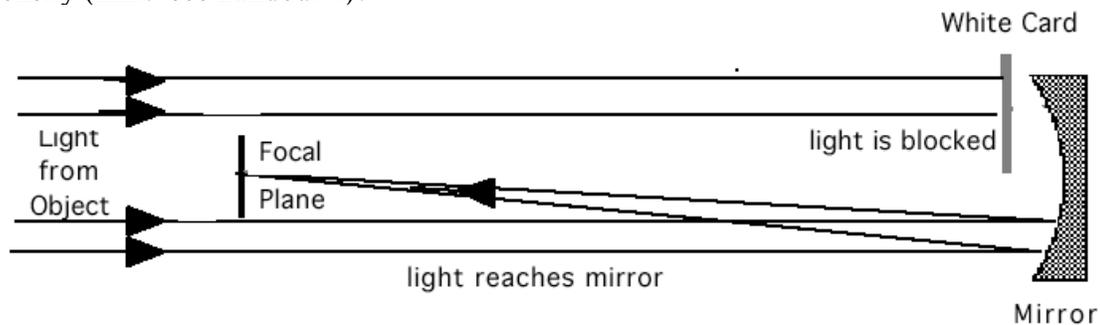
Figure 18[*]: Why isn't there a whole in the image?

2.3 Catadioptric (a combination of refracting and reflecting)

Recall that parabolic mirrors focus incoming light to a focal point but distort off-axis images into footballs, while spherical mirrors don't have this coma problem but neither do they have one focal point. One answer—use a spherical mirror with a refractive **corrector plate** of a certain (complex) curvature to correct the focus, thus creating a **Schmidt-Cassegrain** (Figure 17). It's a very popular design for smaller telescopes; *this is exactly the design of the LX200s we use:* "8-inch (20 cm) f/10 SCT"
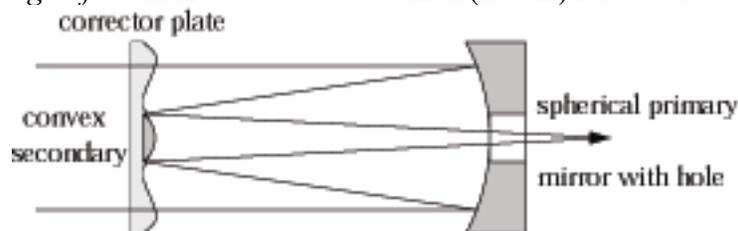
Figure 19: Schmidt-Cassegrain

Advantages:
- Wider undistorted field of view than a Newtonian
- More easily portable
- Mass-producible (spherical mirrors easy; corrector plate made by virtue of cleverness on the part of some telescope genius a few years back)
- Tube is sealed
- Compact tube allows motor drive to track sky motion more reliably

Disadvantages:
- More optical surfaces than a Newtonian
- Larger central obstruction; some decrease in contrast of images results due to diffraction around the central obstruction.
- Need a diagonal mirror to observe objects at high altitude; this left/right swaps the image, making life more confusing…

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[*] K. Gleason, *Accelerated Introduction to Astronomy*, University of Colorado at Boulder, 2001